Wideband photon counting and homodyne detection

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Homodyne detection has been proposed as a means of detecting squeezed coherent radiation. Here the response of a balanced homodyne detector to wideband squeezed coherent states is presented. In order to carry out the analysis the theory of wideband photodetection is reviewed and in order to determine the ultimate performance limits of photoemissive detectors small terms of order $\Delta\omega/\omega_0$ that are usually neglected, where ω_0 is the optical carrier frequency and $\Delta\omega$ is the electronics bandwidth, have been kept. It is shown that the ultimate noise reduction that can be achieved in the noise-power spectrum of a homodyne detector, detecting squeezed coherent radiation, is a factor of 2 worse when photoemissive detectors are used instead of power flux detectors.

I. INTRODUCTION

At present there are a number of experimental efforts[1–3] under way directed toward the generation of squeezed coherent radiation at optical frequencies. Such radiation exhibits quantum fluctuations which are reduced below those of vacuum fluctuations for one of the two quadrature components of the field amplitude.[4,5] By virtue of its ability to detect one amplitude component of the incoming light, homodyne detection has become the method of choice for exhibiting this reduction in quantum fluctuations.[6,7] In homodyne detection the signal light is combined, via a beam splitter, with intense local oscillator light oscillating at the carrier frequency of the signal light. This light is then directed to a photodetector.

The signal delivered by the photodetector results from the constructive or destructive interference of the signal light with the local oscillator light at the photodetector. Noise from intensity fluctuations of the local oscillator may be avoided by using a balanced configuration[8–10] involving two photodetectors and a 50%-50% beam splitter. Such a balanced homodyne detector is depicted in Fig. 1. In this configuration, fluctuations in the local oscillator intensity give rise to a common mode signal which is eliminated by measuring only the difference in the photocurrents generated by the two photodetectors. In contrast, signal light which interferes constructively with local oscillator light at one photodetector will interfere destructively at the other photodetector and hence give rise to a difference mode signal.

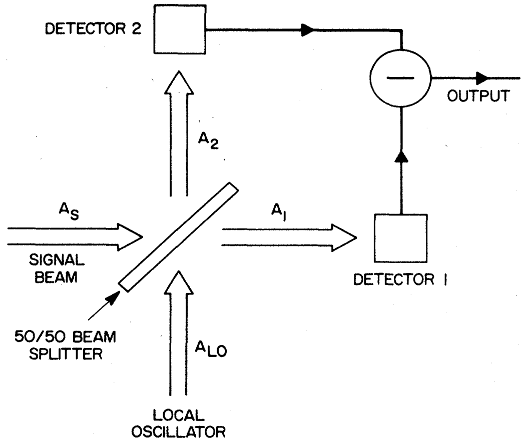

FIG. 1. A balanced homodyne detector.

Theoretical investigations of the quantum-mechanical behavior of balanced homodyne detectors have primarily been restricted to single-mode analysis. In order to avoid potential noise sources near dc, such as $1/f$ noise, or as a matter of convenience, experimentally one is likely to look at frequency components of the homodyne detector's output at 10 MHz to 1 GHz. For such an experimental situation a multimode or wideband analysis of homodyne detection is more appropriate. A wideband analysis of photodetection and homodyne detection has been carried out by Yuen and Shapiro.[6,11] They introduce a photon-units field operator from which they construct an effective photon-flux density operator for the photodetector. Although photon-flux density operators constructed in such a manner behave properly when integrated over volumes large compared with the wavelength λ of light or times long compared with the optical period λ/c,[12] it is not apparent that such operators adequately describe the photoemission process on the finite time scales relevant in wideband homodyne detection.

In this paper an alternate approach to wideband photodetection is taken. Since Fermi's golden rule calculations such as those performed by Glauber[13,14] lead to expressions for the emission probability $P(t)$ that are bilinear[15] in the vector potential $\mathbf{A}(\mathbf{r},t)$,

$$P(t)=\int_{-\infty}^{t} dt' \int_{-\infty}^{t} dt'' S(t'-t'')A(\mathbf{r},t')A(\mathbf{r},t'') , \tag{1.1}$$

we will search for a sensitivity function $S(\tau)$ such that $P(\infty)$ is the number operator. The sensitivity function is in general nonlocal. As a matter of convenience a number of local approximations of $S(\tau)$, such as $\delta(\tau)$, $d\delta(t)/dt$, and $d^2\delta(t)/dt^2$ have been employed in the literature. Most recently such an approximation was implicitly made by Bondurant[16] in his treatment of wideband photodetection. In this paper the local approximation will be avoided.

The approach taken here also differs from usual treatments of photoemission in that the rapidly oscillating terms of (1.1) will be kept. In conventional treatments the product $A(\mathbf{r},t')A(\mathbf{r},t'')$ in (1.1) is replaced by $A^{(-)}(\mathbf{r},t')A^{(+)}(\mathbf{r},t'')$ where $A^{(+)}$ and $A^{(-)}$ are the positive and negative frequency parts of A, respectively. The rapidly oscillating terms are neglected on the grounds that for long observation times they average to zero. These terms may be rigorously neglected when calculating the dc response of a photodetector. In wideband photodetection a finite time scale, the inverse bandwidth of the electronics into which the photocurrent is injected, is introduced. In this case the rapidly oscillating terms will give finite, although usually small corrections. In particular, it will be shown that for a homodyne detector with a local oscillator frequency ω_0 the rapidly oscillating terms will give corrections to the power spectrum of the detector's output at ω that are of order $(\omega/\omega_0)^2$.

Although even for fast photodiodes ω/ω_0 will be of order 10^{-5}, this term must be included if one wishes to determine the ultimate performance of wideband homodyne detectors. At microwave frequencies, assuming mixers constructed from superconductor-insulator-superconductor (SIS) junctions operated in the photon-assisted-quantum-tunneling mode can be described by a photoemissive process,[17] an ω/ω_0 larger than 0.1 is conceivable for a mixer with a 1-GHz intermediate frequency (IF) bandwidth.

After having constructed the photoemission rate operator for a unit-quantum-efficiency photodetector, the results will be extended to photodetectors with less than unit quantum efficiency using techniques introduced by Yuen and Shapiro.[6,8] The response of a balanced homodyne detector to wideband squeezed coherent states is then calculated in the final sections of this paper.

