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Schrödinger wave holography in brain cortex

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An explanation for electroencephalogram (EEG) activity is proposed. Under suitable assumptions concerning the active transport of Na^+-K^+ ions through the glial tissue in brain cortex, a Schrödinger-like equation for ion displacement waves is easily obtained. Theoretical wave-propagator diagrams are in perfect agreement with experimental stimulus-response patterns directly recorded on brain cortexes. Conditions for effective Schrödinger wave holography and its greater efficiency in comparison with D'Alembert wave holography, are briefly discussed. The need for "reference-wave" recruiting device and for a receptor arrangement, for holographic information recovery in proximity of the signal sources, lead to brain-organization models in good agreement with diagrams reported by neurophysiologists.

I. HOLOGRAPHIC MODELS FOR ANIMAL MEMORY

Since the second half of the 1960s some authors[1] have introduced the hypothesis that animal memory works according to a holographic type process. To support this idea several arguments can be given.

(a) *Uniform diffusion of engrams throughout the cerebral cortex.* Thirty years of research, performed by Lashley[2] from 1920 to 1950, have convinced many neuroscientists that memory engrams are not localized in any specific places in animal brains, and that memory blurring essentially depends on the total amount of damaged or extirpated celebral cortex but in no way depends either on the specific areas in which such manipulations are performed, or on the kind of recorded information.

(b) *Associative property.* Observation of animal behavior[3] and psychological analysis of human memory[4] lead us to state that whenever any "fragment" of recorded information is again presented to the attention of a subject, this fragment acts as a memory address for the recovery of the complete information and of any other information associated with it during the subject's life.

(c) *Globality and simultaneity of mnemonic evocations.* Any mnemonic recall acts through a simultaneous presentation of a complex set of associated data (e.g., visual image memories) and more generally of time-varying complex sets of data (e.g., moving-scene memories).

(d) *Memory synopsis.* There is no evidence that mnemonic traces are stored in files, in tree-shaped archives, etc., as computer data are stored in tapes, pages are in books, or books are in libraries. Access time to memories appears to be mainly dependent upon certain emotional strengthening factors and there is no evidence of a scanning time related to the time ordering of life experiences.

All these properties of animal memory can be easily simulated by holographic techniques making use of photographic interference patterns, obtained by superposition of coherent monochromatic light waves, and of information recovery by diffraction processes.[5] On the contrary, they are not *so easily* reproducible by digital techniques based on Boolean or neural network activities.

It would therefore be very interesting to relate such animal memory properties to some kind of wave-propagation phenomenon taking place inside animal brains. It must, however, be kept in mind that a necessary condition for effective holographic information processing is the *linear character* of wave propagations; i.e., the validity of the *superposition principle.* If this condition is not fairly well satisfied, then no holographic process will take place because of the reciprocal destruction of phase information by interfering wave trains.

It is a fact that the cerebral cortexes show wave propagations; i.e., the well-known electroencephalogram (EEG) activity which, indeed, possesses many general qualifications required for a true holographic memory process; in particular, the following.

(i) *Cortical waves are linear.* The validity of the superposition principle is irrefutably evident in all those experiments which are based on time-delayed or frequency-displaced stimulations. One can easily observe linearly superposed patterns and bates that could not take place at all if the superposition principle were not accurately satisfied.[6]

(ii) *Cortical waves are not simple direct effects of the neural activity.* As demonstrated by Li, McLennan, and Jasper in 1952,[7] cortical waves persist with substantially invariated properties when the neural activity is strongly depressed by the administration of anesthetics.

(iii) *Cortical waves appear to be in close relation with mnemonic activity.* A proof of this is their arousal only during "dreams" in sleeping mammalians.

The idea that cortical waves might perform a holographic function was introduced by Barret in 1969.[8] Gabor, however, had already declared his scepticism about it and had proposed and credited an associative memory model based on the simulation of holographic processes by digital techniques.[9] As a consequence many researchers applied their efforts in the direction suggested by him.[10]

This scepticism seems at first very reasonable. Cortical waves appear, in fact, to be defective of what experts in optical holography judge to be an essential requirement for decent information processing; i.e., they lack sufficiently small wavelength components. Gabor himself gave an extensive proof[11] that the maximal amount of information I that a monochromatic light beam of wavelength λ can read out from a plate of area A is given in bits by the relation $I = A/\lambda^2$. Therefore, in order to process large amounts of information by means of devices having small A/λ^2 ratios, one should employ a superposition of light beams covering a dense frequency spectrum. One should also use recording techniques based on photographic emulsions containing a wide repertory of sensitive grains which are highly frequency selective, both in exposition sensitivity and in diffusion power after their blowing up. Such sophisticated procedures are in principle possible, although so far they do not seem to be practically manufacturable. On the other hand it is known, from research on phase relations between differently localized cortex wave recordings, that it is not possible to speak about definite wavelength values, and not even of small wavelengths. The correlation lengths of the EEG seem to have the same order of magnitude as the brain anatomic dimensions[12] and they cover a very low-frequency spectrum range ($\sim$0–100 Hz).[13]

However, the hypothesis that inside the brain cortex an enormous quantity of finely tuned local resonators might be formed, in order that efficient holographic processes could take place, should not appear too peregrin, since it is known that a lot of little resonators are effectively present in the animal auditive sensorial system.[14] Rather, what is very difficult to admit is the existence of efficient focalization devices for holographic information recovery inside animal brains; which are indeed necessary, for information recovery in light or sound holography.

