Coupling of a high-sensitivity superconducting amplifier to a gravitational-wave antenna

P. Carelli
Istituto di Elettronica dello Stato Solido, Consiglio Nazionale delle Ricerche via Cineto Romano 42, 00156 Roma, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, 00185 Roma, Italy

M. G. Castellano
Istituto di Fisica dello Spazio Interplanetario, Consiglio Nazionale delle Ricerche via G. Galilei 00044 Frascati, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, 00185 Roma, Italy

C. Cosmelli
Dipartimento di Fisica, Università La Sapienza, 00185 Roma, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, 00185 Roma, Italy

V. Foglietti
Istituto di Elettronica dello Stato Solido, Consiglio Nazionale delle Ricerche, via Cineto Romano 42, 00156 Roma, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, 00185 Roma, Italy

I. Modena
Dipartimento di Fisica, Università Tor Vergata, Roma, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, 00185 Roma, Italy

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We report the first measurements on a dc superconducting quantum interference device (dc-SQUID) coupled to a gravitational wave detector cooled down to liquid-helium temperature. The overall system is composed of a 2.3-ton aluminum bar and a resonant capacitive transducer coupled to the dc-SQUID amplifier by means of a high-ratio superconducting transformer. The SQUID used in this experiment is a planar device with a multiple-loop geometry having both low noise and good coupling with the external world. The electromechanical equivalent circuit of the system, which is essentially a set of three coupled oscillators, is analyzed in detail in its normal modes and an expression for the effective noise temperature for each mode is given. Measurements are reported on the SQUID alone, on the SQUID plus the high-ratio transformer, and on the SQUID coupled to the antenna in parallel with a field-effect transistor preamplifier.

INTRODUCTION

We have developed a resonant capacitive transducer[1] for detecting the mechanical vibrations of an antenna with the aim of coupling such a detector to a high-sensitivity dc superconducting quantum interference device (dc-SQUID). The purpose of this paper is to discuss the problems relative to such a coupling and to show the first results of a calibration performed with the 2270-kg antenna operating at the CERN laboratories.

In the last few years many workers have developed dc-SQUID's (Refs. 2–4) which can be used as high-sensitivity amplifiers. We want to show that our SQUID's,[5,6] developed by one of us using the fabrication facility of the National Bureau of Standards at Boulder, Colorado, are particularly suitable for this purpose. These SQUID's are planar devices using a multiple-loop geometry which have both low noise and good coupling with the input coil. In fact with this geometry the inductance of the SQUID can be reduced to a few pH, while the coupling with an input coil of a few μH, patterned on the same chip, can be made large enough to match. Besides, the input coil need not be superimposed on the SQUID, in this way avoiding the large spurious capacitive coupling which can be found in other SQUID's.

The resonant capacitive transducer is a low-dissipation capacitance of about 5 nF, mechanically resonating at a frequency of about 900 Hz. The input impedance of the dc-SQUID at such a frequency is essentially an inductance of a few μH. It is clear that there is a large electrical mismatch at the operating frequency. To overcome this problem a high-ratio superconducting transformer was built and inserted between the two impedances to be matched.

Moreover, an appropriate decoupling capacitance must be used, in order to bias the capacitive transducer with the static electric field necessary to its operation. The electrical losses of the decoupling capacitance introduce in the circuit a voltage-noise source, which at present is expected to by far exceed the noise voltage of the SQUID. The ratio between the antenna strain and the resulting flux signal coupled to the SQUID can be evaluated from the equivalent circuit of the system as well as from direct calibration measurements.

In the first part we describe the experimental apparatus, with particular attention to the SQUID and related circuitry. The antenna, the transducer, and the cryogenic apparatus are described in other papers.[7] In the second part we analyze the system in detail, and we calculate its noise temperature for the detection of short bursts of gravitational waves. In the last part we show the experimental results of the calibration.

EXPERIMENTAL APPARATUS

The 2270-kg antenna is a cylindrical aluminum bar 297-cm long suspended inside a vacuum chamber in a helium dewar. The dewar[7] consists of a liquid-nitrogen container, a shield cooled by the helium gas, and a helium container which surrounds the vacuum chamber of the antenna.

The experimental setup including the SQUID amplifier is shown in Fig. 1. The transducer[1] is located on the end face of the antenna and consists of a high-Q aluminum mechanical oscillator, "mushroom" shaped, which forms one side of a 4.5 nF capacitance (C). Its mechanical resonance (first bending mode) has a frequency of 916 Hz at $T=4.2$ K and is equal to the antenna mechanical resonance within 1 Hz. The transducer is polarized through a high-value resistor R_0 (10 GΩ) at a voltage V_0 as high as 300 V, corresponding to a maximum electrical field E of 6 MV/m. Through this electrical field, the vibrations of the capacitor plate are converted into electrical signals. The dc losses in the circuit are so low that once the battery was disconnected no significant voltage decay was observed overnight.