II. THE WIDEBAND UNIT-QUANTUM-EFFICIENCY PHOTODETECTOR

In this section, wideband photoemission theory will be reviewed and a general expression for the photoemission probability for a wideband photodetector will be obtained. To this end, consider the photoemission process of a bound electron moving in some potential well. The electron's Hamiltonian H may be decomposed into the form H_e+H_I where H_I is the interaction Hamiltonian for the electron coupled to the electromagnetic field. Assuming that the electron interacts with electromagnetic radiation via the minimal coupling interaction, and neglecting the A^2 term, H_I in the interaction picture using the notation of Kimble and Mandel[15] can be written in the form

$$H_I(t')=-\frac{e}{m}e^{iH_e(t'-t_0)/\hbar}\mathbf{p}e^{-iH_e(t'-t_0)/\hbar}\cdot\mathbf{A}(\mathbf{r},t') , \tag{2.1}$$

where $\mathbf{p}$ is the electron's momentum at t_0 when the interaction is turned on and $\mathbf{r}$ is the average coordinate of the bound electron. The transverse vector potential $\mathbf{A}(\mathbf{r},t)$ can be split into positive $\mathbf{A}^{(+)}(\mathbf{r},t)$ and negative $\mathbf{A}^{(-)}(\mathbf{r},t)$ frequency components

$$\mathbf{A}(\mathbf{r},t)=\mathbf{A}^{(+)}(\mathbf{r},t)+\mathbf{A}^{(-)}(\mathbf{r},t) , \tag{2.2}$$

where

$$\mathbf{A}^{(-)}(\mathbf{r},t)=\left[\frac{\hbar}{2\pi\epsilon_0}\right]^{1/2}\sum_s\int\frac{d^3k}{\sqrt{\omega_k}}\boldsymbol{\epsilon}_{k,s}A^\dagger_{k,s}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_k t)} \tag{2.3}$$

and

$$\mathbf{A}^{(+)}(\mathbf{r},t)=[\mathbf{A}^{(-)}(\mathbf{r},t)]^\dagger . \tag{2.4}$$

Let $|i\rangle$ denote the initial state of the entire system at $t=t_0$. The transition amplitude to some final state $|f\rangle$ is given by

$$\langle f\mid U(t,t_0)\mid i\rangle$$

where the unitary operator $U(t,t_0)$ has the form

$$U(t,t_0)=1+\frac{1}{i\hbar}\int_{t_0}^{t}H_1(t')dt'+\cdots . \tag{2.5}$$

To be more specific, let the electron be initially in its ground state $|\Psi_0\rangle$ and let $|a\rangle$ denote the initial state of the radiation field, then $|i\rangle=|\Psi_0\rangle|a\rangle$. Let the final state $|f\rangle$ consist of the radiation field state $|b\rangle$ and the electron sitting in an energy eigenstate $|\Psi(E,\Omega)\rangle$ where Ω labels quantum numbers other than the energy. The transition amplitude is then

$$\langle i\mid U(t,t_0)\mid f\rangle=\frac{ie}{m\hbar}\int_{t_0}^{t}dt'\,e^{i[(E-E_0)/\hbar](t'-t_0)}\langle\psi(E,\Omega)\mid\mathbf{P}\mid\psi_0\rangle\cdot\langle f\mid\mathbf{A}(\mathbf{r},t')\mid i\rangle . \tag{2.6}$$

At this point, in the usual treatments of the photoemission process[13–15] the negative frequency $\mathbf{A}^{(-)}(\mathbf{r},t)$ of $\mathbf{A}(\mathbf{r},t)$ is neglected on the grounds that it gives rise to a rapidly oscillating term whose contribution to the integral (2.6) becomes small when the time interval $t-t_0$ becomes long compared with an optical period. In fact, one has rigorously

$$\int_{-\infty}^{\infty}dt\,e^{i[(E-E_0)/\hbar](t'-t)}\langle f\mid\mathbf{A}^{(-)}(\mathbf{r},t)\mid i\rangle=0 . \tag{2.7}$$

The term $\mathbf{A}^{(-)}(\mathbf{r},t)$ can be completely neglected when evaluating the dc response of a photodetector. However, this term may contribute significantly to the video response of a wideband photodetector and hence will be kept in the following analysis. The transition probability that the electron will be excited out of its ground state is given by

$$p(t)=\sum_f|\langle f\mid U(t,t_0)\mid i\rangle|^2 , \tag{2.8}$$

where the generalized sum is carried out over states $|f\rangle$ in which the electron is not in its ground state $|\psi_0\rangle$. The transition probability, realizing that the sum in (2.8) is carried out over a complete set of photon states, can be put into the form

$$p(t)=\sum_{\mu,\nu}\int_{t_0}^{t}dt'\int_{t_0}^{t}dt''k_{\nu\mu}(t'-t'')\langle A_\mu(\mathbf{r},t')A_\nu(\mathbf{r},t'')\rangle\ , \tag{2.9}$$

where the expectation value $\langle A_\mu(\mathbf{r},t')A_\nu(\mathbf{r},t'')\rangle$ is evaluated over the initial state of the field $|a\rangle$. Assuming that the excited electron is detected with unit probability, the sensitivity function $k_{\nu\mu}(\tau)$ is given by

$$k_{\nu\mu}(\tau)=\left|\frac{e}{m\hbar}\right|^2\int dE\int d\Omega\,\sigma(E,\Omega)\langle\psi(E,\Omega)\,|\,\mathbf{P}_\nu\,|\,\psi_0\rangle\langle\psi_0\,|\,\mathbf{P}_\mu\,|\,\psi(E,\Omega)\rangle e^{-i[(E-E_0)/\hbar]\tau}\ , \tag{2.10}$$

where $\sigma(E,\Omega)$ is the density-of-states function. For a photodetector consisting of M independent bound-electron systems the sensitivity function (2.10) is multiplied[15] by M.