After careful consideration of these shortcomings, the only correct conclusion that one is enabled to draw is that cortical waves are unlikely to be of the kind satisfying a D'Alembert equation (i.e., similar to sound or light waves). Rather, as we shall see, they look very much like the waves satisfying a Schrödinger equation. This paves the way for the search for a Schrödinger wave holography model. The exclusion of D'Alembert waves in favor of Schrödinger waves can also be inferred by general arguments concerning the nature of biological processes. Indeed, it would be very difficult to explain on an electrochemical basis how D'Alembert waves could be generated in animal cell tissues. The characteristic feature of a D'Alembert equation is the occurrence of a second time-derivative term of the wave amplitude. From a general physical standpoint the occurrence of such a term is, in all known cases, a consequence of the inertial properties of the matter (this holds also for electromagnetic waves because induction phenomena are related to the relativistic inertial properties of the electromagnetic field). On the contrary, it is not so obvious that a term like this should be found in equations describing pure bioelectrical processes, since the viscosity of a biological medium prevents any long-range propagation of the possible amounts of macroscopic kinetic energy. In local equilibrium thermodynamics there is little place for "acceleration" and only "velocities" are generally permitted. In this realm any phenomenon that could be represented by partial differential equations should be related to a fundamental set of equations, all of which contain only first-order time-derivative terms.

In physics there is only one kind of first-order time-derivative equation governing linear wave propagations: the Schrödinger wave equations. Besides, it is well known that these are of the same kind as those used to describe thermodynamical diffusion (e.g., Fourier equations and Fokker-Planck equations). The former differ from the latter only in a "rotation" of the time axis onto the complex plane. As a consequence also the Schrödinger wave amplitudes must be represented as functions taking values onto a complex plane. This is the same as saying that they must be represented by a pair of real variables.

Fokker-Planck equations were still used by several authors to explain collective behaviors of the neural networks,[15] but these cannot have a meaningful role in explaining a possible cerebral holographic process.

II. ELECTROCHEMICAL SCHRÖDINGER WAVES

We assume the cortical waves to be represented by a pair of local time-dependent variables, $s(x,t)$ and $p(x,t)$, to be interpreted as *macroscopic displacements of sodium and potassium ion concentrations* from certain equilibrium values which shall be specified later.

The idea underlying this assumption is that the wave-propagation medium is the *glial tissue*. This constitutes a strongly connected lattice of cells (*glial cells*; specifically astrocytes) directly intercommunicating through transmembrane electrotonic gap junctions[16] and also indirectly through interstitial pockets filled with the extracellular fluid. Towards the latter various kinds of ion channels and at least one kind of pump (Na^+-K^+ ATPase) have access. In grey matter the number of glial cells is estimated to be as much as 50 times the number of neurons.

The average diameter of a glial cell body is about 18–20 μm, but a lot of randomly bifurcating filaments and veil-like peduncles depart from it, decreasing in number according to a roughly Gaussian distribution with $\approx$45 μm half-width, up to a distance of $\approx$200 μm. Therefore we can calculate that every glial cell keeps in touch with several hundreds lying in its neighborhood.

Until a few years ago the developed glial tissue was mainly and generally thought of as a pure support and supply medium for the neuron network. But during the last decade several bioscientists have been led by accurate experiments to state that peculiar ion transport processes, affecting the bioelectrical activity of the neurons and of the cerebral cortex as a whole, certainly take place inside it. We refer for an account of this subject to the specialized literature.[17]

The possibility that the sodium and potassium ion displacements cooperate in order to generate an electrochemical wave depends essentially on their strict active intercoupling. The main one is certainly due to the Na^+-K^+ ATPase pumps. Each of these performs the transport of two K^+ ions from the extracellular fluid into the intracellular fluid and the inverse transport of three Na^+ ions for one ATP (adenosine triphosphate) molecule hydrolized, and their activity increases with decreasing membrane potential.[18] Another kind of intercoupling is due to the feedback control role exerted by the Ca^{2+} ions through the (Na^{+}-antiports-Ca^{2+})$\rightarrow$(Ca^{2+}-activates-K^{+}) channel system.[19] (A proof that such a double intercoupling can effectively account for wave propagations in living animal tissues has been found by the author. A paper of this subject is in preparation.)

In order to support the idea that self-sustaining linear electrochemical waves may propagate throughout the glial tissue, let us briefly state and discuss the following assumptions.

(i) *Existence of an electrochemical fundamental stationary state.* That is, a local equilibrium thermodynamical state for glial tissue, characterized by invariable and stable local values for ion concentrations. In these conditions the macroscopic electroneutrality must be satisfied and the macroscopic electrical field must be zero valued. We assume by definition the (s,p) local displacements to be zero valued in this state.

This does not mean that the absolute values of the concentrations are microscopically uniform, as would be the case with dead tissue, since the ion concentration ratios, between intracellular and extracellular fluids, remain very different from 1, as a consequence of the pump activity. The (s,p) displacements are understood as "macroscopic values," in the sense that they are average values taken over many-cell-volume units.