One end of the transducer is connected in series to a decoupling capacitance C_d and to the primary L_0 of a high-ratio superconducting transformer. The decoupling capacitance consists of two very flat aluminum plates with a 50-μm Teflon foil in between; the plate diameter is 17 cm and the capacitance 5 nF. The transformer was made by winding 7950 turns of 50-μm niobium wire around a polyvinylchloride (PVC) support. On the external part are five turns of 1-mm Nb wire that form the secondary. The transformer has an external diameter of 4.5 cm, a length of 1.2 cm, and is put inside a double-lead soldered can of 8.7-cm diam and 5.4-cm length. The primary (L_0) and the secondary (L_i) of the transformer have, respectively, inductances of 2.3 H and 1.6 μH, the latter being practically equal to the SQUID input impedance ($L_{\text{in}}=1.5$ μH), with a coupling factor (k) of 0.71. The leads from the secondary are connected via a superconducting connector to the contacts of the input

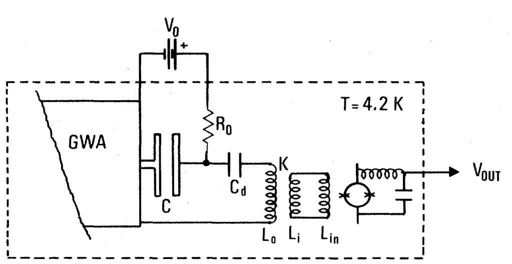

FIG. 1. Scheme of the experimental apparatus. GWA is the gravitational-wave aluminum antenna, C the resonant transducer biased through R_0 at voltage V_0, C_d is a high-Q decoupling capacitance, L_0 is the primary of the high-ratio transformer, L_i is the secondary, and L_{in} the input impedance of the SQUID. Across the SQUID there is a resonating circuit used to match the low impedance of the SQUID with the room-temperature preamplifier.

coil patterned on the SQUID chip. These connections are obtained by pushing the silicon chip against two springs made of niobium wire, realizing in this way the necessary superconducting contact. Other Cu-Be springs provide the remaining contacts for the SQUID: bias current, voltage output, modulation, and feedback flux.

The SQUID used for the experiment is a multiloop thin-film dc-SQUID. The SQUID inductance is made up of 32 square loops 400 μm on a side, connected in parallel. The 20-turn input coil, positioned all around the SQUID, has an inductance of 1.5 μH and a coupling factor of 0.5. The device is described in Ref. 5. Even though the last type of SQUID[6] has better noise performance, at the time of the run it was not completely tested and we felt it was not yet reliable enough to be used in this experiment. In fact the chip must be inserted in the cryostat many weeks before the beginning of the run. In future runs it will be possible to insert the newly developed SQUID in the old SQUID holder.

The SQUID output is matched to the room-temperature preamplifier through a tank circuit resonating at about 100 kHz with a Q of about 100. To linearize the SQUID output a modulation signal at the resonating frequency of the tank is applied to a coil coupled to the SQUID. The output is demodulated with a lock-in, integrated, and fed back to the SQUID through another coil. The chip and its contacts, the superconducting connector, and the tank circuit are inside three cavities milled in a niobium cylinder of 30-mm diam and 30-mm length. Niobium was chosen to provide the necessary shielding from magnetic disturbances. This apparatus was first characterized in a small cryostat and then coupled to the antenna.

In the cryostat at CERN this apparatus is inside the vacuum chamber containing the antenna. The SQUID assembly, the superconducting transformer, the decoupling capacitance, and the polarizing resistor are put on an aluminum block of 11 kg, which provides a mechanical isolation of about 60 dB and thermal contact with the helium bath. The cables connecting the SQUID to the room-temperature electronics are twisted pairs, shielded with thin aluminum foil, and mechanically clamped to the aluminum block. The total length is about 6 m. All leads are filtered at low temperature with homemade feed-through C-R-C filters. At the top of the cryostat the end of the cable is connected to the electronics through low-pass filters. Instead of the standard room-temperature electronics, built on purpose for the SQUID and described in Ref. 8, we used commercial instruments in a similar scheme: a PAR model No. 113 as differential preamplifier, a lock-in amplifier (Ortholoc Model No. SC9505) and a commercial oscillator. Even though the SQUID performance is limited by the PAR model No. 113 noise voltage, the greater flexibility of commercial instruments is undoubtedly an advantage in this calibration of the overall system.

SYSTEM ANALYSIS

The system of the two mechanical oscillators, the antenna, and the transducer, coupled to the SQUID by means of the variable capacitor C and the superconducting transformer can be analyzed as a combination of three different blocks: a mechanical block, a transduction block, and an electrical block. The equivalent circuit[9] is shown in Fig. 2.

The mechanical part is represented in Fig. 2 using an electrical symbology, since the mechanical equations of motion are formally identical to electrical equations, with the correspondence:

mass↔inductance ,

stiffness↔capacitance^{-1} ,

mass/dissipation coefficient↔resistance ,

velocity↔current .

In Fig. 2, the mechanical parameters of the antenna and of the transducer are the reduced masses M_A and m_T, the stiffness is k_A and k_T, and the dissipation coefficients M_A/τ_A and m_T/τ_T; in particular, k_T is the resulting stiffness of the transducer in full operating conditions, i.e., including the reducing effect of the polarizing electrical field E on the overall stiffness of the transducer.