From (2.9) it is convenient to introduce the photon-counting operator $P(t)$ whose expectation value gives the mean number of photons $p(t)$ counted in the time interval $t-t_0$,

$$P(t)=\sum_{\mu,\nu}\int_{t_0}^{t}dt'\int_{t_0}^{t}dt''\,k_{\nu\mu}(t'-t'')A_\mu(\mathbf{r},t')A_\nu(\mathbf{r},t'')\ . \tag{2.11}$$

The rate of photoemission $w(t)$ is given by the time derivative of $p(t)$. It is useful therefore to introduce the photoemission rate operator $W(t)$:

$$W(t)=\frac{dP(t)}{dt}\ . \tag{2.12}$$

The photocurrent operator $I_p(t)$ is then given by

$$I_p(t)\equiv eW(t)\ . \tag{2.13}$$

The properties of the photocurrent delivered by the photodetector to the external world can be determined by evaluating various moments and correlation functions of $I_p(t)$ for the incoming optical field state.

Having obtained the general form for the photon counting operator (2.11) for a unit quantum efficiency (in the sense that every photoelectron emitted is detected), an idealized system for which (2.3), (2.10), and (2.11) can be greatly simplified will now be considered, namely a photodetector embedded in a waveguide.

III. THE PHOTODETECTOR EMBEDDED IN A WAVEGUIDE

For conceptual and computational simplicity, the behavior of a photodetector embedded in an optical waveguide is considered in this section. Such a system could be realized experimentally. Furthermore, one expects that the behavior of this system will not be much different from the more usual experimental situation of a Gaussian light beam directed toward a photodetector and focused in such a way that a minimum in the beam waist occurs at the photodetector's surface. In order that one need only deal with one waveguide mode, the waveguide is taken to be constructed such that all but one of its modes have cutoff frequencies well above optical frequencies. In this case the equations of Sec. II can be simplified to equations that depend only on the z coordinate. The field operator for the field propagating towards the photodetector becomes

$$A(z,t)=a\int_{\omega_c}^{\infty}\frac{d\omega}{(\omega^2-\omega_c^2)^{1/4}}[A(\omega)e^{-i(-k_\omega z+\omega t)}+\text{H.c.}]\ , \tag{3.1}$$

where ω_c is the cutoff frequency for the lowest waveguide mode, a is a normalization constant, and the photon creation and annihilation operators satisfy the usual commutation relations

$$[A(\omega),A(\omega')]=0\ , \tag{3.2}$$

$$[A(\omega),A^\dagger(\omega')]=\delta(\omega-\omega')\ . \tag{3.3}$$

Without loss of generality the photodetector will be located at $z=0$. The time t_0 at which the photodetector was turned on will be taken to be $-\infty$. The photon-counting operator then becomes

$$P(t)=\int_{-\infty}^{t}dt'\int_{-\infty}^{t}dt''k(t'-t'')A(t')A(t'')\ , \tag{3.4}$$

where $A(t)\equiv A(0,t)$ is the field operator at the location of the photodetector. From (2.10) one sees that the photodetector's sensitivity function must have the form

$$k(\tau)=\int_{\omega_0}^{\infty}d\omega\,k(\omega)e^{-i(\omega-\omega_0)\tau}\ , \tag{3.5}$$

where because (3.4) must be Hermitian, $k(\omega)$ must be real.

The function $k(\omega)$ will now be determined by the constraint that it describe a wideband unit-quantum-efficiency photodetector that absorbs every photon in the frequency range ω_l to ω_u and is transparent to photons outside this range. After waiting a sufficiently long time for all the photons in the optical waveguide to arrive at the photodetector, such a photodetector should report the total number of photons in the frequency range ω_l to ω_u that were in the waveguide, that is $P(\infty)$ should be equal to the number operator

$$N=\int_{\omega_l}^{\omega_u}d\omega\,A^\dagger(\omega)A(\omega)\ . \tag{3.6}$$

One has $P(\infty)=N$ when

$$k(\omega)=\frac{1}{(2\pi a)^2}u(\omega-\omega_0-\omega_l)u(\omega_u-\omega+\omega_0)\times[(\omega-\omega_0)^2-\omega_c^2]^{1/2}\ , \tag{3.7}$$

where the waveguide cutoff frequency ω_c has been chosen to lie below the photodetector's cutoff frequency ω_l, and $u(x)$ is the Heaviside unit step function

$$u(x)=\begin{cases}1, & x>0\\ 0, & \text{otherwise}\ .\end{cases} \tag{3.8}$$

Substituting (3.7) into (3.5) one obtains the following form for the response function of a unit-quantum-efficiency photodetector that absorbs every photon incident on it in the frequency range $\omega_u > \omega > \omega_l$:

$$k(\tau)=\frac{1}{(2\pi a)^2}\int_{\omega_l}^{\omega_u} d\omega(\omega^2-\omega_c^2)^{1/2}e^{-i\omega\tau} \; . \quad (3.9)$$

Since the photoemission-rate operator $W(t)$ is the time derivative of the photon-counting operator (3.4) one has

$$W(t)=\int_{-\infty}^{t} dt'[k(t-t')A(t)A(t')+k(t'-t)A(t')A(t)] \; . \quad (3.10)$$

Breaking $A(t)$ into its positive $A^{(+)}(t)$ and negative $A^{(-)}(t)$ frequency components, Eq. (3.10) can be written in the form

$$W(t)=\int_{-\infty}^{t} dt'h(t-t')[A^{(-)}(t)A^{(-)}(t')+A^{(-)}(t)A^{(+)}(t')+A^{(-)}(t')A^{(+)}(t)+A^{(+)}(t')A^{(+)}(t)]$$
$$+\int_{-\infty}^{t} dt'\{k(t-t')[A^{(+)}(t),A^{(-)}(t')]+k(t'-t)[A^{(+)}(t'),A^{(-)}(t)]\} \; , \quad (3.11)$$

where

$$h(\tau)=k(\tau)+k(-\tau) \; . \quad (3.12)$$

The commutators can be evaluated using (3.1) and (3.3). In particular

$$[A^{(+)}(t),A^{(-)}(t')]=a^2\int_{\omega_c}^{\infty}\frac{d\omega}{(\omega^2-\omega_c^2)^{1/2}}e^{-i\omega(t-t')} \; . \quad (3.13)$$

