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Theory of stepped pulses in negative corona discharges

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Theoretical calculations are presented which account for the development of the step observed on the leading edge of the current pulse in a negative point-plane corona (Trichel pulse). The theory explains the step in terms of independent photon and ion secondary processes at the cathode. It is applied specifically to describe the shape of an experimentally determined corona current pulse from a 2-mm diam negative point located 20 mm from a positive plane in oxygen at 6.67 kPa. The secondary-emission coefficients are determined to be $\gamma_p = 2\times 10^{-4}$ (photon secondaries) and $\gamma_i = 0.12$ (ion secondaries).

I. INTRODUCTION

A negative point-plane electrode system operating in electronegative gases gives a corona current which is pulsed over a wide range of applied voltages. These current pulses are called Trichel pulses.[1] For electrodes with a radius of curvature $r_c << 1$ mm, the current pulses in oxygen are relatively simple and regular in form.[2,3] However, when the radius of curvature is larger (say, $r_c > 1$ mm), then the current pulse is found to have a more complex form with a step on the leading edge.[4-6] Moreover, it has been found that even for sharp points, $r_c \sim 0.14$ mm, the "first" pulse in a pulse train exhibits a step on the leading edge.[6] Clearly this step must be an integral part of the pulse formation process, but no explanation has yet been given for the existence of the step.

In a recent paper Morrow[7] outlined a general theory of pulsed negative corona in electronegative gases, which explains most of the features of the Trichel pulse. Because of numerical constraints, the application of the theory was restricted to the first pulse for a system with a large diameter point (10 mm) in oxygen at 6.67 kPa. (The first pulse in a train of Trichel pulses differs from subsequent pulses because it occurs in a region initially free of space charge.) When these conditions were reproduced experimentally it was found that it was possible to measure the first pulse; however, the front of the pulse exhibited a step which was not reproduced by the theory.

One important feature of Morrow's[7] theory is that it includes two distinct processes which cause the emission of secondary electrons at the cathode: one due to photon impact and the other to positive-ion impact (Ref. 7, Fig. 5). The photons and ions are produced in the same region at the same time; however, the ions take longer to reach the cathode and hence to release secondary electrons (Ref. 7, Figs. 5 and 7). Although ion-secondary electrons did not play a significant role in the original calculations, their presence does suggest a possible mechanism for the stepped Trichel pulse.

The aim of the experimental investigation[6] was to measure current waveforms which could be used to verify and extend the theory. Consequently an extensive investigation was made of the form of both the first and normal pulses produced under different conditions.[6] Measurements were made of the waveforms of first and normal pulses produced by a corona discharge in oxygen as a function of gas pressure, point radius, and voltage. Although a stepped Trichel pulse could be observed under the conditions assumed for the calculations reported in Ref. 7, the range of voltage between onset and breakdown was very small. It was thus more convenient to use results from smaller diameter points for comparison with theory.

II. THEORY AND COMPUTATIONS

The theory is based on the time-dependent continuity equations describing drift, diffusion, ionization, attachment, and recombination in one dimension,

$$\frac{\partial N_e}{\partial t} + \frac{\partial N_e W_e}{\partial x} = S_e \ , \tag{1}$$

$$\frac{\partial N_p}{\partial t} + \frac{\partial N_p W_p}{\partial x} = S_p \ , \tag{2}$$

$$\frac{\partial N_n}{\partial t} + \frac{\partial N_n W_n}{\partial x} = S_n \ , \tag{3}$$

where t is time, x is axial position, N_e, N_p, and N_n are electron, positive-ion, and negative-ion densities, respectively, and W_e, W_p, and W_n are the drift velocities for electrons, positive ions, and negative ions, respectively. S_e, S_p, and S_n are source terms including ionization, attachment, and recombination effects as outlined by Morrow.[7] Diffusion is not included, since its inclusion greatly increases the computing time but has a negligible effect on the results.[7] The electric field is determined self-consistently by solving Poisson's equation, taking into account the finite radial size of the charge distribution.[7] The methods used to obtain a numerical solution of these equations to produce a theoretical current waveform have been presented in detail by Morrow.[7]

The boundary conditions used are those previously assumed by Morrow.[7] The boundary condition of greatest significance in this study is the specification of the number of electrons emitted from the cathode, at $x=0$. These are secondary electrons produced by photons N_e^p and ions N_e^i. Thus

$$N_e(0,t) = N_e^p(t) + N_e^i(t) \ , \tag{4}$$

where the doubly indexed variables refer to space and time variables (x and t), and

$$N_e^p(t) = \frac{1}{|W_e(0,t)|}\frac{\gamma_p}{\tau}\int_0^t \exp\left[-\frac{t-t'}{\tau}\right]\int_0^d N_e(x,t')|W_e(x,t')|\alpha^*(x,t')\Omega(x)\exp(-\mu x)\,dx\,dt' \tag{5}$$

and

$$N_e^i(t) = \gamma_i \frac{N_p(0,t)|W_p(0,t)|}{|W_e(0,t)|} \ . \tag{6}$$

Here γ_p is the secondary-electron-emission coefficient due to photon impact, and γ_i the secondary-electron-emission coefficient due to ion impact. The quantity $\Omega(x)$ is the solid angle subtended at the cathode by a disk of charge at x (see Morrow[7]). For the excited state considered, μ is the absorption coefficient, τ the lifetime, and α^* the excitation coefficient. Electron source terms due to photoionization in the gas, detachment from negative ions, and the action of metastable atoms at the cathode have not been included.