The configuration of the mechanical part in Fig. 2 takes into account that in our case the system is sensitive only to the difference between the equivalent displacement of the antenna and of the transducer. The electromechanical transduction block converts the mechanical parameters into electrical parameters by means of the factor $(1/EC)^2$, E being the electrical biasing field of the transducer and C its capacitance; the effect of this part is formally equivalent to inserting an ideal transformer with dimensional transforming ratio EC:1 between the mechanical and the electrical blocks. It is represented in Fig. 2 following the analogy.

Finally, the electrical block is composed of the biasing resistor R_0, the transducer capacitance C, the decoupling capacitance C_d with its loss R_d, the superconducting transformer, and the SQUID.

The two capacitors C and C_d together with L_0 form a third oscillator; so, when the transducer is polarized and coupled to the antenna, the system is a set of three coupled oscillators, the third of which can be tuned to the others by varying on values of C, C_d, and L_0.

Reversing the electromechanical analogy we decouple the resulting system in normal modes. The total system is now equivalent to that shown in Fig. 3, with the electrical part reduced to an equivalent mechanical oscillator. The effect of the superconducting transformer can be taken into account by modifying the primary inductance from L_0 to $L_0'=L_0[1-k^2L_i/(L_i+L_{\rm in})]\cong L_0(1-k^2/2)$ and changing the transforming ratio of the superconducting transformer into an effective value of $N'=(k/2)(L_0/L_i)$. The equations of motion for the antenna, transducer, and electrical oscillator are

$$M_A\ddot{x}+x(m_T\omega_T^2+M_A\omega_A^2)-m_T\omega_T^2y-CE^2z+\frac{M_A\dot{x}}{\tau_A}+\left[\frac{m_T}{\tau_T}+R_cC^2E^2\right](\dot{x}-\dot{y})-R_cC^2E^2=f_x\ ,$$

$$m_T\ddot{y}+m_T\omega_T^2(y-x)+CE^2z+(\dot{y}-\dot{x})\left[\frac{m_T}{\tau_T}+R_cC^2E^2\right]+R_cC^2E^2\dot{z}=f_y\ , \qquad (1)$$

$$C^2E^2L_0'\ddot{z}+zCE^2\left[1+\frac{C}{C_d}\right]+CE^2(y-x)-xCE^2-(\dot{x}-\dot{y})R_cC^2E^2+\dot{z}C^2E^2(R_c+R_d)=f_z\ .$$

Here $x(t)$ and $y(t)$ are the usual mechanical coordinates of the antenna and of the transducer and $z(t)$ is an equivalent mechanical coordinate such that the current $I(t)$ flowing in the primary of the superconducting transformer is given by $ECz(t)$; ω_A and ω_T are the resonant frequencies of the uncoupled antenna and transducer, and R_c is the frequency-dependent resistance in series with C equivalent to R_0:

$$R_c=1/(\omega C)^2R_0\ . \qquad (2)$$

It should be noted that Eqs. (1) have been obtained for the case that the motion of the transducer is parallel to the end face of the antenna; nevertheless, they can be extended with little effort to the real case of a transducer vibrating in the first bending mode.[10] The current in the SQUID is

$$I_{\rm SQUID}=ECN'\dot{z}(t)=EC\frac{k}{2\sqrt{L_0/L_i}}\dot{z}(t)\ . \qquad (3)$$

To calculate the normal modes, we neglect for the moment the dissipation terms and diagonalize the resulting characteristic matrix M associated with Eq. (1). The eigenfrequencies of the system, ω_+, ω_0, and ω_-, can be

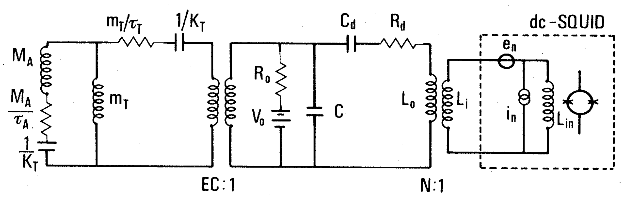

FIG. 2. Electromechanical equivalent circuit of the system. The first network represents the mechanical part of the antenna and of the transducer, the second network is the electrical part of the transducer with the transformer, and the third one represents the input of the SQUID with the equivalent noise generators.

