

Atomic collisions with relativistic heavy ions. IV. Projectile *K*-shell ionization

R. Anholt and W. E. Meyerhof

*Department of Physics, Stanford University, Stanford, California 94305-2196*

H. Gould and Ch. Munger

*Materials and Molecular Research Division, Lawrence Berkeley Laboratory,  
University of California, Berkeley, California 94720*

J. Alonso

*Accelerator and Fusion Research Division, Lawrence Berkeley Laboratory,  
University of California, Berkeley, California 94720*

P. Thieberger and H. E. Wegner

*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973*

(Received 30 July 1985)

Projectile *1s* ionization cross sections are reported for 82-, 140-, and 200-MeV/amu Xe projectiles incident on a variety of thin solid targets between Be and Au. The cross sections were calculated with the plane-wave Born approximation. Possible relativistic wave-function, target-screening, and transverse-excitation effects are discussed. Comparisons of the data with the perturbed-stationary-state theory of Basbas *et al.* and the Glauber approximation are made. Scaled, interpolated Xe+Xe ionization cross sections agree well with measured *p*+H ionization cross sections.

## I. INTRODUCTION

The present paper returns to a question considered in the first part of this series of papers on relativistic heavy-ion-atom collisions: inner-shell ionization.<sup>1</sup> While paper I considered target inner-shell vacancy production, signaled by the emission of target *K* x rays, here we consider projectile *K*-shell ionization, manifested by the charge-state gain by one- or two-electron heavy ions passing through thin foils. Heavy ( $Z \geq 30$ ), relativistic one- or two-electron ions are normally in their ground *1s* or *1s*<sup>2</sup> states. Hence, a measurement of the fraction of zero- or one-electron ions after passage through a thin foil represents a measurement of projectile *1s* ionization.

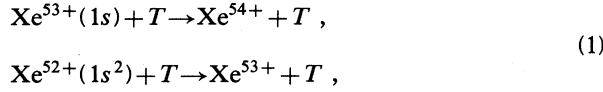
The experimental arrangement is described in paper III of this series.<sup>2</sup> Section II of the present paper discusses the differences between studies of target *K*-shell ionization by measuring target *K* x rays,<sup>1</sup> and of projectile ionization in light and heavy projectiles by measuring charge states.

Ionization cross sections can be calculated with the plane-wave Born approximation<sup>3-5</sup> if the ratio of the ion velocity *v* to the velocity *v<sub>K</sub>* of the *K* electron being ionized is much larger than unity. This is certainly valid for the low-*Z* relativistic projectiles considered in paper II,<sup>6</sup> but is less valid for the high-*Z*, 82- to 200-MeV/amu Xe ions considered here, for which *v/v<sub>K</sub>* is between 1 and 2. Although several minor effects are discussed in Sec. II, the major effect at these velocities is the binding<sup>7</sup> or polarization<sup>8</sup> of the active electron by the perturbing nucleus. The theory of these effects formulated by Basbas and co-workers<sup>9</sup> has been widely applied to measurements of inner-shell vacancy production. The reduction of the ion-

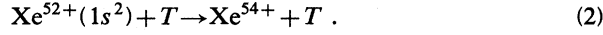
ization cross sections due to the increased binding of the *1s* electron in low-velocity collisions, especially where molecular orbitals are formed, has been established clearly.<sup>10</sup> The polarization of the electronic wave functions brings the inner-shell electrons closer to the perturbing nucleus, which increases the excitation probability.<sup>8</sup> Although the polarization effect on electronic stopping powers (dominated by outer-shell target-electron ionization processes) is well established, that on *K*-shell ionization is not as well understood. At the medium electron velocities ( $v \approx v_K$ ) where the polarization effect should be dominant, a competing process due to the capture of target *K* electrons by the projectile ion obscures the increase in the cross section due to the polarization effect, if target *K*-shell ionization is studied.<sup>11</sup> The advantage of studying projectile ionization is that the equivalent process (capture of a projectile *K* electron by the target) is absent due to the lack of vacancies in the target atom.

The measured cross sections are compared with theories of *K*-shell ionization including binding and polarization effects in Sec. III. We also make comparisons with theories that have been applied to symmetric *p*+H collisions. Since we have new measurements of equilibrium projectile charge states,<sup>2</sup> and a theory of target *K*-electron capture for relativistic heavy ions has now been formulated,<sup>12</sup> we return to the question of target *K*-electron capture contributions to target inner-shell vacancy production, which was left open in paper I. A difference between the relative importance of the binding and polarization effects on target ionization and on projectile ionization is suggested by the present data, but the uncertainties due to target electron capture do not allow us to draw a definite conclusion.

Evidence is obtained in this work for single-electron ionization in the collisions ( $T$ =target)



and for double-electron ionization:



We show in Sec. III C that the double-ionization cross section can be calculated from the product of one-electron ionization probabilities<sup>13,14</sup> integrated over impact parameters.