(ii) *Continuity equations for (s,p) displacements.* Because of the number conservation of ions (irrespective of the dissociation-recombination processes to which they are subjected in cells and in extracellular fluid), we assume for $s(x,t)$ and $p(x,t)$ the exact validity of the continuity equations:

$$\frac{\partial s(x,t)}{\partial t}+\nabla\cdot\mathbf{J}_1(x,t)=0\ ,$$
$$\frac{\partial p(x,t)}{\partial t}+\nabla\cdot\mathbf{J}_2(x,t)=0\ , \tag{2.1}$$

where $\mathbf{J}_1(a,t)$ and $\mathbf{J}_2(x,t)$ stand, respectively, for the sodium and potassium macroscopic current densities. (∇ and $\nabla\cdot$ are, respectively, the gradient and divergence operators.) According to the laws of electrochemical cinetics we assume $\mathbf{J}_1,\mathbf{J}_2$ not to depend directly upon quantities other than combinations of ion concentration gradients, macroscopic electrical field, and activity of the pumps, with coefficients at most dependent on the local absolute values of the ion concentrations, temperature, and time invariant local properties of the medium. These functional dependences undergo a great simplification when the interdependence among such quantities is taken into account.

(iii) *Maximum efficiency principle for biological phenomena.* That is, natural selection provided for no useless energy dissipation occurring when no useful work has to be done. This means that *pump activity reaches its minimum during the fundamental state* and that *current densities are in a good approximation irrotational both in fundamental and in excited states.* If it were not so, then useless ATP energy dissipation would take place producing currents that would be ineffective for (s,p) displacements.

(iv) *Homogeneous and isotropic electrochemical macroscopic properties of glial tissue.* When observed under the microscope, the cerebral cortex appears as a six-layered wafer, of ≈ 2 mm thickness, constituted by ascending and descending neurons embedded in glial tissue. Its extension amounts to $\approx 1.4\ m^2$ for a human brain (when completely displayed) and its composition appears to be quite homogeneous throughout its entire extension.[20]

(v) *The sodium and potassium macroscopic currents promoted by the interstitial pockets are linear combinations of (s,p) gradients.* This assumption seems reasonable but awaits experimental confirmation. Theoretically it is a consequence of the following assumptions: macroscopic electroneutrality, absence of external charge sources, macroscopic irrotationality of electrical currents, and dependence of other kinds of ion concentration on the (s,p) displacements or electrochemical passivity of other kinds of ions.

(vi) *Permanent variations of glial tissue properties, necessary for hologram formation, only slightly affect the local impedence of the membranes to the (s,p) ion currents.* In order for slow wave holographic processes to take place efficiently, the existence of a great number of small local resonators, densely distributed all over the medium, seems to be providential. At first such an assumption might not appear plausible.

We should notice, however, that recent experiments on the voltage response of cultured glial cell membranes to imposed step currents show the typical transitory behavior of damped low-frequency oscillations; conversely the current responses to imposed step voltages show the typical behavior of a damped low-frequency LC circuit.[21] [This all happens as though the voltage-dependent potassium channels, which suddenly open when the membrane potential goes over a threshold value (≈ -40 mV), had an exceptionally high inductive impedance. An explanation for this phenomenon, based on a "turbine" model for this kind of channel, is in preparation by the author.]

We assume that the holograms are formed by slight variations of the internal viscosity of such resonators. Maximum efficiency is assured provided the viscosity variations are inversely proportional to the cumulative local energy dissipations.

Since these effects are supposed to be purely perturbative—i.e., they can be regarded as infinitesimal local modifications of the medium—we will take them into account later, when the hologram formation and reading out are treated mathematically.

Putting all these assumption together, we may state the expressions of the $\mathbf{J}_1,\mathbf{J}_2$ currents in the form

$$\mathbf{J}_1(x,t)=\alpha_1(x)\nabla s(x,t)+\beta_1(x)\nabla p(x,t)\ ,$$
$$\mathbf{J}_2(x,t)=\alpha_2(x)\nabla s(x,t)+\beta_2(x)\nabla p(x,t)\ , \tag{2.2}$$

where $\alpha_1,\alpha_2,\beta_1,\beta_2$, are coefficients only slightly depending on the time-invariant local properties of the medium.

Substitution of (2.2) into (1.2) gives the system of equations:

$$\frac{\partial s}{\partial t}+\nabla\cdot(\alpha_1\nabla s+\beta_1\nabla p)=0\ , \tag{2.3a}$$

$$\frac{\partial p}{\partial t}+\nabla\cdot(\alpha_2\nabla s+\beta_2\nabla p)=0\ , \tag{2.3b}$$

which certainly admits oscillatory solutions provided the matrix

$$M=\begin{bmatrix}\alpha_1(x) & \beta_1(x)\\ \alpha_2(x) & \beta_2(x)\end{bmatrix} \tag{2.4}$$

has complex conjugate eigenvalues. We shall assume this to be the case and that the oscillatory solutions are very slightly damped.

Assuming in first approximation α,β to be exactly uniform throughout the medium, we can easily find two complex-conjugate linear combinations of s and p, say $\psi(x,t)$ and $\psi^*(x,t)$, satisfying the generalized Schrödinger equations:

$$\frac{\partial\psi}{\partial t}=(\epsilon+i\sigma)\nabla^2\psi,\quad \frac{\partial\psi^*}{\partial t}=(\epsilon-i\sigma)\nabla^2\psi^*,\quad 0<\epsilon<<\sigma \tag{2.5}$$

($i=\sqrt{-1}$, $\nabla^2\equiv\nabla\cdot\nabla$ is the Laplace operator). These equations must be corrected by the addition of small perturbative terms proportional to ψ,ψ^* gradients when α,β inhomogeneities are taken into account. We shall consider them later when the problem of hologram formation is discussed.