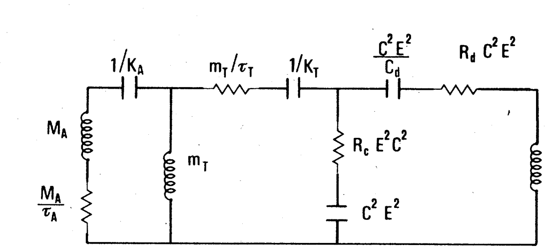

FIG. 3. Mechanical equivalent circuit for the system of the three-coupled-oscillators (antenna, transducer, and electrical) resonating circuit.

calculated from the solution of $\det M=0$. The corresponding eigenvectors have components a_x^i, a_y^i, and a_z^i (where $i=+1,0,-1$ from now on will be the mode index) and are normalized as follows:

$$M_A(a_x^i)^2+m_T(a_y^i)^2+C^2E^2L_0'(a_z^i)=1 \; .$$

They are given by

$$a_x^i=\left[m_T\left[\frac{-M_A}{m_T}+\frac{M_A}{m_T}\frac{\omega_A^2}{\omega_i^2}\right] + \frac{L_0'}{E^2}[m_T\omega_T^2+(\omega_i^2-\omega_T^2)(-M_A\omega_i^2+M_A\omega_A^2)/\omega_i^2]^2 + M_A\right]^{-1/2} ,$$

$$a_y^i=a_x^i\left[\frac{-M_A}{m_T}+\frac{M_A\omega_A^2}{m_T\omega_i^2}\right] , \tag{4}$$

$$a_z^i=a_x^i[m_T\omega_T^2+(1-\omega_T^2/\omega_i^2)(-M_A\omega_i^2+M_A\omega_A^2)/CE^2] \; .$$

The displacements $x(t)$, $y(t)$, and $z(t)$ can now be written in the most general case as a superposition of the three normal modes, excited with amplitude n_i:

$$x(t)=\sum_{i=0,+,-}\eta_i a_x^i e^{j\omega_i t} ,$$

$$y(t)=\sum_{i=0,+,-}\eta_i a_y^i e^{j\omega_i t} , \tag{5}$$

$$z(t)=\sum_{i=0,+,-}\eta_i a_z^i e^{j\omega_i t} .$$

For the current in the SQUID we get the following expression, which will be useful later:

$$I_{\text{SQUID}}=EC\frac{k}{2\sqrt{L_0/L_i}}\dot z(t)=EC\sqrt{L_0/L_i}(\dot\eta_+(t)a_z^+ + \dot\eta_0(t)a_z^0+\dot\eta_-(t)a_z^-)k/2 \; , \tag{6}$$

where the time-dependent part has been included in the η_i. By means of the coefficients a_x^i, a_y^i, and a_z^i, we build up the orthogonal transformation matrix A:

$$A=\begin{bmatrix} a_x^+ & a_x^0 & a_x^- \\ a_y^+ & a_y^0 & a_y^- \\ a_z^+ & a_z^0 & a_z^- \end{bmatrix} , \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix}=A\begin{bmatrix} \eta_+ \\ \eta_0 \\ \eta_- \end{bmatrix} \tag{7}$$

which, applied to the equations of motion (1), diagonalizes the characteristic matrix M and decouples the system equations into

$$\ddot\eta_i(t)+\omega_i^2\eta_i(t)=0 \tag{8}$$

which have the solution $\eta_i(t)=\eta_i e^{j\omega_i t}$. If the system is forced, the same transformation A must be applied to the forcing terms of Eq. (1) in order to get the forcing terms for the decoupled equations:

$$\begin{bmatrix} F_+ \\ F_0 \\ F_- \end{bmatrix}=A^T\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} , \tag{9}$$

where A^T is the transpose of A. The dissipation matrix D [which contains the coefficients of the dissipative terms in Eq. (1)] will not be diagonalized exactly by the transformation A, but, due to the high quality factors of the three oscillators, the off-diagonal terms are expected to be small, while the normal mode frequencies remain the same. So each mode can be characterized by an overall $Q^i=\omega_i\tau_i$, where the τ_i's are given by the diagonal terms of the transformed matrix $D'=A^TDA$:

$$1/\tau_i=\frac{M_A(a_x^i)^2}{\tau_A}+\frac{m_T(a_x^i-a_y^i)^2}{\tau_T}+C^2E^2R_d(a_z^i)^2 + R_cC^2E^2(a_x^i-a_y^i-a_z^i)^2 \; . \tag{10}$$

Due to the finite value of the Q^i's, in the frequency domain each mode is spread from a $\delta(\omega)$ to a Lorentzian curve centered on the normal-mode frequency and having a width of ω_i/Q^i. We can then rewrite Eq. (7) introducing the dissipative and the forcing terms as follows:

$$\ddot\eta_i(t)+\frac{\dot\eta_i(t)}{\tau_i}+\omega_i^2\eta_i(t)=F_i(t) \; . \tag{11}$$

In the following sections we will evaluate for each mode the signal and the noise arising from the oscillators and from the SQUID.