The present calculations pertain to the case of a negative 2-mm sphere located 20 mm from a positive plane in oxygen at 6.67 kPa. The channel diameter, which defines the radial extent of the discharge, was chosen to be 1 mm (see Morrow[7]). Calculations for such a small diameter point were made possible by using a finer mesh spacing to resolve the cathode fall region (1.33 μm as opposed to 4 μm previously) and using more grid points (300 as opposed to 181 previously).

Thus the basic method used to compute the current waveform remains unchanged and most of the input parameters are the same as used before by Morrow.[7] The only parameter changes made were the following: (1) the value of γ_p was reduced from 10^{-3} to 2×10^{-4}, which is well within the expected range of this parameter;[8] (2) the value of γ_i was increased from 0.01 to 0.12 and kept constant as the electric field varied;[9] and (3) since positive-ion secondaries play a crucial role in determining the shape of the current pulse, it was found necessary to use a more accurate representation of the ion mobility at 6.67 kPa, namely,[10,11]

$$\mu^+ = 3.192\times10^{-3} - 1.976\times10^{11}|E|/N \ \mathrm{m^2V^{-1}s^{-1}} \ , \tag{7}$$

$$E/N < 10^{-20}\ \mathrm{V\,m^2} \ ,$$

$$= 1.216\times10^{-3}\ \mathrm{m^2V^{-1}s^{-1}}, \ E/N > 10^{-20}\ \mathrm{V\,m^2} \ , \tag{8}$$

where E is the electric field and N the neutral-gas number density.

III. RESULTS AND DISCUSSION

An experimentally measured current waveform is presented in Fig. 1 for the first pulse in a system with a 2-mm diameter point and a 20-mm gap. The applied voltage was 2.8 kV, and the pressure of oxygen was 6.67 kPa. The computed current waveform is also shown in Fig. 1 and the computed pulse is seen to have two peaks rather than a step on the leading edge. The computed curve has been shifted in time so that the first peak coincides with the first maximum for the experimental trace. The amplitude has not been normalized and the agreement with the measured pulse height is excellent. The agreement in the general shape of the pulse is also satisfactory, especially when it is noted that no allowance is made for the finite bandwidth of the experimental measurement system. The time between the first and second maxima also agrees well with the experiment and can be changed by adjusting the secondary coefficients as described below. The rise of the first pulse is faster than found in the experiment, but again this may be due to the finite bandwidth of the measurements.

The details of the calculated electric field distribution and of the electron, positive- and negative-ion-density distributions are all similar to those found by Morrow[7] for a 10-mm diameter point, except that the structure occurs closer to the cathode.

The stepped Trichel pulse can thus be reproduced by the theoretical methods used previously. The mechanism for the production of such a pulse can be understood by reference to the curves in Fig. 1 showing secondary electron production as a function of time. (Note that the scales for each contribution differ by a factor of 20.) In the early stages of the discharge, before space-charge effects become dominant ($t < 10$ ns), the flux of secondary electrons due to photons is sufficient to produce a self-sustained discharge. The replenishment criterion, where each electron provides a successor to itself at the cathode to create a self-sustained discharge,[7] is satisfied and the current grows rapidly. This self-sustained discharge continues until space-charge effects reduce the field near the cathode to such an extent that the replenishment criterion is no longer satisfied (Ref. 7, Fig. 4). The circuit current then reaches a peak at $t = 10$ ns and begins to fall. At $t = 20$ ns the secondary electron flux begins to rise rapidly due to the arrival of the positive ions, and the circuit current passes through a minimum. The number of secondary electrons produced by ions is almost 2 orders of magnitude greater than that produced by photons, and the current rises again due to amplification of these

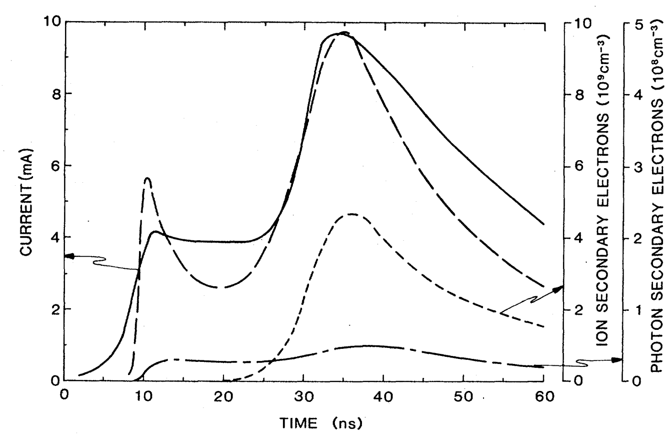

FIG. 1. Comparison of the computed current waveform with the experimentally measured waveform for the first pulse. Also shown separately are the computed number of secondary electrons due to photons and the number of secondary electrons due to ions. Experimentally measured waveform (——); computed waveform (— — —); photon secondary electrons (—-—-); ion secondary electrons (– – –).

