Resistive ballooning modes in helical axis stellarators

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The helical nature of the magnetic axis induces a curvature component that drives resistive ballooning modes, which scale as the resistivity to the third power in a helical axis stellarator.

Stellarator configurations that have a helical magnetic axis have been demonstrated theoretically to possess very favorable ideal magnetohydrodynamic stability properties.[1-5] In this paper, we investigate the driving mechanism and determine the scaling of resistive ballooning-mode activity for a helical axis stellarator in a geometry with helical symmetry.

The incompressible resistive ballooning mode equation for arbitrary magnetic confinement geometry is[6]

$$(\mathbf{B}\cdot\nabla)\left[\frac{k_\perp^2/B^2}{(1+n^2\eta k_\perp^2/\gamma)}(\mathbf{B}\cdot\nabla)V_\perp\right]+2\frac{dp}{d\psi}\left[\frac{\mathbf{B}\times\mathbf{k}_\perp\cdot\boldsymbol{\kappa}}{B^2}\right]V_\perp -\rho_m\gamma^2\frac{k_\perp^2}{B^2}V_\perp=0 \;, \quad (1)$$

where $\boldsymbol{\kappa}$ is the magnetic field line curvature, ψ is the helical flux function, η is the resistivity, and n is the longitudinal mode number. We investigate this equation in a (ρ,θ,ϕ) flux coordinate system in which the magnetic field lines are straight. The radial coordinate is ρ, the poloidal angle is θ, and the anglelike variable $\phi=hZ$ identifies the ignorable longitudinal coordinate. The condition that the magnetic field lines be straight requires $\sqrt{g}\,(\mathbf{B}\cdot\nabla\phi)=q(\rho)\psi'$, from which we obtain that the Jacobian $\sqrt{g}$ is

$$\sqrt{g}=\frac{q\psi'}{h^2F(\rho)}\left\{1+\epsilon^2\left[X^2+Y^2+\frac{1}{q}\left[X\frac{\partial Y}{\partial\theta}-Y\frac{\partial X}{\partial\theta}\right]\right]\right\} \;, \quad (2)$$

where $F(\rho)$ is the longitudinal magnetic field in the covariant representation, X is the distance from the geometric axis to the projection of some point in the plasma to the midplane, and Y is the distance from that point to the midplane. These distances are normalized to the minor radius a. With this expression for $\sqrt{g}$, we can construct expressions for B^2, $k_\perp^2$, and $\mathbf{B}\times\mathbf{k}_\perp\cdot\boldsymbol{\kappa}$, which we expand in the smallness of the parameter $\epsilon=ah$ to obtain the reduced ballooning-mode equation

$$\frac{\partial}{\partial\theta}\left[\frac{[F^2/(\psi')^2]\alpha_\perp(\rho,\theta)}{1+(n^2/S_R)(\eta/\gamma)[F^2/(\psi')^2]\alpha_\perp(\rho,\theta)}\frac{\partial V_\perp}{\partial\theta}\right]-\frac{\gamma^2q^2}{(\psi')^2}\alpha_\perp(\rho,\theta)V_\perp-\frac{\beta_0p'}{(\psi')^2}[K_p(\rho,\theta)+K_s(\rho,\theta)(\theta-\theta_k)]V_\perp=0 \;, \quad (3)$$

where $\beta_0=2p_0/(h^2F_e^2)$, and where we have normalized the pressure to p_0 (its value at the magnetic axis), ψ to $a\epsilon F_e$, and F to F_e (its value at the edge of the plasma). The resistivity is normalized to its value on axis; the growth rate γ is expressed in units of the poloidal Alfvén frequency $h^2F_e/\sqrt{\rho_m}$, and S_R is the magnetic Reynolds number. The function $\alpha_\perp(\rho,\theta)$, which represents $k_\perp^2$ for $\epsilon \ll 1$, is

$$\alpha_\perp(\rho,\theta)=\left[\left[\frac{\partial X}{\partial\rho}\right]^2+\left[\frac{\partial Y}{\partial\rho}\right]^2\right]-2\frac{q'}{q}\left[\frac{\partial X}{\partial\rho}\frac{\partial X}{\partial\theta}+\frac{\partial Y}{\partial\rho}\frac{\partial Y}{\partial\theta}\right](\theta-\theta_k)+\left[\frac{q'}{q}\right]^2\left[\left[\frac{\partial X}{\partial\theta}\right]^2+\left[\frac{\partial Y}{\partial\theta}\right]^2\right](\theta-\theta_k)^2+O(\epsilon^2) \;. \quad (4)$$

The function of $K_p(\rho,\theta)$,

$$K_p(\rho,\theta)=q^2\left[X\frac{\partial X}{\partial\rho}+Y\frac{\partial Y}{\partial\rho}\right]+\frac{2\psi'q^2}{F}-\left[\frac{\partial X}{\partial\rho}\frac{\partial^2X}{\partial\theta^2}+\frac{\partial Y}{\partial\rho}\frac{\partial^2Y}{\partial\theta^2}\right]+O(\epsilon^2) \;, \quad (5)$$

is related to the normal curvature, and the function $K_s(\rho,\theta)$,

$$K_s(\rho,\theta)=-\left[\frac{q'}{q}\right]\left[q^2\left[X\frac{\partial X}{\partial\theta}+Y\frac{\partial Y}{\partial\theta}\right]-\left[\frac{\partial X}{\partial\theta}\frac{\partial^2X}{\partial\theta^2}+\frac{\partial Y}{\partial\theta}\frac{\partial^2Y}{\partial\theta^2}\right]\right]+O(\epsilon^2) \;, \quad (6)$$

is related to the geodesic curvature.