SIGNAL CALCULATION

A short burst of gravitational waves (GW) impinging on the antenna is equivalent to a forcing term $f_0\delta(t)$ for the motion equation of the antenna. The corresponding forcing terms for the normal-mode equations are [Eq. (9)]

$$F^+(t)=f_0\delta(t)a_x^+ ,$$

$$F^0(t)=f_0\delta(t)a_x^0 , \tag{12}$$

$$F^-(t)=f_0\delta(t)a_x^- .$$

Equations (12) with these forcing terms have the solution

$$\eta_i(t)=f_0a_x^i\sin(\omega_i t)e^{-t/2\tau_i}/\omega_i \; . \tag{13}$$

Inserting the $\eta_i(t)$ in Eq. (6) and recalling that the exponential term is almost constant during an oscillation period, we get the required expression for the signal current in the SQUID. In a range of frequency near any normal mode, the initial signal energy in the SQUID is simply

$$E_{\text{SQUID}}^i=L_{\text{in}}(I_{\max}^i)^2/2 = (ECk/2)^2L_0f_0^2(a_x^i)^2(a_z^i)^2/2 \; . \tag{14}$$

Recalling that ΔE, the mechanical energy deposited by the GW into the antenna, is equal to $f_0^2/2M_A$,[11] we can evaluate the effective β_i for each mode, β_i being defined as the ratio of the electrical energy available at the input of the amplifier to the mechanical energy deposited into the antenna, as

$$\beta_i = E^i_{\rm SQUID}/\Delta E = (ECk/2)^2 L_0 M_A (a^i_x)^2 (a^i_z)^2 \ . \quad (15)$$

It should be noted that β_i will be the same for the three modes only if the three oscillators have the same resonant frequency.

Using β_i we can easily write the signal energy into the SQUID as a function of the "temperature" T_x of the signal:

$$E^i_{\rm SQUID} = \beta_i \Delta E = k_B T_x \beta_i \ , \quad (16)$$

k_B being the Boltzmann constant.

NOISE CALCULATION

First we calculate the resonant noise due to the Brownian motion of the three oscillators. If all the elements of the equivalent circuit are at the same thermodynamic temperature T, we can use the equipartition theorem to calculate the mean thermal energy associated with each mode:

$$\omega_i^2 \langle \eta_i^2(t) \rangle /2 = \langle \dot{\eta}_i^2(t) \rangle /2 = k_B T/2 \quad (17)$$

and, using (6), we can get the integrated value of the narrow-band noise current in the SQUID

$$I^i_{\rm nb} = E^2 C^2 (a^i_z)^2 k_B T (k/2) L_0 / L_i \quad (18)$$

in units of A^2/Hz. From the analysis of the circuit of Fig. 2 it can be shown that the voltage noise e_n of the SQUID contributes to the noise at the input of the SQUID with a wide-band term

$$I^2_{\rm wb}(e_n) = e_n^2/(2\omega_i L_{\rm in})^2 \quad (19)$$

in units of A^2/Hz and with a resonant term that is usually negligible. In fact one must compare the term $e_n k^2 L_0 / 4L_i$ with the Johnson noise due to the loss R_d of the decoupling capacitance. However, in order to take advantage of the very low noise of the SQUID, the capacitance losses must be negligible with respect to the SQUID voltage noise; in this case the current due to e_n should be calculated directly from the circuit of Fig. 2. The contribution of the SQUID current noise is only a wide-band noise of spectrum i_n^2.

It should be noted that in the previous evaluation of the wide-band noise at each mode we have neglected the contributions due to the Lorentzian "tails" coming from the nearest peaks. An evaluation of this contribution shows that it is negligible in our case, but of course this may not be true in other experimental situations.

NOISE TEMPERATURE

The noise energy through $L_{\rm in}$ will have at each mode a resonant term

$$N^i_{\rm nb} = L_{\rm in}(I^i_{\rm nb})^2 \quad (20)$$

in units of J and a wide band term

$$N_{\rm wb} = L_{\rm in}[i_n^2 + I^2_{\rm wb}(e_n)] \quad (21)$$

in units of J/Hz. Now, if we apply a vector difference procedure separately for each mode with integration time t (Ref. 12), we can obtain the effective temperature $T_{\rm eff}$ of the system, $T_{\rm eff}$ defined as usual as the temperature T_x for which the signal-to-noise ratio S/N is equal to 1.

So for each mode we can write

$$\beta^i k_B T^i_{\rm eff} = N^i_{\rm nb} t/\tau_i + N_{\rm wb}/t \quad (22)$$

and using the optimum integration time

$$t_i^{\rm opt} = 2\tau_i (N_{\rm wb}/N^i_{\rm nb}) \quad (23)$$

we have for $T^i_{\rm eff}$ a minimum value

$$T^i_{\rm eff} = [(I^i_{\rm nb})^2 I^2_{\rm wb}]^{1/2} / \beta_i k_B \tau_i \ . \quad (24)$$

It should be noted that, since we want to observe just one normal mode in our bandwidth $1/t$, the following relation must be satisfied:

$$t_i^{\rm opt} \geq t^{\rm min} = 1/\Delta\omega_{\rm min} \ , \quad (25)$$

$\Delta\omega_{\rm min}$ being the smaller between $(\omega_0 - \omega_-)$ and $(\omega_+ - \omega_0)$. In Fig. 4 a calculation for the effective noise temperature as a function of L_0 is shown for each of the three normal modes of the system. In this example the transducer is tuned to the antenna resonant frequency so that the two