The integral containing the commutators vanishes as can be established by noting that for a $k(\tau)$ of the general form (3.5) one has

$$\int_{-\infty}^{\infty} d\tau k(\tau)e^{-i\omega\tau}=0 \quad (3.14)$$

for $\omega>0$. Hence it has been shown that $W(t)$ can be written in the normal ordered form

$$W(t)=\int_{-\infty}^{t} dt'h(t-t')$$
$$\times[A^{(-)}(t)A^{(-)}(t')+A^{(-)}(t)A^{(+)}(t')$$
$$+A^{(-)}(t')A^{(+)}(t)+A^{(+)}(t')A^{(+)}(t)] \; . \quad (3.15)$$

It will now be shown that, due to the finite response time of the electronics into which the photocurrent is injected, the photodetector is blind to the terms $A^{(-)}(t)A^{(-)}(t')$ and $A^{(+)}(t')A^{(+)}(t)$ in this expression. The operator for the photocurrent generated by the photoemitter is obtained by multiplying the photoemission-rate operator by the charge of the electron:

$$I_p(t)=eW(t) \; . \quad (3.16)$$

This current is filtered by the response function $H(t)$ of the electronics into which this current is injected. The operator $I(t)$ for the current ultimately detected is given by

$$I(t)=\int_{-\infty}^{\infty} dt'H(t'-t)I_p(t') \; , \quad (3.17)$$

where in order to be causal the response function must satisfy

$$H(t'-t)=0 \;\text{ for } t'>t \; . \quad (3.18)$$

Introducing the Fourier transforms

$$H(\omega)=\int_{-\infty}^{\infty} H(t)e^{i\omega t}dt \; ,$$
$$I_p(\omega)=\int_{-\infty}^{\infty} I_p(t)e^{i\omega t}dt \; , \quad (3.19)$$

Eq. (3.17) can be put into the form

$$I(t)=\frac{1}{2\pi}\int_0^{\infty} d\omega[H(\omega)I_p(\omega)e^{-i\omega t}+\text{H.c.}] \; . \quad (3.20)$$

The filter function $H(\omega)$ will have a high-frequency cutoff ω_f due to the finite bandwidth of the electronics. As an example, consider the photodiode detector in Fig. 2. The photodiode is modeled as a current source $I_p(t)$. This photocurrent is in parallel with the diode's capacitance C. The signal is delivered to an amplifier or recorder having an input impedance R. The response function for this system is

$$H(\omega)=\frac{1}{1-i\omega RC} \; . \quad (3.21)$$

The circuit thus has a characteristic cutoff frequency $\omega_f=1/RC$ determined by the RC time constant of the electronics. Above this frequency the response of the electronics to the photocurrent falls off with frequency as $1/\omega$. The cutoff frequency for fast photodiodes lies typically in the range of 100–1000 MHz, more than 5 orders of magnitude below visible light frequencies. Furthermore, when other effects such as lead inductance and amplifier bandwidths are included the high-frequency falloff $H(\omega)$ will be much greater than $1/\omega$.

From Eq. (3.16) and (3.19) one has

$$I_p(\omega)=eW(\omega) \; . \quad (3.22)$$

The Fourier component $W(\omega)$ of $W(t)$ can be obtained from (3.15) and has the form

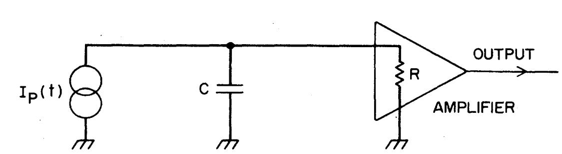

FIG. 2. The ac equivalent circuit of a photodiode whose photocurrent $I_p(t)$ is delivered to an amplifier having an input impedance R. The current source $I_p(t)$ is in parallel with the diode's capacitance C.

$$W(\omega)=\int_{\omega_c}^{\infty} d\omega' F(\omega,\omega')A^{\dagger}(\omega')A(\omega+\omega') + \frac{u(\omega-2\omega_c)}{2} \times \int_{\omega_c}^{\omega-\omega_c} d\omega' F(\omega,-\omega')A(\omega-\omega')A(\omega') , \tag{3.23}$$

where

$$F(\omega,\omega')=\frac{2\pi a^2}{(\omega'^2-\omega_c^2)^{1/4}[(\omega+\omega')^2-\omega_c^2]^{1/4}} \times \int_0^{\infty} d\tau\, h(\tau)(e^{-i(\omega+\omega')\tau}+e^{i\omega'\tau}) . \tag{3.24}$$

It is now readily apparent that if the filter function $H(\omega)$ cuts off in such a manner that the electronics cannot respond to signals at frequencies above twice the waveguide cutoff frequency ω_c, the second term of (3.23) does not contribute to $I(t)$ since the Heaviside function $u(\omega-2\omega_c)$ is zero over the passband of $H(\omega)$. Hence one may write

$$I(t)=e\int_{-\infty}^{\infty} dt' H(t'-t)\int_{-\infty}^{t'} dt'' h(t'-t'') \times [A^{(-)}(t')A^{(+)}(t'') + A^{(-)}(t'')A^{(+)}(t')] , \tag{3.25}$$

where it is understood that $2\omega_c$ lies above the passband of $H(t)$. From this expression it is immediately seen that $\langle 0|I^n(t)|0\rangle=0$, that is, when no photons are present no photocurrent is delivered by the detector. Throughout the rest of this paper $W(t)$ will be taken to be

$$W(t)=\int_{-\infty}^{t} dt' h(t-t') \times [A^{(-)}(t)A^{(+)}(t')+A^{(-)}(t')A^{(+)}(t)] \tag{3.26}$$

with the understanding that $2\omega_c$ will lie above the detector's passband. It will also be convenient to take $H(\omega)$ to be a step-function filter of bandwidth $\Delta\omega$ so that (3.20) can be written as

$$I(t)=\frac{e}{2\pi}\int_0^{\Delta\omega} d\omega[W(\omega)e^{-i\omega t}+\text{H.c.}] , \tag{3.27}$$

where

$$W(\omega)=\int_{\omega_c}^{\infty} d\omega' F(\omega,\omega')A^{\dagger}(\omega')A(\omega+\omega') \tag{3.28}$$

and it is understood that $\Delta\omega<2\omega_c$.