Any solution of Eqs. (2.5) can be put in the form[22]

$$\psi(x,t)=\int_{V_M}K(x,t;x',t')\psi_0(x')dV(x')\quad(t>t') \tag{2.6}$$

with $\psi^*(x,t)$ the complex-conjugate expression of $\psi(x,t)$, and $\psi_0(x')$ and $\psi_0^*(x)$ the initial wave amplitudes at time t'; the integration being extended over the entire volume of the medium. K and K^* are the propagation functions (propagators) for ψ and ψ^*, respectively. These can be thought of as representing the amplitude responses at point x and time t, consequent to the application of a δ-like impulsive stimulus at a point x' and at a time t'. The explicit expression for K is

$$\begin{aligned}K&=[2\pi(\epsilon+i\sigma)(t-t')]^{-d/2}\\&\quad\times\exp[-(x-x')^2/4(\epsilon+i\sigma)(t-t')]\end{aligned} \tag{2.7}$$

where $(x-x')^2$ is the squared distance between points x and x' and K^* is the complex conjugate of K (d is the dimension of the medium). Assuming that at time $t'=0$ the medium is perturbed by a short-duration stimulus, distributed around point $x'=0$ in a Gaussian-like fashion with half-width α [this should simulate the effect of a sudden (s,p) displacement generated by an assonic arborization at a single spike run-end] we find for any real linear combination $V(x,t)$ of ψ and ψ^* the expression

$$\begin{aligned}V(x,t)&=\frac{C}{2}(2\pi)^{d/2}[e^{i\varphi}\psi(x,t)+e^{-i\varphi}\psi^*(x,t)]\\&=C[(\alpha^2/2+\epsilon t)^2+(\sigma t)^2]^{-d/2}\exp\left[-\frac{x^2(\alpha^2/2+\epsilon t)}{4[(\alpha^2/2+\epsilon t)^2+(\sigma t)^2]}\right]\\&\quad\times\cos\left[\frac{x^2\sigma t}{4[(\alpha^2/2+\epsilon t)^2+(\sigma t)^2]}-\frac{d}{2}\arctan\frac{\sigma t}{\alpha^2/2+\epsilon t}+\varphi\right],\end{aligned}$$

where C and φ are real constants. We suppose V to represent the *electrical* potential response recorded at time t by a micropipette electrode placed at point x.

In Fig. 1 various patterns of V are plotted for fixed x and varying t according to different values of α,φ param-

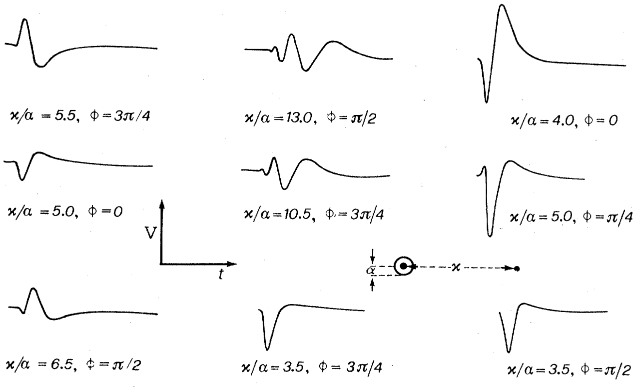

FIG. 1. Theoretical voltage responses to an impulsive stimulus. Common $d=3$ and $\epsilon=0$ are assumed; x is the distance of the detector from the signal source, α the half-magnitude of the Gaussian-shaped source size. Because of the scale covariance, variations of α only affect V and t scales when x/α is held fixed.

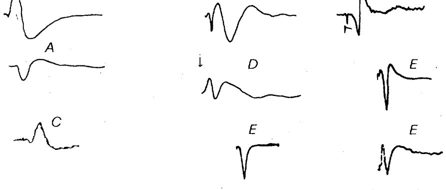

FIG. 2. Experimental potential responses directly detected on mammalian brain cortexes. References for the curves are as follows: *A*, Ref. 12, Pt. A, p. 305; *B*, Ref. 12, Pt. A, 291; *C*, Ref. 12, Pt. A, p. 190; *D*, Vanderwolf and Ossenkopp, in *Neuronal Plasticity and Memory Formation,* Ref. 26, p. 605; *E*, Clare and Bishop [EEG Clin. Neurophysiol. **8**, 583 (1956)].

eters. In Fig. 2 we report some characteristic stimulus-evoked responses usually recorded by the direct application of electrodes on mammalian brain cortexes.

III. SCHRÖDINGER WAVE HOLOGRAPHY

Schrödinger wave holograms (SWH's) are superior to D'Alembert wave holograms (DAWH's) at least for the following reasons.

(a) Differently from DAWH's, for which only plate recordings are permitted, SWH's require, as a recording support, the entire medium through which the wave propagation takes place. This property is essentially due to the different ways in which the two kinds of waves depend on their respective initial conditions. This allows for much more information to be stored by SWH's than by DAWH's.

(b) Schrödinger waves (SW's) generated by periodic signals propagate with velocities proportional to the square roots of their respective frequencies, whereas D'Alembert waves all propagate with the same speed (obviously not in dispersing media). Then SW Fourier time-transforms can easily be obtained by exploiting the time spreading effect. This allows also for practically immediate recovery of holographic information.