## II. THEORIES OF INNER-SHELL IONIZATION

### A. The plane-wave Born approximation

Our discussion of theories of inner-shell ionization by relativistic projectiles will contrast effects on target inner-shell vacancy production with effects on projectile ionization in low- $Z$  and high- $Z$  projectiles, which are summarized in Table I. The starting point for all three is the nonrelativistic plane-wave Born approximation (PWBA) in which the ionization cross section is calculated using<sup>15</sup>

$$\sigma_{1s} = \frac{8\pi a_0^2 Z_x^2 \alpha^2}{\beta^2} \int_0^\infty d\epsilon \int_{q_0}^\infty \frac{dq}{q^3} |F(q)|^2. \quad (3)$$

Here,  $Z_x$  is the atomic number of the perturbing nucleus (the projectile charge for target ionization or the target charge for projectile ionization),  $v = \beta c$  is the ion velocity,

$c$  is the speed of light,  $\alpha$  is the fine-structure constant,  $a_0$  is the Bohr radius,  $\epsilon$  is the kinetic energy of the ionized electron,  $q_0 = (E_K + \epsilon)/v$  is the minimum momentum transfer needed to ionize the electron,  $E_K$  is the  $K$ -electron binding energy,  $F(q)$  is the ionization form factor given by

$$F(q) = \langle \epsilon | \exp(i\mathbf{q} \cdot \mathbf{r}) | 1s \rangle, \quad (4)$$

and  $|\epsilon\rangle$  and  $|1s\rangle$  are continuum and  $1s$  electronic wave functions. We shall consistently calculate single-ionization cross sections per  $K$  electron in this paper. Target  $K$ -vacancy production cross sections are defined per atom, hence are factors of 2 larger.<sup>3,4</sup>

### B. Wave-function effects

The form factor  $F(q)$  is usually calculated with non-relativistic, hydrogenic  $1s$  and continuum wave functions. For target  $K$ -vacancy production in neutral target atoms, one must approximate the many-electron wave functions by using a reduced effective target charge  $Z^* = Z_t - 0.3$  for the  $K$  shell,<sup>15</sup> and one accounts for the difference between the ideal hydrogenic binding energy  $\frac{1}{2}Z^{*2}$  a.u. and the actual one  $E_K$  by introducing a parameter  $\theta_K$ , which is the ratio between the two. Then the ionization cross section is given by<sup>3</sup>

$$\sigma_{1s} = 4\pi \left[ \frac{a_0 \alpha Z_x}{\beta Z^*} \right]^2 f(\eta_K, \theta_K), \quad (5)$$

where  $\eta_K = \beta^2 / (\alpha Z^*)^2$ ,

$$f(\eta_K, \theta_K) = \int_{\theta_K}^\infty dW \int_{Q_0}^\infty \frac{dQ}{Q^2} |F_K(Q, W)|^2, \quad (6)$$

TABLE I. Effects on  $K$ -shell ionization.

| Effect                                                 | On target ionization                               | On projectile ionization                  |                                                   |
|--------------------------------------------------------|----------------------------------------------------|-------------------------------------------|---------------------------------------------------|
|                                                        |                                                    | Low- $Z$                                  | High- $Z$                                         |
| First-order theory                                     | PWBA                                               | PWBA                                      | PWBA                                              |
| Electronic wave function                               | $\psi_0 \sim e^{-Z^*r}$ ;<br>use $Z^* = Z_t - 0.3$ | $\psi_0 \sim e^{-Zr}$ ;<br>$Z \equiv Z_p$ | Dirac; $Z = Z_p$<br>$\psi_0 \sim r^{s-1} e^{-Zr}$ |
| Binding energy                                         | $E_K = \theta_K \frac{1}{2} Z^{*2}$                | $E_K \equiv \frac{1}{2} Z^2$              | $E_K = (1-s)mc^2$ ,<br>$s^2 = 1 - \alpha^2 Z^2$   |
| Wave-function distortion                               | polarization + binding effect                      | $v/v_K > 10$<br>no correction             | $v/v_K \sim 1$ to 2<br>corrections needed         |
| Relativistic velocity $\beta \cdot \alpha$ interaction | calculate with $Z^*, \theta_K$                     | negligible                                | calculate with Dirac wave functions               |
| Perturbing nucleus screening                           | bare projectiles<br>no screening                   | cross-section reductions                  | $Z_t^2 \rightarrow Z_t^2 + Z_p$                   |
| Coulomb deflection                                     | negligible                                         | negligible                                | negligible                                        |
| Experimental effects                                   | target $K$ -electron capture                       |                                           |                                                   |

$W = (E_K + \epsilon) / (\frac{1}{2} Z^2)$ ,  $Q_0 = W^2 / 4\eta_K$ , and  $F_K(Q, W)$  is given by Khandelwal *et al.*<sup>3</sup> Tables of the function  $f(\eta_K, \theta_K)$  or related quantities are available<sup>3,4</sup> from which target  $K$ -vacancy production cross sections can be calculated. With relativistic ions, one must calculate  $\eta_K$  using the ion velocity  $\beta^2$ , not the ion energy  $(\gamma - 1)Mc^2$ , where  $\gamma^{-2} = 1 - \beta^2$ , as is usually prescribed.<sup>3,4,14</sup> For low- $Z$  projectile ionization, where hydrogenic wave functions and energies are valid, these tables can be used by taking  $\theta_K = 1$  and  $Z_x = Z_t$ .