Equation (3.26) will be used as the basic model for a photodetector in the remainder of this paper. Although this expression is bilinear and normal ordered in $A^{(-)}(t)$ and $A^{(+)}(t)$, it differs from the expression one would have obtained had one followed the usual custom of neglecting the negative frequency component of $A(t)$ in Eq. (2.6). In particular one would have obtained

$$W(t)=\int_{-\infty}^{t} + dt'[k(t-t')A^{(-)}(t)A^{(+)}(t') + k(t'-t)A^{(-)}(t')A^{(+)}(t)] . \tag{3.29}$$

Although the terms neglected by using (3.29) generally turn out to be small, it will be shown in later sections that these terms are finite in the case of wideband homodyne detection and must be kept if one wishes to determine the ultimate performance of a homodyne detector responding to squeezed coherent radiation.

IV. PHOTODETECTORS WITH LESS THAN UNIT QUANTUM EFFICIENCY

In this section, a photodetector with less than unit quantum efficiency will be modeled. Less than unit quantum efficiency results from the presence of loss mechanisms by which photons can be absorbed in a photodetector without generating an observable photocurrent. By the fluctuation-dissipation theorem there must be equilibrium noise fluctuations associated with this loss. It will now be determined how this noise effects the photodetector's performance.

As shown in Fig. 3, a detector with less than unit quantum efficiency can be modeled by a unit-quantum-efficiency detector in front of which one has placed a beam splitter that only lets a fraction η of the incoming light pass to the detector. In order to prevent photons from entering the photodetector via the other port of the beam splitter a blackbody absorber has been placed at that port. The blackbody absorber will be a source of equilibrium noise which at optical frequencies will consist essentially of vacuum fluctuations for a room-temperature absorber. Let $A_S(t)$ denote the incoming light and $A_N(t)$ denote the equilibrium radiation emitted by the absorber. Assuming a frequency-independent reflectivity, the light field $A(t)$ entering the detector is given by

$$A(t)=\eta^{1/2}A_S(t)+(1-\eta)^{1/2}A_N(t) . \tag{4.1}$$

Employing the optical waveguide model of Sec. III, the fields $A_S(t)$ and $A_N(t)$ both have the form (3.1):

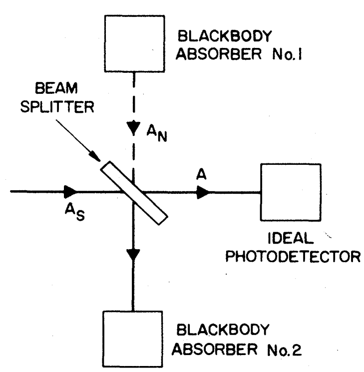

FIG. 3. Simulating a photodetector with less than unit quantum efficiency via a unit-quantum-efficiency photodetector and a beam splitter. Blackbody absorber 1 prevents stray light from entering the photodetector via the unused port of the beam splitter. Blackbody absorber 2 absorbs those photons of the incoming signal A_S that are not counted by the photodetector.

$$A_S^{(-)}(t)=a\int_0^\infty \frac{d\omega}{(\omega^2-\omega_c^2)^{1/4}}A_S^\dagger(\omega)e^{i\omega t}\,,$$
$$A_N^{(-)}(t)=a\int_0^\infty \frac{d\omega}{(\omega^2-\omega_c^2)^{1/4}}A_N^\dagger(\omega)e^{i\omega t}\,, \tag{4.2}$$

where the creation and annihilation operators have the usual commutation relations

$$[A_S(\omega),A_S(\omega')]=0\,, \tag{4.3}$$
$$[A_S(\omega),A_S^\dagger(\omega')]=\delta(\omega-\omega')\,, \tag{4.4}$$
$$[A_N(\omega),A_N(\omega')]=0\,, \tag{4.5}$$
$$[A_N(\omega),A_N^\dagger(\omega')]=\delta(\omega-\omega')\,. \tag{4.6}$$

Because the two light fields $A_S(t)$ and $A_N(t)$ are independent, one has

$$[A_S(\omega),A_N(\omega')]=[A_S(\omega),A_N^\dagger(\omega')]=0\,. \tag{4.7}$$

Substituting the positive and negative frequency components of (4.1) into (3.26) one has

$$\begin{aligned}W(t)=&\eta\int_{-\infty}^{t}dt'h(t-t')[A_S^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_S^{(+)}(t)]\\&+\eta^{1/2}(1-\eta)^{1/2}\int_{-\infty}^{t}dt'h(t-t')[A_S^{(-)}(t)A_N^{(+)}(t')+A_N^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_N^{(+)}(t)+A_N^{(-)}(t')A_S^{(+)}(t)]\\&+(1-\eta)\int_{-\infty}^{t}dt'h(t-t')[A_N^{(-)}(t)A_N^{(+)}(t')+A_N^{(-)}(t')A_N^{(+)}(t)]\,.\end{aligned} \tag{4.8}$$

Such an approach, in which (4.1) is substituted into the expression for a unit-quantum-efficiency photodetector in order to obtain the response of a photodetector with quantum efficiency η, has been used by Yuen and Shapiro.[6,8] The second and third terms of (4.8) arise from the presence of the equilibrium noise field. The first noise term arises from the interference between the vacuum field and the signal field. The second noise term is a dark current term which counts the photons emitted from the blackbody absorber or loss when the absorber's temperature is greater than zero. When the equilibrium noise field consists of vacuum fluctuations the expectation value of the two noise terms vanish and the expectation value of the photoemission-rate operator becomes

$$\langle W(t)\rangle=\eta\left\langle\int_{-\infty}^{\infty}dt'h(t-t')[A_s^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_S^{(+)}(t)]\right\rangle\,. \tag{4.9}$$

As expected, the mean counting rate is reduced by the efficiency factor η. The first noise term of (4.8), which represents the interference of the vacuum field with the signal field, contributes to the variance or fluctuations about the mean counting rate $\langle W(t)\rangle$ as can be seen by evaluating $\langle W^2(t)\rangle$:

$$\begin{aligned}\langle W^2(t)\rangle=&\eta^2\left\langle\left[\int_{-\infty}^{t}dt'h(t-t')[A_S^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_S^{(+)}(t)]\right]\right\rangle\\&+\eta(1-\eta)\int_{-\infty}^{t}dt'\int_{-\infty}^{t}dt''h(t-t')h(t-t'')\\&\times\{\langle A_s^{(-)}(t)A_S^{(+)}(t')\rangle[A_N^{(+)}(t'),A_N^{(-)}(t)]+\langle A_S^{(-)}(t)A_S^{(+)}(t)\rangle[A_N^{(+)}(t'),A_N^{(-)}(t')]\\&+\langle A_S^{(-)}(t')A_S^{(+)}(t')\rangle[A_N^{(+)}(t),A_N^{(-)}(t)]\\&+\langle A_S^{(-)}(t')A_S^{(+)}(t)\rangle[A_N^{(+)}(t),A_N^{(-)}(t')]\}\,.\end{aligned} \tag{4.10}$$

Since the commutators for the noise field operators are nonzero, vacuum fluctuation noise contributes significantly to the variance of $W(t)$ for quantum efficiencies η less than unity.