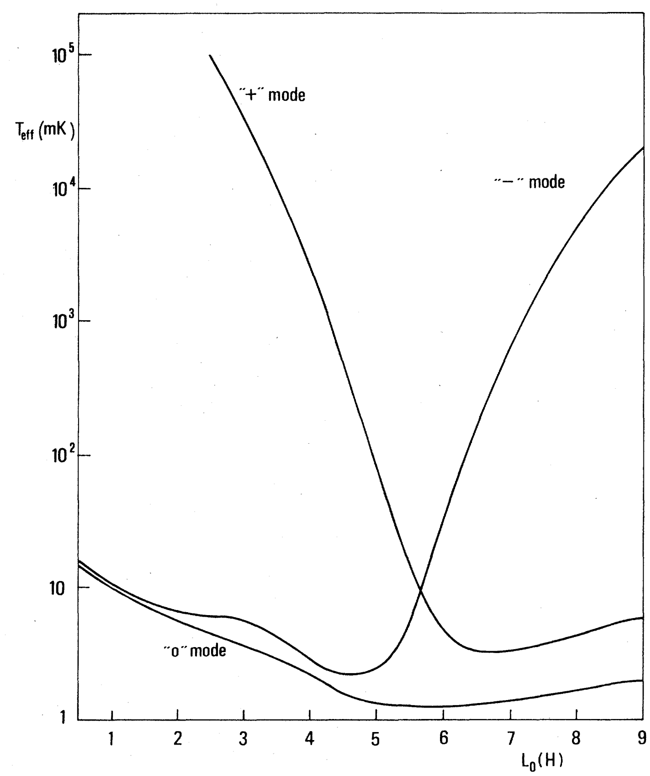

FIG. 4. Effective noise temperature as a function of L_0 evaluated from (23) for the three modes of a typical system. The following values have been used: $M_A = 1135$ kg, $m_T = 0.348$ kg, $Q_A = 20\times10^6$, $Q_T = 3\times10^6$, $\nu_A = 900$ Hz, $\nu_T = 904$ Hz, $C = 8.5$ nF, $E_0 = 3\times10^6$ V/m, $R_0 = 5$ GΩ, $C_d = 100$ nF, $R_d = 0.4$ Ω, $K = 0.77$, $L_i \cong L_{\rm in} \simeq 1.5$ μH, $I_n = 1.3$ pA/$\sqrt{\rm Hz}$, $V_n = 0.58$ fV/$\sqrt{\rm Hz}$.

mechanical modes are an indistinguishable mixture of the two uncoupled oscillators. We can see that for L_0 less than 1 H the "electrical" mode is still strongly decoupled from the two mechanical modes. When L_0 approaches the value of 3 H, a condition is reached for which Eq. (25) is not satisfied. As a consequence the integration time cannot be chosen equal to the optimum value which minimizes $T_{\rm eff}$ and we have a flection point in the curve of the effective temperature for the two mechanical modes. When L_0 is greater than 3 H the electrical oscillator is more strongly coupled to the mechanical oscillators; the resulting greater splitting of the mode frequencies leads to a lower value for $t_{\rm min}$ so that Eq. (25) can be satisfied and $T_{\rm eff}$ is minimized. For $L_0=5.7$ H the three oscillators are tuned within a few Hz. In this case we have a "privileged" mode (the central mode) with a noise temperature lower than the other modes. This effect is due to the fact that the three uncoupled oscillators have different initial Q's.

The noise temperatures evaluated in Fig. 4 are calculated using a set of values already obtained for each section of the apparatus. The best noise temperature obtained for a resonant-wave gravitational antenna is approximately 20 mK (Ref. 13) with a SQUID amplifier and 200 mK (Ref. 14) with a field-effect transistor (FET) amplifier.

EXPERIMENTAL RESULTS

First, the isolated SQUID was tested separately in a small cryostat to check its intrinsic noise. The preamplifier used in this case was a homemade low-noise preamplifier with a voltage noise of 1.2 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ and current noise less than 10 $\mathrm{fA}/\sqrt{\mathrm{Hz}}$. The maximum critical current was 105 μA, the minimum 68 μA, and the maximum responsivity 100 $\mu\mathrm{V}/\phi_0$. The Q of the tank circuit was 70 and the minimum measured noise obtained with a biasing current slightly larger than the maximum critical current was less than $1\times10^{-6}\ \phi_0/\sqrt{\mathrm{Hz}}$ at a frequency of about 900 Hz. This value is lower than the value previously published for these same SQUID's.[5]

A test run was performed in the CERN cryostat with the SQUID not coupled to the transducer and the antenna, but connected only to an electrical resonating circuit similar to that of the final experiment, i.e., a high-ratio transformer and a decoupling capacitance. The transformer used was slightly different from the final one, with a 3 μH secondary and with a primary of 0.6 H. The decoupling capacitance was a high-quality commercial mica capacitor (20 nF). This run was performed to check for problems, such as the shielding against spurious fields. For example, the magnetic field of the CERN protosynchrotron (PS), situated in a building 100 m away, produced in the SQUID a short pulsed signal of about $\frac{1}{2}\phi_0$ about once per second. A double lead shield around the transformer subsequently eliminated this problem.