For high- $Z$  ions and target atoms, one should use Dirac electronic  $1s$  and continuum wave functions.<sup>16</sup> At low ion velocities, the use of Dirac wave functions is known to enhance high- $Z$   $K$ -shell ionization cross sections.<sup>17</sup> The factor  $r^{s-1}$  in the  $1s$  and continuum wave functions gives a weak divergence at small  $r$  because  $s = (1 - \alpha^2 Z^2)^{1/2}$ ,  $\alpha = 1/137.037$  is less than unity, which contracts the radial electronic density distribution. This contraction enhances the electronic momentum distribution at large momentum  $q$ . Since the ionization form factor is just the Fourier transform of the product of the  $1s$  and continuum electronic wave functions, the form factor at large  $q$  is enhanced, leading to larger ionization cross sections. For the present high-velocity ions, however, the minimum momentum transfer  $q_0 \sim E_K / \beta c$  is small, so one can approximate the form factor with

$$F(q) \sim i q \cdot \langle \epsilon | \mathbf{r} | 1s \rangle. \quad (7)$$

In this case, the size of the radial wave functions, proportional to the expectation value of the electron coordinate  $r$ , affects the magnitude of the form factor. Hence, the contraction of the relativistic wave functions leads to smaller ionization cross sections at large ion velocities.

$K$ -shell ionization cross sections were calculated for Xe,

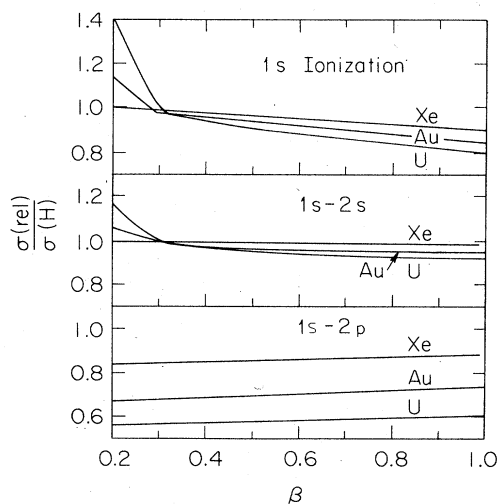


FIG. 1. Ratios of  $1s$ - $2s$  and  $1s$ - $2p$  excitation and  $1s$  ionization cross sections for Xe, Au, and U projectiles calculated with Dirac relativistic wave functions and with nonrelativistic wave functions.

Au, and U projectiles using the plane-wave Born formulation of Jamnik and Zupancic.<sup>16</sup> All multipoles up to  $l = 3$  were included. The ratios of cross sections calculated using Dirac wave functions and energies to those using nonrelativistic wave functions and energies are shown in Fig. 1 for  $1s$  ionization,  $1s$ - $2s$  excitation, and  $1s$ - $2p$  excitation. The excitation cross sections are relevant to studies of projectile x-ray production, which are discussed in a later paper in this series. The use of relativistic electronic wave functions clearly gives smaller ionization cross sections at high velocities. Part of this reduction is due to wave-function effects, but part is due to the use of the larger Dirac binding energies  $[(1 - s)mc^2 = 132 \text{ keV for U}]$  than nonrelativistic ones ( $Z^2/2 \text{ a.u.} = 115 \text{ keV for U}$ ). The dipole  $1s$ - $2p$  excitation cross sections tend to be more reduced than the monopole  $1s$ - $2s$  ones. Dipole  $1s$  ionization is dominant, but all multipoles up to  $l = 3$  contribute to the total ionization cross section. At small  $\beta$ , the relativistic cross sections increase, consistent with earlier calculations.<sup>17</sup>

### C. Distortion effects

For the high- $Z$  targets and high- $Z$  projectile ions used, the increased binding of the  $1s$  electron and/or the polarization of the inner-shell wave functions affect the  $K$ -shell ionization cross sections.<sup>7-10</sup> At low velocities, where the target and projectile electrons form molecular orbitals, the increased binding of the  $1s$  electron makes it more difficult to excite the electron, which gives smaller ionization cross sections.<sup>7,10</sup> At high velocities, molecular orbitals may not be formed, but the electron clouds are nevertheless strongly distorted by the perturbing potential. Since the potential is attractive, the electronic density is redistributed toward the perturbing nucleus, bringing the electron and perturber closer together, and increasing the probability of excitation.<sup>8</sup> For the relevant intermediate velocities ( $v \sim v_K$ ), Basbas *et al.*<sup>9</sup> have developed a formulation that interpolates between the low- and high-velocity regimes. At the heart of the theory a cutoff impact parameter  $b_c = c_K a_K$  ( $a_K$  is the Bohr radius of active electron and  $c_K$  is a constant) is assumed, below which binding effects are dominant and above which polarization effects dominate. At low velocities, ionization occurs mainly at impact parameters  $b < a_K$ , so binding effects should give reduced cross sections. At high velocities, ionization can occur at large impact parameters  $b > a_K$ , so the polarization effect gives increased cross sections. By changing the cutoff impact parameter, one changes the relative weighting of the binding and polarization effects. A value of  $c_K = 1.5$  was chosen in Ref. 9 to fit existing experimental target-atom ionization data. The measurements described in the following section suggest the need to use a larger value of  $c_K$  for projectile ionization for large perturbing charges, thus deemphasizing the polarization effect. However, the expressions for binding and polarization effects developed in Ref. 9 are valid strictly only in the region of small perturbations,  $Z_p \ll Z_t$  for target ionization or  $Z_t \ll Z_p$  for projectile ionization. In some of the present cases, where  $Z_p$  is approximately equal to  $Z_t$ , the applicability of this theory is no longer certain.