The photodetector with less than unit quantum efficiency modeled here will be used in Sec. V to discuss balanced homodyne detection.

V. BALANCED HOMODYNE DETECTION

The balanced homodyne detector is depicted in Fig. 1. Incoming signal light $A_S(t)$ is combined with local oscillator light $A_{\mathrm{LO}}(t)$ via a 50%-50% beam splitter. The two resulting beams are directed to two photodetectors. It is the difference mode current generated by these two photodetectors that is measured. In this section the operator for this difference mode current is constructed and some of its properties discussed. In order to simplify the analysis it is assumed that both photodetectors have the same quantum efficiency η. Let $A_S(t)$ and $A_{\mathrm{LO}}(t)$ denote, respectively, the field operators for the incoming signal and the local oscillator light. At the 50%-50% beam splitter these two light beams are combined to produce two light beams $A_1(t)$ and $A_2(t)$ directed towards photodetectors 1 and 2, respectively. The unitary transformation performed by the 50%-50% beam splitter is taken to be

$$\begin{aligned} A_1(t) &= \frac{1}{\sqrt{2}}[A_S(t)+A_{\mathrm{LO}}(t)] , \\ A_2(t) &= \frac{1}{\sqrt{2}}[A_S(t)-A_{\mathrm{LO}}(t)] . \end{aligned} \tag{5.1}$$

It is immediately apparent from this transformation that when the local oscillator light and signal light interfere constructively at photodetector 1 they interfere destructively at photodetector 2. This interference gives rise to the difference mode photocurrent. Let $A_{N1}(t)$ and $A_{N2}(t)$ denote, respectively, the equilibrium noise fields associated with the losses in the photodetectors 1 and 2, then the photoemission-rate operator $W_1(t)$ for photodetector 1 can be written as

$$\begin{aligned} W_1(t) = \int_{-\infty}^{t} dt' h(t-t') \Bigg[& \frac{\eta}{2}[A_{\mathrm{LO}}^{(-)}(t)A_{\mathrm{LO}}^{(+)}(t')+A_{\mathrm{LO}}^{(-)}(t')A_{\mathrm{LO}}^{(+)}(t)+A_S^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_S^{(+)}(t)] \\ & +\frac{\eta}{2}[A_S^{(-)}(t)A_{\mathrm{LO}}^{(+)}(t')+A_{\mathrm{LO}}^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_{\mathrm{LO}}^{(+)}(t)+A_{\mathrm{LO}}^{(-)}(t')A_S^{(+)}(t)] \\ & +\left[\frac{\eta-\eta^2}{2}\right]^{1/2}[A_S^{(-)}(t)A_{N1}^{(+)}(t')+A_{N1}^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_{N1}^{(+)}(t)+A_{N1}^{(-)}(t')A_S^{(+)}(t)] \\ & +\left[\frac{\eta-\eta^2}{2}\right]^{1/2}[A_{\mathrm{LO}}^{(-)}(t)A_{N1}^{(+)}(t')+A_{N1}^{(-)}(t)A_{\mathrm{LO}}^{(+)}(t')+A_{\mathrm{LO}}^{(-)}(t')A_{N1}^{(+)}(t)+A_{N1}^{(-)}(t')A_{\mathrm{LO}}^{(+)}(t)] \\ & +(1-\eta)[A_{N1}^{(-)}(t)A_{N1}^{(+)}(t')+A_{N1}^{(-)}(t)A_{N1}^{(+)}(t')] \Bigg] . \end{aligned} \tag{5.2}$$

A similar expression can be obtained for the photoemission-rate operator $W_2(t)$ for photodetector 2. By detecting the difference mode current, that is, measuring $W_1(t)-W_2(t)$, the terms bilinear in A_{LO} or A_S will cancel since they give rise to common mode signals. Hence the homodyne detector becomes insensitive to fluctuations in the local oscillator intensity $A_{\mathrm{LO}}^{(-)}(t)A_{\mathrm{LO}}^{(+)}(t)$. Introducing the common mode $A_{\mathrm{NS}}(t)$ (symmetric mode) and difference mode $A_{\mathrm{NA}}(t)$ (antisymmetric mode) noise operators

$$\begin{aligned} A_{\mathrm{NS}}(t) &= \frac{1}{\sqrt{2}}[A_{N1}(t)+A_{N2}(t)] , \\ A_{\mathrm{NA}}(t) &= \frac{1}{\sqrt{2}}[A_{N1}(t)-A_{N2}(t)] , \end{aligned} \tag{5.3}$$