TABLE I. General trends in pulse shape with changes in various parameters. Increasing quantities are represented by +; decreasing quantities are represented by −.

	I_1	I_2	I_3	I_2/I_3	t_r	t_{pp}[a]
V	+	+	+		−	−
D	+	+	+	+	−	−
τ	−	−	−	−	+	+
γ_p	+	+	+	+	−	−
γ_i			+	−		

[a]I_1 is the maximum current at the first peak ($t \sim 10$ ns, Fig. 1), I_2 is the minimum current between peaks ($t \sim 20$ ns), I_3 is the maximum current at the second peak ($t \sim 33$ ns), t_r is the rise time of the first peak, t_{pp} is the time between current maxima, D is the diameter of the discharge channel, and V is the applied voltage.

electrons in the high-field region, despite the fact that the high-field region is now very narrow (Ref. 7, Fig. 2). In this case the replenishment criterion is no longer satisfied and the discharge now behaves as though it were externally maintained, with the ions as the external source of secondary electrons. This stage continues until the number of positive ions in the cathode-fall region becomes depleted (Ref. 7, Fig. 7) and the electron field in the region falls (Ref. 7, Fig. 2), leading to a decrease in the flux of ions at the cathode and the gradual decay of the current.

Thus the theory clearly describes and explains most of the features of the stepped Trichel pulse waveform. A more severe test of the theory is to determine if the theory can reproduce the pulse-shape changes that are observed as the macroscopic parameters are varied; the trends found so far are described below. The sensitivity of the theoretical calculations to changes in the microscopic parameters is also of vital interest; some of these trends are also described.

An outstanding problem is to explain why subsequent pulses for a fine point do not have a step on the leading edge. The fact that there is a step on the first pulse even for fine points is, of course, consistent with the present theory. The absence of the step in subsequent pulses may, for a fine point, reflect the fact that a significant number of positive ions are still arriving at the point at the time of the second pulse. Morrow[7] has shown that positive ions are produced far out in the gap and take a long time to reach the point. The more convergent field for a fine point may concentrate these positive ions sufficiently to swamp the γ_p effect.

The experimental study[6] explored the changes in the first-pulse behavior with changes in gas pressure, point radius, corona current, applied voltage, and cathode surface material. Not all these effects have yet been investigated theoretically; however, the calculations do reproduce some of the changes. For example, as the pressure increases the theory predicts that (1) the time between current peaks decreases, (2) the rise time of both current peaks decreases, and (3) the minimum current between the current maxima increases relative to the maximum current. The increase in amplitude of the first peak of the current waveform with a gold-coated cathode may be explained by assuming that gold gives a higher γ_p.

While a detailed presentation of these studies is beyond the scope of this paper, some trends are briefly summarized in Table I. It might be anticipated that a closer study of these trends could be used to refine the theory and, in particular, to calibrate certain material coefficients whose values are presently uncertain.

IV. CONCLUSIONS

The theory gives for the first time an explanation of the stepped Trichel pulse. Moreover, it is clear that close agreement between theory and experiment is possible and work is proceeding toward this end. It is believed that the dominant processes have been included in the theory; however, further consideration needs to be given to the effects of photoionization, detachment, and metastable atoms. At present the applied voltage is assumed to be constant, and it may be of some value to include the external circuit in future calculations. To obtain a more accurate fit between theory and experiment it will be necessary to determine the frequency response of the experimental apparatus and convolve this with the calculated ideal curve to obtain a more realistic reproduction of the measurement curve.

The step on the leading edge of the Trichel pulse observed for large diameter points can be explained in terms of two independent secondary-electron-emission processes, one due to photon impact and the other to positive-ion impact. While the photons and ions are produced in the same region at the same time, the ions take longer to reach the cathode; consequently their contribution to the current is delayed in time, producing a second peak. The existing theory[7] explains this phenomenon with minimal changes in certain parameters to values which are well within the published range.

In addition, the good agreement between theory and experiment may give us the possibility of measuring γ_p and γ_i independently for a range of gases, pressures, and electrode surfaces. For example, the point-conditioning process discussed by Bugge[2] and Cross, Morrow, and Haddad[6] may reflect changes in γ_p and γ_i with time. It would be necessary to determine the radius of the discharge near the cathode, but this could possibly be done photographically.

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