### D. Relativistic velocity effects

Most of the effects discussed in Secs. II A–II C occur for nonrelativistic projectiles with energies less than about 20 MeV/amu. At relativistic velocities, the perturbing Hamiltonian consists of the Coulomb interaction between the perturbing nucleus and electron and, in addition, a magnetic current-current interaction. The longitudinal part of the current-current interaction combines coherently with the Coulomb potential to give the cross section calculated with the nonrelativistic PWBA.<sup>5</sup> The transverse interaction between the current of the perturbing nucleus  $Z_x e \beta c$  and that of the electron  $e \alpha$  gives a cross section that increases as  $\ln \gamma^2 - \beta^2$  at large projectile kinetic energies  $(\gamma - 1)Mc^2$ . At high energies, the longitudinal part is constant, since it only depends on the ion velocity, which approaches a constant  $c$ . In the present cases, where  $\gamma \leq 1.2$ , the transverse contribution increases the ionization cross sections by less than 4%. For high- $Z$  target and projectile ionization, the transverse contribution should be calculated using Dirac wave functions (and including distortion effects), but because of the negligible magnitude of the transverse cross sections, the contributions were calculated with just the first-order plane-wave Born approximation.

### E. Screening effects

For projectile ionization by neutral target atoms, one must account for the screening of the perturbing nucleus by the target electrons.<sup>6,18–20</sup> This effect does not occur for target  $K$ -shell ionization by nearly bare projectiles. Paper II showed that the target screening reduced ionization and excitation cross sections for low- $Z$  projectiles by significant factors. The transverse cross sections were so much more reduced than the longitudinal ones, that they could be neglected for low- $Z$  projectiles with  $E_p \leq 2000$  MeV/amu.

To account for electronic screening on projectile ionization, Eq. (3) has to be modified as follows:<sup>6,18</sup>

$$\sigma_{1s} \sim \int_0^\infty d\epsilon \int_{q_0}^\infty \frac{dq}{q^3} |F(q)|^2 \left[ \left[ Z_t - |F_t(q, Z_t)| \right]^2 + Z_t - \sum_{i=1}^{Z_t} |F_{it}(q, Z_t)|^2 \right], \quad (8)$$

where

$$F_{it}(q, Z_t) = \langle \psi_i | \exp(i\mathbf{q} \cdot \mathbf{r}) | \psi_i \rangle \quad (9)$$

is the target form factor for the  $i$ th target electron occupying orbital  $\psi_i$ ,  $F_t = \sum_i F_{it}(q)$ , and the sum is over all bound target electrons. The first term in the brackets in Eq. (8) is due to the normal perturbing potential represented by  $Z_x^2$  in Eq. (3). The screening correction  $-|F_t(q, Z_t)|$  approaches  $Z_t$  at low  $q$  (corresponding to a fully screened potential) and vanishes at large  $q$ . The “antiscreeing” term,  $Z_t$ , accounts for the ionization of projectile electrons by  $Z_t$  separate target electrons. The final term is an antiscreeing correction factor that vanishes at large  $q$  and approaches  $Z_t$  at small  $q$ , canceling

out the antiscreeing term.<sup>6</sup> The net effect of these terms is that for small  $q$ , corresponding to excitation at large impact parameters where the projectile would see a neutral target atom, the perturbing charge is nearly zero, but for large  $q$ ,  $\sigma_{1s}$  varies as  $Z_t^2 + Z_t$ , where  $Z_t^2$  comes from the electron-target nucleus Coulomb potential and  $Z_t$  comes from  $Z_t$  separate electron-electron interactions.

For the present  $\sim 100$ -MeV/amu Xe ions,  $q$  is sufficiently large so that the calculated cross sections vary as  $Z_t^2 + Z_t$ . The binding and polarization effects affect the cross-section term proportional to  $Z_t^2$ , but not that associated with the  $Z_t$  separate electron-electron interactions. However, the theory of McGuire *et al.*<sup>18</sup> neglects kinetic energy constraints on the electron contributions. In 82-MeV/amu Xe collisions, the target electrons have a kinetic energy of  $\sim 45$  keV in the projectile frame, which is barely sufficient to ionize the Xe  $K$ -shell electron (binding energy 42 keV). Therefore, this electron contribution ( $Z_t \sigma_e$ ) is smaller than  $Z_t \sigma_B$ , where  $\sigma_B$  is the first Born cross section. We calculated this contribution using the formulas of Rudge and Schwartz<sup>21</sup> for electron-induced ionization cross sections  $\sigma_e$ , which agree reasonably well with experimental results near the threshold ionization energy.<sup>22</sup>