the difference mode photoemission-rate operator

$$W_D(t) \equiv W_1(t)-W_2(t) \tag{5.4}$$

can be put into the form

$$\begin{aligned} W_D(t) = \int_{-\infty}^{t} dt' h(t-t')\{ & \eta[A_S^{(-)}(t)A_{\mathrm{LO}}^{(+)}(t)+A_{\mathrm{LO}}^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_{\mathrm{LO}}^{(+)}(t)+A_{\mathrm{LO}}^{(-)}(t')A_S^{(+)}(t)] \\ & +\eta^{1/2}(1-\eta)^{1/2}[A_{\mathrm{LO}}^{(-)}(t)A_{\mathrm{NS}}^{(+)}(t')+A_{\mathrm{NS}}^{(-)}(t)A_{\mathrm{LO}}^{(+)}(t')+A_{\mathrm{LO}}^{(-)}(t')A_{\mathrm{NS}}^{(+)}(t)+A_{\mathrm{NS}}^{(-)}(t')A_{\mathrm{LO}}^{(+)}(t)] \\ & +\eta^{1/2}(1-\eta)^{1/2}[A_S^{(-)}(t)A_{\mathrm{NA}}^{(+)}(t')+A_{\mathrm{NA}}^{(-)}(t)A_S^{(+)}(t')+A_S^{(-)}(t')A_{\mathrm{NA}}^{(+)}(t)+A_{\mathrm{NA}}^{(-)}(t')A_S^{(+)}(t)] \\ & +(1-\eta)[A_{\mathrm{NS}}^{(-)}(t)A_{\mathrm{NS}}^{(+)}(t')+A_{\mathrm{NS}}^{(-)}(t')A_{\mathrm{NS}}^{(+)}(t)+A_{\mathrm{NA}}^{(-)}(t)A_{\mathrm{NA}}^{(+)}(t')+A_{\mathrm{NA}}^{(-)}(t')A_{\mathrm{NA}}^{(+)}(t)]\} . \end{aligned} \tag{5.5}$$

If the noise consists of vacuum fluctuations, the expectation value of W_D becomes relatively simple:

$$\langle W_D(t)\rangle = \eta\int_{-\infty}^{t} dt' h(t-t')\langle A_S^{(-)}(t)A_{\mathrm{LO}}^{(+)}(t')+A_{\mathrm{LO}}^{(-)}A_S^{(+)}(t')+A_S^{(-)}(t')A_{\mathrm{LO}}^{(+)}(t)+A_{\mathrm{LO}}^{(-)}(t')A_S^{(+)}(t)\rangle . \tag{5.6}$$

This is just the expectation value of the difference mode signal generated by the interference of the signal light with the local oscillator light. The first two noise terms of (5.5), due, respectively, to the interference of the local oscillator light with the symmetric noise field and the interference of the signal light with the antisymmetric noise field, will contribute to the expectation value of $\langle W_D^2(t)\rangle$.

It will be convenient from now on to work with the difference mode current operator

$$I_D(t)=\frac{e}{2\pi}\int_0^{\Delta\omega}d\omega[W_D(\omega)e^{-i\omega t}+\text{H.c.}]\ , \tag{5.7}$$

where the Fourier transform $W_D(\omega)$ of (5.5) is given by

$$\begin{aligned}W_D(\omega)=\int_{\omega_c}^{\infty}d\omega' F(\omega,\omega')\{&\eta[A_{\text{LO}}^{\dagger}(\omega')A_S(\omega+\omega')+A_S^{\dagger}(\omega')A_{\text{LO}}(\omega+\omega')]\\&+\eta^{1/2}(1-\eta)^{1/2}[A_{\text{LO}}^{\dagger}(\omega')A_{\text{NS}}(\omega+\omega')+A_{\text{NS}}^{\dagger}(\omega')A_{\text{LO}}(\omega+\omega')]\\&+\eta^{1/2}(1-\eta)^{1/2}[A_S^{\dagger}(\omega')A_{\text{NA}}(\omega+\omega')+A_{\text{NA}}^{\dagger}(\omega')A_S(\omega+\omega')]\\&+(1-\eta)[A_{\text{NS}}^{\dagger}(\omega')A_{\text{NS}}(\omega+\omega')+A_{\text{NA}}^{\dagger}(\omega')A_{\text{NA}}(\omega+\omega')]\}\ .\end{aligned} \tag{5.8}$$

It will now be assumed that the local oscillator light is in a spectrally pure coherent state $|\,\text{LO}\rangle$ specified by

$$A_{\text{LO}}(\omega)\,|\,\text{LO}\rangle=Ae^{-i\phi}\delta(\omega-\omega_0)\,|\,\text{LO}\rangle\ , \tag{5.9}$$

where A is a positive real number specifying the local oscillator's field amplitude, ϕ specifies the local oscillator's phase, and ω_0 is the frequency of the local oscillator light. In terms of the local oscillator's power P_{LO}, A is given by

$$A=\left[\frac{\pi P_{\text{LO}}}{\hbar\omega_0}\right]^{1/2}. \tag{5.10}$$

The expectation value of $W_D(\omega)$ with this local oscillator state is

$$\langle W_D(\omega)\rangle=A\eta\langle F(\omega,\omega_0)e^{i\phi}A_S(\omega_0+\omega)+F(\omega,\,\omega_0-\omega)e^{-i\phi}A_S^{\dagger}(\omega_0-\omega)\rangle\ . \tag{5.11}$$

Hence

$$\langle I_D(t)\rangle=\eta A\frac{e}{2\pi}\left\langle\int_0^{\Delta\omega}d\omega\{[F(\omega,\omega_0)e^{i\phi}A_S(\omega_0+\omega)+F(\omega,\omega_0-\omega)e^{-i\phi}A_S^{\dagger}(\omega_0-\omega)]e^{-i\omega t}+\text{H.c.}\}\right\rangle\ . \tag{5.12}$$

The expectation value of $I_D^2(t)$ is

$$\begin{aligned}\langle I_D^2(t)\rangle=&\eta^2A^2\left[\frac{e}{2\pi}\right]^2\left\langle\left[\int_0^{\Delta\omega}d\omega\{[F(\omega,\omega_0)e^{i\phi}A_S(\omega_0+\omega)+F(\omega,\,\omega_0-\omega)e^{-i\phi}A_S^{\dagger}(\omega_0-\omega)]e^{-i\omega t}+\text{H.c.}\}\right]^2\right\rangle\\&+\eta(1-\eta)A^2\left[\frac{e}{2\pi}\right]^2\int_0^{\Delta\omega}d\omega[F(\omega,\omega_0)F^*(\omega,\omega_0)+F^*(\omega,\omega_0-\omega)F(\omega,\,\omega_0-\omega)]\\&+\eta\left[\frac{e}{2\pi}\right]^2\int_0^{\Delta\omega}d\omega\int_{\omega_c}^{\infty}d\omega'\int_0^{\Delta\omega}d\omega''\int_{\omega_c}^{\infty}d\omega'''\langle F(\omega,\omega')e^{-i\omega t}A_S^{\dagger}(\omega')\\&\qquad\times[F(\omega'',\omega''')e^{-i\omega''t}A_S(\omega''+\omega''')\delta(\omega+\omega'-\omega''')\\&\qquad\quad+F^*(\omega'',\omega''')e^{i\omega''t}A_S(\omega''')\delta(\omega+\omega'-\omega''-\omega''')]\\&\qquad+F^*(\omega,\omega')e^{i\omega t}A_S^{\dagger}(\omega+\omega')\\&\qquad\times[F(\omega'',\omega''')e^{-i\omega''t}A_S(\omega''+\omega''')\delta(\omega'-\omega''')\\&\qquad\quad+F^*(\omega''+\omega''')e^{i\omega''t}A_S(\omega''')\delta(\omega-\omega''-\omega''')]\rangle\ .\end{aligned} \tag{5.13}$$