The input-circuit resonanting frequency was 1590 Hz and had the tendency to oscillate with an apparent Q of the order of 10^5. The overall noise, mainly because of the PS pulses and the electrical oscillation at 1590 Hz, was so high that we were not able to lock the SQUID up to the frequency of interest (about 1 kHz). On the other hand the system, which in a previous test in a small cryostat had been very sensitive to mechanical vibrations (via the magnetic flux trapped in the superconducting shield), in the CERN cryostat was unaffected by this kind of disturbance, due to the good mechanical insulation.

Then the SQUID was connected to the antenna via the transducer with the scheme of Fig. 1. This run was made to check the overall system.

To allow the use of a low-noise FET amplifier in addition or as an alternative to the SQUID, in this test the biasing resistance R_0 of the transducer was put at room temperature. In this way it was possible to use the FET amplifier in a scheme similar to the usual one[1] except for the presence of L_0. However, since the biasing cables of the transducer were not filtered by any high resistance at low temperature, they could introduce noise into the system.

A very small piezoelectric ceramic attached to the antenna and an auxiliary capacitor plate were used to excite the antenna and calibrate the overall system. During this run the three modes of the system were (with a bias voltage of 230 V on the transducer)

$$\nu_-=912.04 \text{ Hz};\quad Q=2\times10^5 \text{ (mechanical mode)}\,,$$

$$\nu_0=933.18 \text{ Hz};\quad Q=8\times10^4 \text{ (mechanical mode)}\,,$$

$$\nu_+=2316.5 \text{ Hz};\quad Q=6\times10^3 \text{ (electrical mode)}\,.$$

A mechanical problem produced a detuning of about 10 Hz between the transducer resonant frequency and that of the antenna, and also a low value for the Q of the transducer. However, for performing the calibration, these effects are not important. The electrical Q was lower than the desired value and large effort will be devoted in the future to increase this value by at least an order of magnitude. Such a high value for the Q of the input circuit may appear in disagreement with the optimum value suggested by the theory of a dc-SQUID used as amplifier reported in Ref. 15. In that paper it was shown that the optimum noise temperature for a dc-SQUID connected to a resonating circuit is obtained with a very low Q of the order, in our case, of a few units. However, the definition of optimum noise temperature used in that paper is different from that used in the gravitational case. In fact, in Ref. 15 the optimization concerns a signal-to-noise ratio where the signal is the Johnson noise of the input circuit and the noise is the intrinsic noise of the SQUID. On the contrary, in a gravitational-wave experiment, the signal is produced by the gravitational wave impinging on the antenna and the noise is the intrinsic noise of the SQUID as well as the Johnson noise of all the resistances in the input equivalent circuit.

With the transducer not biased it is possible to investigate the electrical mode using either the output of the FET amplifier or the output of the SQUID. The electrical mode seemed to be easily excited by electrical disturbances. Proper shielding of the cables at the output of the cryostat and a proper choice of the bandwidth of the SQUID feedback loop partially solved this problem.

The calibration procedure was the following. With the transducer biased at 93 V and connected to the FET amplifier, one of the mechanical modes was excited by sending a sinusoidal voltage signal of known amplitude to the small piezoelectric ceramic. From the voltage output of the FET amplifier the energy released from the excitation signal to the system was calculated. Then, switching to the SQUID and recording its open loop output, without changing anything else, one would get the required calibration of the system. Unfortunately, the leads to the piezoelectric ceramic introduced so much noise that the SQUID could not operate properly when they were connected to the excitation voltage. So the measurement was done in the decay mode, i.e., disconnecting the excitation, recording the SQUID output versus time for a few minutes (Fig. 5) and extrapolating to get the value at $t=0$. Moreover, the slope of Fig. 5 provided a measurement of the quality factor of the excited mode. The sensitivity of the SQUID at the operating frequency was checked by sending to the SQUID a reference signal of known amplitude and of frequency within 10 Hz from the calibration frequency.

An energy excitation equal to one $k_B T$ ($T=4.2$ K) on one mechanical mode corresponded to $5\times10^{-5}\phi_0$ inside the SQUID in the resonance bandwidth. The calibrated value is within a factor of 2 of the expected value. The calculation was made using the separately measured circuital elements.

With the feedback loop closed and with the transducer unbiased, a SQUID wide-band noise of $2.5\times10^{-5}\phi_0/\sqrt{\mathrm{Hz}}$ was measured at about 1 kHz. This noise is more than an order-of-magnitude worse than that measured with the SQUID alone and is due to the unfiltered cables connecting the transducer to the auxiliary FET amplifier; in the future we will not need those cables.

With the transducer biased the mechanical modes were excited by the feedback signal. This effect could be due to the direct coupling between the feedback coil and the input coil of the SQUID. This coupling introduces in the input circuit a frequency-dependent term[16] that at the mechanical resonance may give a positive feedback. A detailed analysis of the various terms shows a possible instability in the system. In the design of the next SQUID we will try to reduce such coupling by patterning a proper feedback loop on the chip and using the particular geometry of the multiloop SQUID.