### F. Experimental effects

When one measures target  $K$ -vacancy production by bare projectiles, contributions due to the capture of target  $K$  electrons by the projectile are present<sup>11</sup> [as well as secondary processes discussed in paper I (Ref. 1)]. These contributions are difficult to estimate since one must calculate both the electron-capture cross sections and the number of vacancies in the projectile into which the target electrons can be captured. Assuming equilibrium target thicknesses and that only zero-, one-, or two-electron projectiles are present, the capture contribution to target  $K$ -vacancy production is given by

$$\sigma_{K \text{ capt}} = (\sigma_{KK} + \sigma_{KH})F_0 + (\frac{1}{2}\sigma_{KK} + \sigma_{KH})F_1 + \sigma_{KH}F_2, \quad (10)$$

where  $F_n$  is the measured equilibrium fraction of projectiles carrying  $n$  electrons,  $\sigma_{KK}$  is the capture cross section for single-electron capture from a fully occupied target  $K$  shell to an empty projectile  $K$  shell, and  $\sigma_{KH}$  is that where the  $K$  electron goes into  $L$ -shell and higher projectile orbitals. Equation (10) assumes that the projectile is in its ground state (which is approximately valid for Xe projectiles used in this work, as shown in a later paper in this series). We included all charge fractions with  $n > 2$  in the  $n = 2$  fraction, since  $\sigma_{KH}$  is not significantly reduced by the presence of one or two  $2s$  electrons (which only occurs for a small fraction of 82-MeV/amu Xe ions incident on high- $Z$  targets).<sup>2</sup>

When one determines projectile ionization cross sections by passing one- or two-electron, high- $Z$  projectiles through a thin foil and measuring the zero- or one-electron fractions, capture and projectile excitation play no role. The target atom cannot capture projectile electrons efficiently due to the lack of bound-state vacancies.

For high- $Z$  projectiles, excitation leads to the immediate radiative decay back to the ground state, because of the very short lifetime of excited states compared to the time between ionizing collisions. Therefore, the projectile does not change charge unless  $1s$  ionization occurs. We made numerical simulations for the present collisions using an 11-state model discussed in a later paper in this series, which includes excitation processes. The simulations show that as long as small target thicknesses are used, ionization cross sections measured using thin targets are exactly equal to the  $1s$  ionization cross sections.

### III. RESULTS

#### A. Projectile ionization

In all the cases studied the  $\text{Xe}^{52+}$  single-electron ionization cross section was equal to twice the  $\text{Xe}^{53+}$  ionization cross section within the experimental uncertainties shown in Figs. 2–4. To obtain the most accurate  $1s$  single-electron ionization cross section, we took the weighted average of the measured  $\text{Xe}^{53+}$  and one-half of the  $\text{Xe}^{52+}$  single-electron ionization cross sections. From the linear target-thickness dependence of the  $\text{Xe}^{54+}$  yield for a  $\text{Xe}^{52+}$  projectile, we could also search for two-electron ionization.

Figure 2 shows the single-electron and  $\text{Xe}^{52+}$  double-

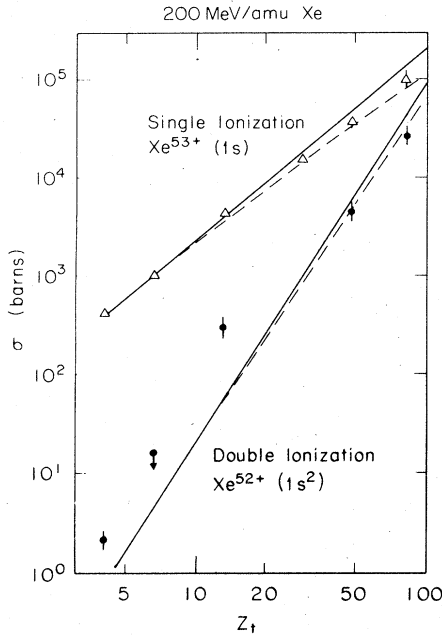


FIG. 2. Single-ionization  $1s$  cross sections and double-ionization cross sections for  $\text{Xe}^{52+}$  ( $1s^2$ ) plotted against target atomic number for 200-MeV/amu Xe. In the plane-wave Born approximation, the single-ionization cross sections increase as  $Z_t^2 + Z_t$  (solid line). Wave-function distortion effects reduce the single-ionization cross sections (dashed lines). The computed double-ionization cross sections (solid lines) are a factor of  $0.26 Z_t^2 / Z_p^2$  below the single-ionization ones (dashed lines). The distortion effects further reduce the double-ionization cross sections, as described in the text. The lower limit of this reduction on double ionization is shown by the dashed lines.

electron ionization cross sections for 200-MeV/amu projectiles. We discuss double-electron ionization in Sec. III C. The solid lines in Fig. 2 were calculated using relativistic electronic wave functions and including transverse excitation and target-screening effects, but not the wave-function distortion effects. The calculated cross sections increase roughly as  $Z_t^2$ . At low  $Z_t$ , the calculations are in good agreement with experiment, but are higher at large  $Z_t$ . We hypothesize that the discrepancy at large  $Z_t$  is due to distortion effects. To examine the wave-function distortion effects more carefully, we obtained reduced cross sections using

$$\sigma_{\text{red}} = (\sigma_{\text{meas}} - Z_t \sigma_e) Z_t^{-2}, \quad (11)$$

where  $\sigma_{\text{meas}}$  are the measured cross sections, and  $Z_t \sigma_e$  are the electron-induced contributions.<sup>21</sup>

The theory of Basbas *et al.*<sup>9</sup> with  $c_K = 1.5$  predicts that for these collisions, the reduced cross sections increase with  $Z_t$ , because the polarization effects are more important than binding effects (thin solid lines in Fig. 3). Clearly, this theory disagrees with the bulk of the high- $Z_t$  data. To obtain better agreement, we semiempirically increased the cutoff impact parameter  $b_c = c_K a_K$  to reduce the polarization effect and enhance the binding effect. Using  $c_K = 3$  brings the theory into better agreement with experiment (dashed lines in Figs. 2–4). We found by trial and error that no improvement is obtained with other values of  $c_K$ ; the value  $c_K = 3$  gives the best overall compromise fit.