(c) The maximum amount of information a SW can propagate is given in bits by the ratio between the entire volume of the medium and the volume of the minimum size in which a signal emission takes place. Assuming this emission size to be of the order of magnitude of a minimal axonic arborization in human cortex ($\approx 2\times 10^{-2}$ mm^3), we can estimate as much as 10^6—10^8 bits of information for one impulsive local stimulus. This quantity spreads throughout the medium and decreases in time at a rate depending on the damping constant values [represented by ϵ in Eqs. (2.5)] and by other absorption phenomena that probably take place at the boundaries of the glial tissue. This should be compared with the ratio A/λ^2 quoted in Sec. I.

(d) Diffraction of SW's through a single fixed hologram can evoke time-varying information, i.e., moving-scene memories; these can effectively be recorded in fixed SW holograms. On the contrary, only time invariant information can be recorded by DAWH's.

(e) Any item of time-varying information that is injected into the medium through a system of independent pointlike sources inserted in it, can easily be recovered from SWH's in proximity of the same sources. On the contrary, information recovery from DAWH's is only possible by focalization devices for the diffracted waves.

Let us now see how SW holography should run. Equation (2.7) characterizes the propagation of SW's through a perfectly homogeneous and isotropic medium with adsorbing boundaries. If such an ideal medium were infinitely extended then Eqs. (2.5) could be solved for plane-wave solutions of the form

$$\psi_k(x,t)=A(\mathbf{k})\exp[i(\mathbf{k}\cdot\mathbf{x})-(\epsilon+i\sigma)k^2\tau] \qquad (3.1)$$

[with $\psi_k^*(x,t)$ the complex conjugate of this expression], where $\mathbf{k}$ is an arbitrary real vector linked to the wave-number unimodular vector $\mathbf{n}$ and to the wavelength λ by the relation $\mathbf{k}=2\pi\mathbf{n}/\lambda$. Equation (3.1) describes a wave with definite frequency $\nu=\sigma k^2/2\pi=2\pi\sigma/\lambda^2$ and with a maximum amplitude $|\psi_k|$ exponentially decreasing in time with a time decay constant $\tau=1/\epsilon k^2=\sigma/2\pi\epsilon\nu=\lambda^2/4\pi^2\epsilon$. Notice how the wave damping increases with increasing frequency and decreasing wavelength.

At any time $t>0$ functions (3.1) constitutes an infinite and complete set of mutually orthogonal complex functions satisfying the relations

$$\int_{V_\infty}\psi_{k_1}^*(x,t)\psi_{k_2}(x,t)dV(x)$$
$$=(2\pi)^3\,|A(\mathbf{k}_1)|^2\exp(-t/2\tau)\delta(\mathbf{k}_1-\mathbf{k}_2)\,, \qquad (3.2)$$

where $\delta(\mathbf{k}_1-\mathbf{k}_2)$ is the δ Dirac distribution defined in $\mathbf{k}$ space.

Such ideal conditions certainly cannot be realized in a true cell medium; if this were so, then infinite amounts of information could be holographically processed by them. In practice one must consider quite different behaviors for a similar set of true wave functions, mainly because of the following reasons.

(i) The true medium is finite and irregularity bounded.

(ii) Its macroscopic properties and features cannot be extrapolated below the scale of a few average cell diameters. At this scale the (s,p) displacement equations would be better described by discrete sets of first-order total time-derivative equations than by partial differential equations. We can take this into account assuming that the spectrum of permitted frequencies is cut off beyond a certain value $\nu_c=2\pi\sigma/\lambda_c^2$, where λ_c is on the order of magnitude of a few average cell diameters.

(iii) At an intermediate scale local random inhomogeneities of the medium must be taken into consideration. As could be demonstrated by Wentzel-Kramers-Brillouin approximation methods,[23] to the extent to which the local variations of α,β in Eqs. (2.3) are small compared with their mean values, such inhomogeneities, while appreciably affecting the local phases of the wave amplitudes, have only small effects on their local moduli. Because of this, in place of functions (3.1), a maximal finite set of mutually orthogonal periodic functions, with almost uniform moduli and random variable phases, can be assumed:

$$\psi_i(x,t)=[A_i+\epsilon_i(x)]\exp[i\varphi_i(x)-(\tau_i^{-1}+i\omega_i)t]\,, \qquad (3.3)$$

where ϵ_i are real functions of negligible magnitude with respect to A_i and $\psi_i^*(x,t)$ is the complex conjugate of $\psi_i(x,t)$; index i runs over a set S of values having a power nearly equal to V_M/λ_c^3 ($\approx 10^9$ for human brain cortex) and $\varphi_i(x)$ are random phase functions having mean-square gradients $\langle[\nabla\varphi_i]^2\rangle$ nearly equal to ω_i/σ. One should notice that in a frequency interval $\Delta\omega$ a number of nearly equal ω_i values increasing with $\omega^{3/2}$ should fall into place.

Because of their nice random oscillatory properties, functions (3.3) satisfy, for any *generic* smooth function $F(x)$, the equations

$$\int_{V_M} F(x)\psi_i^*(x,t)\psi_j(x,t)dV(x) = \delta_{ij}\,|\,A_i\,|^2\exp(-2t/\tau_i)\int_{V_M} F(x)dV(x)+\epsilon_{ij}[F]\ , \tag{3.4}$$

where δ_{ij} is the Kronecker symbol and the $\epsilon_{ij}[F]$ are negligible terms functionally depending on F. One must consider Eqs. (3.4) to be well satisfied also when $F(x)$ is a *generic* wave function or a product of a finite number of such.