An approximate solution of Eq. (3) can be obtained for a model equilibrium with circular flux surfaces represented by $X=X_m+\rho\cos\theta$ and $Y=\rho\sin\theta$, where X_m is the distance from the magnetic axis to the geometric axis. We invoke the electrostatic approximation and apply the two-length scale expansion, following the procedure carried out for tokamaks,[7] to obtain the envelope equation:

$$a_R\gamma\frac{\partial^2V_0}{\partial\theta^2}-\left[\frac{\gamma^2q^2}{(\psi')^2}[1+s^2(\theta-\theta_k)^2+w]\right]V_0+\frac{2u^2[1+s^2(\theta-\theta_k)^2]V_0}{a_R\gamma+[\gamma^2q^2/(\psi')^2][1+s^2(\theta-\theta_k)^2]+w}=0 \;, \quad (7)$$

where $s = pq'/q$ is the magnetic shear, $u = -0.5\beta_0 p' q^2 X_m / (\psi')^2$, $w = \beta_0 p'[\rho(1+q^2) + 2\psi' q^2/F]/(\psi')^2$, and $a_R = S_R / n^2\eta$. This approximation is valid for mode structures very extended in θ. The asymptotic analysis of Eq. (7) yields the width L of the mode given by

$$L \simeq \left[\frac{S_R}{n^2\eta} \frac{(\psi')^2}{\gamma q^2 s} \right]^{1/4} . \tag{8}$$

The tokamak analysis of Hender *et al.*[7] can be further applied to Eq. (7) to obtain the dispersion relation

$$\frac{\gamma^2 q^2}{(\psi')^2} \left[2\frac{\gamma^2 q^2}{(\psi')^2} + 2w + a_R \gamma \right] = 2u^2 . \tag{9}$$

The regime dominated by the resistive ballooning mode corresponds to $2w/a_R << \gamma << a_R(\psi')^2/(2q^2)$, which yields the scaling

$$\gamma = \left[\frac{2u^2(\psi')^2}{q^2 a_R} \right]^{1/3} = \left[\frac{\beta_0 p' q X_m}{\psi'} \right]^{2/3} \left[\frac{n^2\eta}{2S_R} \right]^{1/3} . \tag{10}$$

Noncircularity effects introduce additional factors in Eq. (10) without altering the basic scaling $(\beta_0^2\eta)^{1/3}$ and compressibility effects become important when the growth rate is small and comparable to the frequency of sound waves.

In Fig. 1, we illustrate a numerical solution of Eq. (1) with the parameters described in the figure caption and compare the growth rates obtained with those from the dispersion relation given by Eq. (10). The scalings are virtually identical except for a small offset due to a weakly unstable ideal mode.

The resistive ballooning modes in stellarator configurations with a helical magnetic-axis dimension comparable to the minor radius are driven by the component of the curvature induced by the helical motion of the magnetic axis about the geometric axis. These modes scale as the resistivity to the third power.

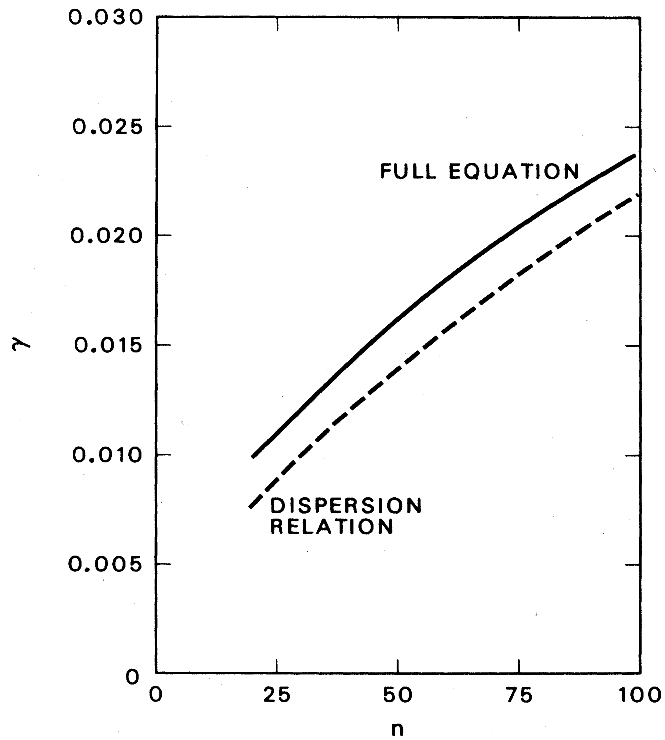

FIG. 1. Growth rate γ as a function of the mode number from the full equation (solid curve) and from the dispersion relation (dashed curve) on the flux surface $\rho = 0.45$ for a straight stellarator configuration with a circular boundary and $\beta_0 = 0.4\%$, $X_m = 0.5$, $\epsilon = 0.2$, and $S_R = 10^4$.

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[1] R. Gruber, S. Semenzato, F. Troyon, T. Tsunematsu, W. Kerner, P. Merkel, and W. Schneider, Comput. Phys. Commun. **24**, 363 (1981).
[2] P. Merkel, J. Nuhrenberg, R. Gruber, and F. Troyon, Nucl. Fusion **23**, 1061 (1983).
[3] M. Mond, Phys. Fluids **26**, 3327 (1983).
[4] D. A. Monticello, R. L. Dewar, H. P. Furth, and A. Reiman, Phys. Fluids **27**, 1248 (1984).
[5] D. C. Barnes and J. R. Cary, Phys. Fluids **27**, 2522 (1984).
[6] D. Correa-Restrepo, Z. Naturforsch. **37A**, 848 (1982).
[7] T. C. Hender, B. A. Carreras, W. A. Cooper, J. A. Holmes, P. H. Diamond, and P. L. Similon, Phys. Fluids **27**, 1439 (1984).