The theory of Basbas *et al.*<sup>9</sup> is usually applied to calculate target inner-shell vacancy production where  $Z_x \ll Z$  ( $Z_p \ll Z_t$ ). In near-symmetric collisions at the present relativistic velocities ( $v/v_K \sim 1$ ), a large number of

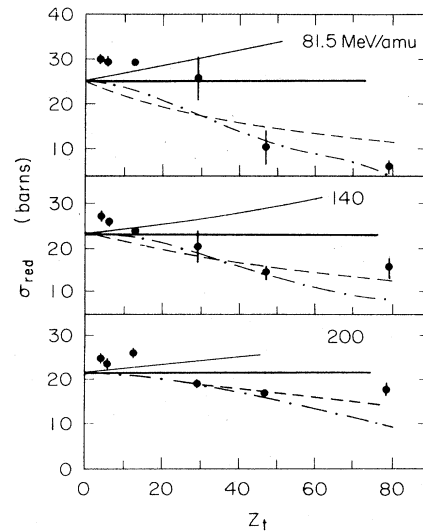


FIG. 3. Reduced projectile ionization cross sections for 81.5-, 140-, and 200-MeV/amu Xe ions plotted against target atomic number. The thick solid lines are PWBA calculations, and the thin solid lines were calculated with the theory of Basbas *et al.* (Ref. 9) for binding and polarization effects using  $c_K = 1.5$  (solid lines) and  $c_K = 3$  (dashed lines). The chain curves are Glauber-approximation calculations.

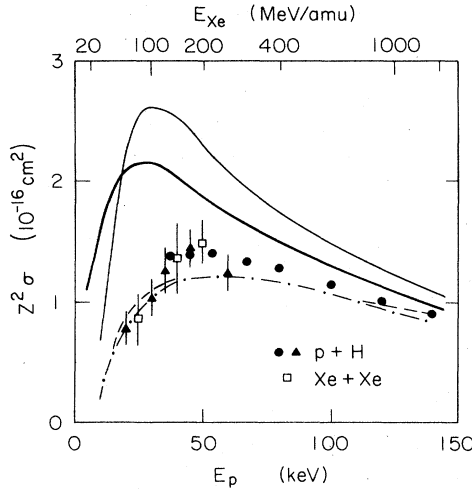


FIG. 4. Scaled  $p + H$  and  $Xe + Xe$  1s ionization cross sections plotted against proton kinetic energy. The PWBA (thick solid line), Basbas theory using  $c_K = 1.5$  (thin solid line) and  $c_K = 3$  (dashed line), and Glauber theory (chain curve) results are shown. The  $p + H$  data points are from Park (Ref. 26) (triangles) and Shah and Gilbody (Ref. 25) (closed circles). Some of Park's points for  $E_p > 50$  keV have been omitted.

theories have been developed to calculate ionization in  $H^+ + H$  collisions.<sup>23,24</sup> We can obtain reduced cross sections for  $Xe + Xe$  collisions by interpolating between measurements for  $Z_t = 47$  and 79. By plotting  $Z^2\sigma$  ( $Z = Z_t = Z_p$ ) versus the proton kinetic energy, we can then compare  $Xe + Xe$  with  $p + H$  ionization cross sections<sup>25,26</sup> at the same value of  $v/v_K$ . The Xe energy scale in Fig. 4 is related to the  $p + H$  one using

$$E_{Xe} = 932 \left[ \frac{1}{(1 - \beta^2)^{1/2}} - 1 \right], \quad (12)$$

$$\beta = \frac{54}{137.037} (0.04025 E_p)^{1/2},$$

where  $E_{Xe}$  is in MeV/amu and  $E_p$  is the proton energy in keV. The Xe energy scale ends at  $E_p = 160$  keV where  $\beta$  approaches unity. This type of scaling is exact for symmetric collisions ( $Z = Z_x$ ) in the PWBA (Ref. 3) and in molecular perturbed stationary-state calculations for one-electron systems.<sup>27</sup> Since the target electron antiscreening effects have been removed in deriving the reduced cross sections, and the electronic relativistic effects and transverse excitation are negligible, this scaling should be nearly exact in the present  $Xe + Xe$  collisions. The measured  $Xe + Xe$  points are clearly in good agreement with the measurements of Shah and Gilbody<sup>25</sup> and Park.<sup>26</sup> We also show in Fig. 4 the Basbas calculations using  $c_K = 1.5$  (thin solid line) and  $c_K = 3$  (dashed line). Those using  $c_K = 3$  are in reasonable agreement with the measured  $p + H$  and  $Xe + Xe$  cross sections, but the original theory with  $c_K = 1.5$  significantly overestimates the  $p + H$  cross sections.