As we shall see, Eqs. (3.4) play an essential role in SW holography. We shall name functions (3.3) *random normal functions,* and the oscillatory perturbations of the medium described by them *random normal modes.*[24]

IV. HOLOGRAPHIC INFORMATION PROCESSING BY RECRUITING OF RANDOM NORMAL MODES

Although SWH information could be simply stored by slight variations of the local permeabilities of the medium, we are inclined to believe that the recording process taking place inside the animal brains is based on the existence of a lot of small resonators dispersed throughout the cortex, so that the holograms could be formed by slight variations of their damping constants. We imagine that this expedient was favored by natural selection because of its greater efficiency.

Indeed it is possible to prove that the latter kind of hologram formation has, with respect to the former, the following advantages.

(i) Great improvement of signal-to-noise ratios in hologram evocation.

(ii) Larger capability of storing information by superposition of holograms.

(iii) Highest selectivity in evoking the best impressed holograms. Indeed, in the case of resonators, the strengthening of a hologram increases exponentially with the number of times the hologram recordings are repeated, whereas in the former such strengthening only increases linearly.

Obviously, it is not necessary to specify that such a hypothesis is purely speculative. In our opinion this seems to be a sufficient reason for freeing our imagination in order to guarantee that everything works as in the best possible world. Therefore let us state the following assumptions.

(a) The wave propagation medium contains a statistically uniform distribution of small oscillators densely covering the entire wave frequency spectrum.

Wave propagations are supposed to be locally affected very slightly by oscillators. Despite this, large cumulative effects can result in the diffused waves when the perturbative contributions coming from the oscillators lying all over the medium sum up *coherently* (i.e., "constructively" according to the superposition principle).

One should notice that the ratio between the local intensity of a monochromatic wave component coherently diffused by a set of N equal oscillators and the intensity of one *incoherently* diffused by them (i.e., coming from them with random phase relations) is equal to N.

If only $M=\alpha N$ $(0<\alpha<1)$ among the N oscillators give rise to coherent diffusion, then such a ratio amounts to $\alpha^2 N$. We can see that, provided N is large enough (say 10^{13}), a negligible fraction of slightly, but coherently, diffusing oscillators can produce intense effects.

We suppose the holograms to be formed by variation of the internal viscosities of the local oscillators: i.e., by variation of their damping constants. This allows them to store, cumulatively, the maximum amount of information. In order for the holographic evocations to take place efficiently, we assume that the values of the damping constants are subjected to slight modifications which are in each place inversely proportional to the amount of the energy cumulatively dissipated by the oscillators themselves during the wave propagations. We could characterize this memory formation as a sort of "running in" effect.

It would be easy to prove that such holographic recordings are very sharply selective for the time-Fourier components of the wave propagations; much more than the oscillator bandwidths might suggest.

In order to express mathematically the effect produced by a small resonator subjected to the action of a wave, several models might equally well be hypothesized. We propose tentatively the following one: a resonator is constituted by the two parts of cell membrane which enclose an interstitial pocket; for the sake of simplicity we assume the latter to be spherically shaped. This assumption is comforted by the experimental fact, anticipated in assumption (vi) of Sec. II, that the glial cell membranes show effectively damped oscillatory behavior, precisely in the range of EEG frequencies, under the action of potassium-ion step currents.

From a macroscopical standpoint one should imagine several hundred resonators lying in a volume unit. We can also assume that the "force" acting on each of them is proportional to the potassium-ion flux leaking through its surface. By applying the theory of linear filters[25] we deduce that the source of the disturbance to the wave propagation due to a set of resonators placed in the neighborhood of a point x is correctly represented by adding to the left-hand side of Eq. (2.3b) the following term:

$$\eta(x,t)=\int_{-\infty}^{+\infty} G(x;t-t')\frac{\partial p(x,t')}{\partial t'}dt'\ , \tag{4.1}$$

where $G(x,t-t')$ is a suitable transfer Green function vanishing at any $t<t'$.

The appropriate expression of G, obtained by Fourier-transform methods, is

$$G(x;t-t')=\sum_{j\in I(x)}\int_{-\infty}^{+\infty}\frac{g\exp[-i\omega(t-t')]}{\bar{\omega}_j^2-\omega^2+i\gamma(x,\bar{\omega}_j)}d\omega \tag{4.2}$$

[$I(x)$ is set of oscillators lying in the volume unit centered at x], where g, $\gamma(x,\bar{\omega}_j)$, and $\bar{\omega}_j$ are, respectively, the coupling constant, the damping constants, and the proper frequencies of the oscillators lying in the volume unit centered at x. For the sake of simplicity we have assumed g to be the same for all oscillators.

Supposing the spectrum of the proper frequencies $\bar{\omega}_j$ to be randomly distributed in such a way that a number

$N_i \cong N_0 \bar{\omega}_i \, \Delta\omega$, where N_0 is a convenient integer, falls into the frequency interval $(\bar{\omega}_i, \bar{\omega}_i + \Delta\bar{\omega})$, we find that only the imaginary part of the integral on the right-hand side (rhs) of (4.2) gives an appreciable contribution to G. Therefore, after insertion of (4.2) into (4.1), we obtain for η the expression

$$\eta(x,t) \cong -\sum_{j \in I(x)} \int_{-\infty}^{+\infty} \frac{g N_0 \bar{\omega}_j \, \Delta\bar{\omega} \exp(-i\omega t) \gamma(x,\bar{\omega}_j) \tilde{p}(x,\omega)}{(\bar{\omega}_j^2 - \omega^2)^2 + [\omega \gamma(x,\bar{\omega}_j)]^2} d\omega \, , \tag{4.3}$$

where $\tilde{p}(x,\omega)$ is the Fourier transform of $p(x,t)$:

$$\tilde{p}(x,\omega) = \int_{-\infty}^{+\infty} p(x,t') \exp(-i\omega t') dt' \, . \tag{4.4}$$