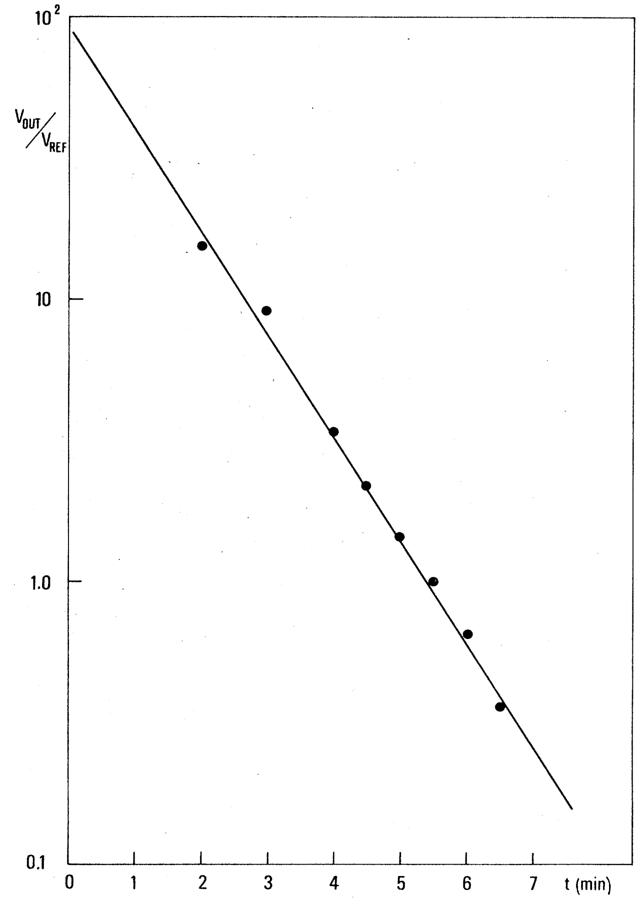

FIG. 5. Free decay of one mechanical mode excited by means of a small piezoelectric ceramic glued onto the antenna. Figure shows the linearized output of the dc-SQUID normalized to the reference signal as a function of time.

CONCLUSIONS

We have coupled a dc-SQUID to the resonant capacitive transducer used in the gravitational experiments being carried out at CERN laboratories. In this preliminary test we have tested the overall system and found the ratio between the normal-mode energy and the flux in the SQUID. In the next test we will try to overcome the problems encountered, particularly the stability problems for the feedback loop of the SQUID and the electrical Q of the input circuit.

A detailed analysis of the behavior of the detection system was done by decoupling it in normal modes. This allowed us to evaluate the noise temperature for each of the three normal modes and choose the best operating conditions.

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[1]P. Rapagnani, Nuovo Cimento **5C**, 385 (1982).
[2]M. W. Cromar and P. Carelli, Appl. Phys. Lett. **38**, 723 (1981).
[3]F. Wellstood, C. Heiden, and J. Clarke, Rev. Sci. Instrum. **55**, 952 (1984).
[4]C. D. Tesche, K. H. Brown, A. C. Callegari, M. M. Chen, J. H. Greiner, H. C. Jones, M. B. Ketchen, K. K. Kim, A. W. Kleinsasser, H. A. Notarys, G. Proto, R. H. Wang, and T. Yogi, in *Proceedings of the 17th International Conference on Low Temperature Physics,* edited by U. Eckern, A. Schmid, W. Weber, and H. Wuhl (North-Holland, Amsterdam, 1984), Pt. II, p. 263.
[5]P. Carelli and V. Foglietti, IEEE Trans. Magn. **MAG-19**, 299

(1983).

[6]P. Carelli and V. Foglietti, IEEE Trans. Magn. **MAG-21**, 421 (1985).

[7]E. Amaldi, P. Bonifazi, F. Bronzini, P. Carelli, M. G. Castellano, G. Cavallari, E. Coccia, C. Cosmelli, V. Foglietti, R. Habel, I. Modena, Y. Ogawa, G. V. Pallottino, G. Pizzella, and P. Rapagnagni, Nuovo Cimento **7C**, 338 (1984).

[8]P. Carelli and V. Foglietti, J. Appl. Phys. **53**, 7592 (1982).

[9]M. G. Castellano and C. Cosmelli, Nuovo Cimento **7C**, 9 (1984).

[10]Y. Ogawa and P. Rapagnani, Nuovo Cimento **7C**, 21 (1984).

[11]G. V. Pallottino and G. Pizzella, Nuovo Cimento **4C**, 237 (1981).

[12]R. P. Giffard, Phys. Rev. D **14**, 2487 (1976).

[13]S. P. Boughn, W. M. Fairbank, R. P. Giffard, J. N. Hollenrhost, E. R. Mapoles, M. S. McAshan, P. S. Michelson, H. J. Paik, and R. C. Taber, Astrophys. J. **261**, L19 (1982).

[14]G. Pizzella (private communication).

[15]J. Clarke, C. D. Tesche, and R. P. Giffard, J. Low Temp. Phys. **37**, 405 (1979).

[16]R. P. Giffard, R. A. Webb, and J. C. Wheatley, J. Low Temp. Phys. **6**, 533 (1972).