Of the many theories that have been developed to calculate ionization in  $p + H$  collisions, that giving the overall best agreement with experiment is probably the Glauber approximation. (See Park<sup>23</sup> and McGuire<sup>24</sup> for further

comparisons.) One of the main things the Glauber approximation<sup>24,28</sup> for 1s ionization does is that the unitarity of the ionization amplitude is preserved. For symmetric collisions near  $v \sim v_K$ , the first-order semiclassical approximation<sup>14</sup> predicts ionization probabilities that are greater than  $\frac{1}{2}$  in small impact-parameter collisions. Such large probabilities deplete the initial 1s occupation amplitude  $a_0(t)$ , assumed to be unity for all times  $t$  in first-order theories like the PWBA, thus leading to smaller ionization probabilities and cross sections. It is not clear how the physical ideas behind this approximation can be compared with the physical ideas of binding and polarization in the theory of Basbas *et al.*<sup>9</sup>

Figure 3 compares Glauber calculations of the reduced ionization cross sections with experiment (chain curve). The Glauber theory agrees well with experiment (and with our empirical modification of the theory of Basbas *et al.*<sup>9</sup> using  $c_K = 3$ ) for  $Z_t > 20$ . The main disagreement occurs for  $Z_t < 20$ , where the data points are higher than the Born calculations. The Glauber cross sections always lie below the Born ones, possibly indicating the lack of elements in the theory that can be physically connected with the polarization effect. The present low- $Z_t$  data points do not agree with the original Basbas theory either. These points are most affected by target antiscreening. If the full Born electron-electron contribution  $Z_t\sigma_B$  were subtracted from the measured cross sections instead of  $Z_t\sigma_e$  ( $\sigma_e$  incorporating threshold effects), the reduced cross sections would be in better agreement with the Born and Glauber calculations at low  $Z_t$  (e.g., the 82-MeV/amu  $Xe + Be$  reduced cross sections are reduced by 25%). These considerations suggest the possibility that the discrepancy at low  $Z_t$  may be due to our lack of a complete theory of target screening and antiscreening near the electron ionization threshold velocity.

## B. Target ionization

Figure 5 compares the Basbas theory<sup>9</sup> to measurements of target  $K$ -vacancy production by 82- and 197-MeV/amu Xe ions. The cross sections for fixed projectile charge and velocity fall off rapidly with  $Z_t$  between Ni and U. In order to plot the cross sections on a linear scale comparable to Fig. 3, we multiplied the  $K$ -vacancy production cross sections by  $(Z_t/Z_p)^4$ . The resulting cross sections have a peak at the value of  $Z_t$  where the ion velocity is equal to the target  $K$ -electron velocity ( $Z_t = 54$  at 82 MeV/amu and  $Z_t = 77$  at 197 MeV/amu). This peak approximately reflects the peak seen at  $\eta_K \approx 1$  [Eq. (5)] in reduced ionization cross sections plotted against  $\eta_K$  ( $\eta_K$  increases from right to left in Fig. 5, however).<sup>15,29</sup> The capture contributions were calculated using measured equilibrium fractions and the eikonal theory of nonradiative capture.<sup>2</sup> The solid and dashed lines in Fig. 4 were calculated using  $c_K = 1.5$  and 3, respectively, and include the capture contribution (shown as dash-dot lines). For target  $K$ -vacancy production, the experimental results lie closer to the original theory of Basbas *et al.*<sup>9</sup> with  $c_K \approx 1.5$ , except at low  $Z_t$ .

In principle, a theory of the binding and polarization effects should consistently predict target and projectile

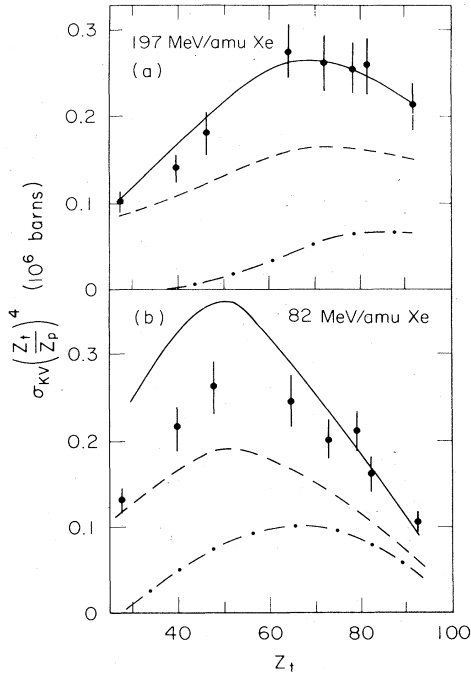


FIG. 5. Target  $K$ -vacancy production cross sections for 82- and 197-MeV/amu Xe projectiles, multiplied by  $(Z_t/Z_p)$  (Ref. 4). The calculated capture contributions to the total  $K$ -vacancy production cross section are shown by the chain curves. The solid and dashed lines were calculated using the Basbas theory of polarization and binding effects (Ref. 9) using  $c_K = 1.5$  and 3, respectively, and include the capture contributions.

ionization cross sections; one should not be required to use a different cutoff impact parameter for the two different cases. Target-nucleus screening effects on the distortion effects on projectile ionization may differ from those on target ionization, but our calculations show that screening is negligible at high perturbing charge. We are not prepared to conclude that a discrepancy exists between the results for projectile and target ionization though. We estimate that the theoretical capture cross sections in Fig. 5 are uncertain within a factor of 2.<sup>2</sup> If the capture cross sections were doubled, the Basbas theory using  $c_K = 3$  would fit the data better. Also, the independent-electron theory used to calculate the capture contribution, Eq. (10), neglects processes like the capture of target  $L$  electrons followed by target  $K$  to  $L$  excitation, which may contribute significantly to the measured target  $K$ -vacancy production cross sections.<sup>30</sup>