Assuming the frequency spectrum of p to be discrete; that is,

$$\tilde{p}(x,\omega) = \sum_i [\tilde{p}_i(x) \delta(\omega - \omega_i) + \tilde{p}_i^*(x) \delta(\omega + \omega_i)] \, , \tag{4.5}$$

where $\delta(\omega \pm \omega_i)$ are Dirac δ distributions; and substituting this expression into the rhs of Eq. (4.3), we obtain

$$\eta(x,t) \cong -\sum_{j \in I(x)} \sum_i \left[\frac{g N_0 \bar{\omega}_j \, \Delta\bar{\omega} \exp(-i\omega_i t) \omega_i^2 \gamma(x,\bar{\omega}_j) \tilde{p}_i(x)}{(\bar{\omega}_j^2 - \omega_i^2)^2 + [\omega_i \gamma(x,\bar{\omega}_j)]^2} + \text{c.c.} \right] . \tag{4.6}$$

If the damping constants γ remain small enough, then the overwhelming part of the rhs of (4.6) is given by

$$\eta(x,t) \cong -\sum_{j \in I(x)} \left[\frac{g N_0 \omega_j \, \Delta\bar{\omega} \exp(-i\omega_j t) \tilde{p}_j(x)}{\gamma(x,\omega_j)} + \text{c.c.} \right] , \tag{4.6'}$$

where the convention of taking $i = j$ when $\omega_i \cong \bar{\omega}_j$ is made.

The conditions for holography are fulfilled when damping constants γ are such as to satisfy the following relationships:

$$\gamma^{-1}(x,\omega_j) = C \, | \tilde{\psi}_H(x,\omega_j) |^2 \, , \tag{4.7}$$

where C is a suitable positive real constant and $\psi_H(x,\omega_j)$ is the time-Fourier transform of the wave function which is used for the hologram recording.

Inserting Eq. (4.7) into the rhs of Eq. (4.6) and performing a few formal changes, we obtain

$$\eta(x,t) = C \frac{\partial}{\partial t} \sum_{j \in I(x)} | \tilde{\psi}_H(x,\omega_j) |^2 \times [\tilde{p}_j(x) \exp(-i\omega_j t) - \text{c.c.}] \, . \tag{4.8}$$

Since $\tilde{p}_j(x)$ is a real linear combination of the time Fourier transforms of the wave functions ψ, ψ^*, describing the propagation in act, we can also state Eq. (4.8) in the following form:

$$\eta(x,t) \cong -i\eta_0 \sum_{j \in I(x)} \omega_j \, | \tilde{\psi}_H(x,\omega_j) |^2 \times \{ \tilde{\psi}(x,\omega_j) \exp[-i(\omega_j t + \theta)] + \text{c.c.} \} \, , \tag{4.9}$$

where η_0 and θ are convenient real numbers.

(b) The waves propagating during the formation of a hologram are the superposition of a certain *reference wave* $\psi_R(x,t)$, having sufficiently high amplitudes and other suitable properties to be dealt with later, with the *information wave* $\psi_I(xt)$ directly generated by a system of information carrying local sources embedded in the medium.

This means that the wave function $\psi_H(x,t)$, whose time-Fourier transform appears in Eq. (4.8), is assumed to have the expression

$$\psi_H(x,t) = \psi_R(x,t) + \psi_I(x,t) \, . \tag{4.10}$$

(c) Information recovery is obtained by diffractive scattering of a reference wave on a hologram previously formed by association of an identical reference wave with an information wave.

(d) Any reference wave is a superposition of normal modes whose frequency spectra are statistically distributed all over the frequency spectrum range of the signal to be reproduced.

In (iii) of Sec. III we have estimated that a set S of $\approx 10^9$ mutually orthogonal oscillation modes could be excited inside the entire human brain cortex; we also supposed that the frequency spectrum is completely degenerate. In principle this assumption is legitimated by the small random irregularities of the wave-propagation medium. However, since the EEG frequency spectrum range is poorly extended, we can estimate that an enormous amount of nearly equal frequency values fall into the bandwidth of each resonator; therefore, we can assume the total wave frequency spectrum to be partitioned in a collection of mutually disjointed subsets $J_i \in S$ of frequency values nearly equal to ω_i. According to this, the normal modes are conveniently classified by double labeling. Then we can give mathematical form to the initial statement by assuming that any reference wave $\psi_R(x,t)$ will take the general form

$$\psi_R(x,t) = \sum_i A_R(\omega_i) \sum_{j \in J_i'(R)} \exp\{ i [\varphi_{ij}(x) + \theta_{ij}^R - \omega_i t] \} \, , \tag{4.11}$$

where j summations extend to small subsets $J_i'(R) \subset J_i$ of equal frequency modes and θ_{ij}^{R} are real numbers accounting for the initial phases; A_R are real amplitudes only dependent on ω_i in a manner that will be established later. We have eliminated from Eq. (4.10) the τ_i^{-1}-dependent damping factors appearing in Eqs. (3.3) since we suppose the reference waves to be sustained by suitable sets of periodic signal sources over a long time. It is easy to verify that an enormous quantity of mutually orthogonal reference waves covering quite densely the frequency spectrum range can still be produced.