### C. Double ionization

In the independent-electron approximation,<sup>13</sup> the probability of simultaneously ionizing two  $1s$  electrons in a collision with impact parameter  $b$  is just the square of the single-electron ionization probability  $P(b)$  (per electron). In the semiclassical approximation the double-ionization cross section is given by

$$\sigma(1s^2) = \int_0^\infty db \, 2\pi b P^2(b). \quad (13)$$

Using semiclassical approximation tables of the reduced impact-parameter dependence of  $P(b)$ ,<sup>14</sup> the ratio of the double-ionization cross section to the single-ionization one is given by

$$\frac{\sigma(1s^2)}{\sigma(1s)} = D \frac{Z_t^2}{Z_p^2}, \quad (14)$$

where  $D$  is equal to 0.36, 0.30, and 0.26 for 82-, 140-, and 200-MeV/amu Xe projectiles.

The semiclassical calculations of Hansteen *et al.*<sup>14</sup> are, like the plane-wave Born approximation ones, based on first-order perturbation theory. Target-screening, transverse-excitation, Dirac wave-function effects should not affect the ratio of the double-ionization to single-ionization cross sections significantly. Binding and polarization do affect the ratio, however, since the ionization probability at every impact parameter is reduced.<sup>31</sup> Since the constant  $D$  can be viewed as the average value of the ionization probability over the range of impact parameters contributing to the ionization cross section, a reduction of the probability at every impact parameter  $b$  due to distortion effects, should reduce the ratio of double ionization to single ionization. The lower limit to this reduction is the ratio of the single-ionization cross section calculated with binding and polarization to that without this correction ( $\sigma_{\text{red}}/\sigma_B$  in Fig. 3). The application of this reduction factor gives the dashed curves in Fig. 2.

For mylar targets in 200-MeV/amu Xe collisions, only an upper limit to the measured double-ionization cross sections could be obtained. The data for Ag and Au targets are in agreement, within large experimental uncertainties, with the theory including the distortion effects.

It would be desirable to increase the accuracy of the double-ionization measurements, which was determined here by poor counting statistics due to a limited amount of counting time. The ratio of the double-ionization to single-ionization cross sections indirectly tests the validity of the semiclassical-approximation calculations of  $P(b)$  for relativistic heavy ions. It is presently technically not feasible to measure directly the impact-parameter dependence of projectile ionization at the Bevalac accelerator, so the double-ionization cross section is the only means available to obtain information about the impact-parameter dependence of the ionization probability.

## IV. CONCLUSIONS

Measurements of projectile ionization cross sections using one-electron, high- $Z$ , nearly relativistic heavy ions represent the cleanest method of studying wave-function distortion effects at intermediate velocities ( $v \approx v_K$ ). Uncertainties in target  $K$  x-ray measurements due to x-ray fluorescence yields, contributions of secondary processes, and of  $K$ -electron capture by the projectile are completely absent when one measures projectile charge-changing cross sections at small target thicknesses.

The most significant results of the present measurements are that, for the first time, one can probe diverse theories of ionization ranging from those that have been used exclusively in near-symmetric collisions such as

$p + H$  to those where the perturbing charge  $Z_x$  is much less than  $Z$ . In particular, one can examine theories such as that of Basbas *et al.*<sup>9</sup> which have previously been used only for  $Z_x \ll Z$  to see whether they are also applicable if  $Z_x$  is approximately equal to  $Z$ . Likewise, one can test theories like the Glauber approximation in highly asymmetric collisions where  $Z_x \ll Z$ . Our measurements using Xe ions are for velocities where  $v \sim v_K$ , as is the case for  $\sim 25$  keV protons. This region of  $Z_p$  and  $v$  is ideal because electronic relativistic effects are relatively unimportant, the required projectile velocity is not so relativistic that transverse excitation is significant, and the momentum transfer is large enough that electronic screening of the perturbing target nucleus is nearly negligible. In this region, the scaled Xe + Xe cross sections agree well with  $p + H$  ones.

The present results for projectile  $K$ -shell ionization suggest that the Basbas theory with  $c_K = 1.5$  underestimates the binding effect at large perturbing charges. A larger binding-polarization cutoff impact parameter is needed for projectile ionization, which is inconsistent with the present and many other measurements of target  $K$ -shell ionization, but is consistent with  $p + H$  ionization. The fact that the binding effect is underestimated or the polarization effect is overestimated at large  $Z_x$  may be due to

the breakdown of the theory at large perturbing charge. The theory frequently truncates terms in  $Z_x$  beyond second order. Possibly the truncated higher-order terms for the binding effect outweigh those for the polarization effect. Possibly, the apparent good agreement between target ionization at nonrelativistic velocities and the original theory of Basbas *et al.*<sup>9</sup> is due to inaccurate estimates of target electron-capture contributions. The present evidence of projectile double-electron ionization is reasonably consistent with calculations in the independent-electron approximation.

#### ACKNOWLEDGMENTS

This work was supported in part by National Science Foundation Grant No. PHY-83-13676 (Stanford University), by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 (Lawrence Berkeley Laboratory), and by the U.S. Department of Energy under Contract No. DE-AC02-76CH00016 (Brookhaven National Laboratory). We are indebted to Jim McGuire for making Glauber calculations of  $1s$  ionization, shown in Fig. 3.

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