For any two time-Fourier transforms of such reference wave functions—say $\tilde{\psi}_R(x,\omega_i)$ and $\tilde{\psi}_{R'}(x,\omega_i)$—strong orthogonality equations similar to (3.4) still hold:

$$\int_{V_M} F(x)\tilde{\psi}_R^*(x,\omega_i)\psi_{R'}(x,\omega_i)dV(x) \cong N_R(\omega_i)[A_R(\omega_i)]^2\delta_{R,R'}\int_{V_M} F(x)dV(x) , \qquad (4.12)$$

where $F(x)$ is any *generic* smooth function, $N_R(\omega_i)$ is the number of modes falling into $J_i'(R)$, and $\delta_{R,R'}$ is the Kronecker delta. In the following we shall assume

$$A_R(\omega_i)=C_R[\omega_i N_R(\omega_i)]^{-1/2} , \qquad (4.13)$$

where C_R is a real constant. This ensures that the coefficient appearing in front of the rhs of (4.12) is simply inversely proportional to ω_i. We need this condition in order that the holographically stored items of information may be faithfully reproduced by reference-wave diffraction.

Now we have almost all we need in order to show how the "ghosts" of holographically recorded information can be evoked. All is accomplished when we consider that any solution of the oscillator-perturbed wave equations can be exactly put in the form

$$\psi(x,t)=\psi_0(x,t)+\int_{V_M}\int_{-\infty}^{t} K_0(x,t;x',t') \times\eta(x',t')dt'\,dV(x) , \qquad (4.14)$$

where ψ_0 and K_0 are, respectively, the wave function and the propagator kernel relative to a medium deprived of all oscillators and $\eta(x',t')$ is the source contribution of the oscillators given by Eq. (4.1) or, more specifically, by Eq. (4.9). Then, provided we assume ψ_0 to be a reference wave $\psi_{R'}$ and the wave function ψ_H appearing in Eq. (4.9) satisfies Eq. (4.10), we obtain for $\psi(x,t)$ the integral equation

$$\psi(x,t)=\psi_{R'}(x,t)-i\eta_0\int_{V_M}\int_{-\infty}^{t} K_0(x,t;x',t')\sum_i |\tilde{\psi}_R(x,\omega_i)+\tilde{\psi}_I(x,\omega_i)|^2\omega_i\{\tilde{\psi}(x,\omega_i)\exp[-i(\omega_i t+\theta)]+\text{c.c.}\}dt'\,dV(x) . \qquad (4.15)$$

This equation can be solved in series of η_0 powers by iterative methods. The first-order approximation (i.e., the Born approximation) is obtained by simple substitution of $\psi_{R'}(x,\omega_i)$ in place of $\psi(x,\omega_i)$ into the integral expression at rhs of Eq. (4.15). Then taking account of Eqs. (4.12) and (4.13) and of the propagator properties, we can easily verify the following statements.

1. If $\psi_{R'}$ is orthogonal to ψ_R then $\psi(x,t)\cong\psi_R(x,t)$; i.e., no appreciable disturbance is performed by the local oscillators.

2. If $\psi_{R'}$ is proportional to ψ_R then we find

$$\psi(x,t)\cong\alpha\psi_R(x,t)+\beta\psi_I(x,t)+\beta^*\psi_I^{\text{ad}}(x,t) , \qquad (4.16)$$

where α,β are non-negligible complex constants, and ψ_I^{ad} is the "advanced" wave function relative to ψ_I. The latter is the function describing a wave propagation similar to the one described by ψ_I, but constituted by anticausal or convergent-to-source waves. Both ψ_I and ψ_I^{ad} do simultaneously contribute to the recovery of recorded information.

3. If two or more information waves participate with their respective recruited reference waves in the formation of a hologram, then the recruiting of one of such reference waves, possibly exerted by the representation of a fragment of information, gives rise to a self-strengthening evocation of all other associated information. This works equally well for time-invariant and time-varying items of information.

We can prove this by solving Eq. (4.15) beyond the Born approximation.

To perform all these operations a set of randomly localized sources for the emission of reference waves and a noiselike signal-recruiting device, like that pictured in Fig. 3, appears to be unavoidable. Since the waves diffracted by holograms reproduce in shape the original information waves, and these reach their maximum amplitudes in exactly the neighborhoods of the signal sources, then the recovery of information is possible through a set of threshold receptors applied in close relationship with the set of the signal sources. It is interesting to compare this scheme with certain organization diagrams reported by neurophysiologists.[26]

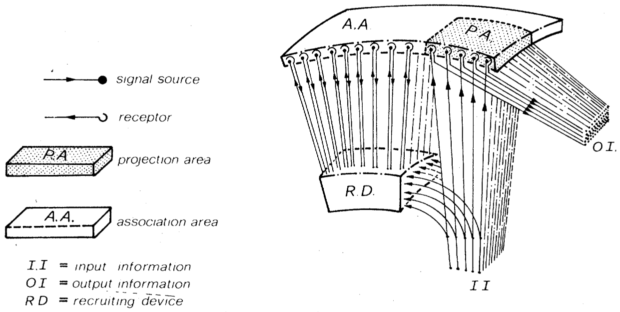

FIG. 3. Recruiting device RD is supposed to sustain the reference-wave emission through a system of sources dispersed all over the cortex (AA + PA) and regulated by feedback control lines. Switching on and modulation of the reference waves depend upon the items of input information. Recovery of information is performed by receptors placed in the vicinity of the information-wave sources embedded in projection area PA.

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