The first two terms of this expression are proportional to the local oscillator power A^2. The last term is independent of the local oscillator and can be made negligible for sufficiently large local oscillator power. From now on it will be assumed that the last term of (5.13) can be neglected. Equation (5.13) then reduces to

$$\langle I_D^2(t)\rangle = \eta^2 A^2 \left[\frac{e}{2\pi}\right]^2 \left\langle \left[\int_0^{\Delta\omega} d\omega \{ [F(\omega,\omega_0)e^{i\phi}A_S(\omega_0+\omega) + F(\omega,\omega_0-\omega)e^{-i\phi}A_S^\dagger(\omega_0-\omega)]e^{-i\omega t} + \text{H.c.}\} \right]^2 \right\rangle$$
$$+\eta(1-\eta)A^2\left[\frac{e}{2\pi}\right]^2 \int_0^{\Delta\omega} d\omega [F(\omega,\omega_0)F^*(\omega,\omega_0) + F^*(\omega,\omega_0-\omega)F(\omega,\omega_0-\omega)] \ . \tag{5.14}$$

The second term arises from the noise associated with the losses of the photodetectors with less than unit quantum efficiency. By comparing (5.11) with the first term of (5.14) one sees that a homodyne detector measures the operator

$$I_S(t) = \int_0^{\Delta\omega} d\omega \{ [F(\omega,\omega_0)e^{i\phi}A_S(\omega_0+\omega) + F(\omega,\omega_0-\omega)e^{-i\phi}A_S^\dagger(\omega_0-\omega)]e^{-i\omega t} + \text{H.c.}\} \ . \tag{5.15}$$

In order to get some insight into the physical quantity this operator represents, consider the incoming field operator (3.1) written in component form:

$$A_S(t) = A_S'(t)\cos(\omega_0 t+\phi) + A_S''(t)\sin(\omega_0 t+\phi) \ , \tag{5.16}$$

where the two field component operators $A_S'(t)$ and $A_S''(t)$ are given by

$$A_S'(t) = a\int_0^{\Delta\omega} d\omega \left[\left[\frac{A(\omega_0+\omega)e^{i\phi}}{[(\omega_0+\omega)^2-\omega_c^2]^{1/4}} + \frac{A^\dagger(\omega_0-\omega)e^{-i\phi}}{[(\omega_0-\omega)^2-\omega_c^2]^{1/4}} \right] e^{-i\omega t} + \text{H.c.} \right] \tag{5.17}$$

and

$$A_S''(t) = a\int_0^{\Delta\omega} \left[\left[-\frac{iA(\omega_0+\omega)e^{i\phi}}{[(\omega_0+\omega)^2-\omega_c^2]^{1/4}} + \frac{iA^\dagger(\omega_0-\omega)e^{-i\phi}}{[(\omega_0-\omega)^2-\omega_c^2]^{1/4}} \right] e^{-i\omega t} + \text{H.c.} \right] \tag{5.18}$$

where the integration has been restricted to the detector bandwidth. It is readily apparent that $A_S'(t)$ and $A_S''(t)$ are Hermitian and hence, in principle, observable operators. The component operator $A_S'(t)$ has a form similar to $I_S(t)$. When the bandwidth $\Delta\omega$ is made sufficiently narrow such that $[(\omega_0 \pm \omega)^2-\omega_c^2]^{1/4}$ can be approximated by $(\omega_0^2-\omega_c^2)^{1/4}$ and $F(\omega,\omega_0)$ and $F(\omega,\omega_0-\omega)$ by $F(0,\omega_0)$, the operators $I_S(t)$ and $A_S'(t)$ are in fact proportional to each other. Hence it has been shown that a balanced homodyne detector measures one field component of the incoming field. The particular component it detects is determined by ϕ, the phase of the local oscillator light.

Before describing the response of a balanced homodyne detector to squeezed coherent radiation, it will be useful to compute the response of the detector in the absence of incoming light, that is, when the incoming light is described by the vacuum state. From (5.12) and (5.14) one has

$$\langle I_D(t)\rangle = 0 \tag{5.19}$$

and

$$\langle I_D^2(t)\rangle = \eta A^2 \left[\frac{e}{2\pi}\right]^2 \times \int_0^{\Delta\omega} d\omega [\,|F(\omega,\omega_0)|^2 + |F(\omega,\omega_0-\omega)|^2] \ . \tag{5.20}$$

So, although the mean difference mode current $I_D(t)$ is zero, the shot noise or fluctuation in this current is not. This shot noise arises from the interference of the local oscillator light with the incoming vacuum fluctuations, and the vacuum fluctuations associated with the losses of the photodetectors. In Sec. VI will be described states of the radiation field, called squeezed coherent states, whose fluctuations in $A_S'(t)$ and $I_D(t)$ are less than those of the vacuum state.

VI. WIDEBAND SQUEEZED COHERENT STATES

In Sec. V the component operators $A_S'(t)$ and $A_S''(t)$ for an optical field were introduced. These observable operators are noncommuting having the commutation relation

$$[A_S'(t), A_S''(t)] = 2ia^2\int_0^{\Delta\omega} d\omega \left[\frac{1}{[(\omega_0+\omega)^2-\omega_c^2]^{1/2}} + \frac{1}{[(\omega_0-\omega)^2-\omega_c^2]^{1/2}} \right] \tag{6.1}$$

and thus satisfy the Heisenberg uncertainty relation

$$\Delta A_S'(t)\Delta A_S''(t) \ge a^2\int_0^{\Delta\omega} d\omega \left[\frac{1}{[(\omega_0+\omega)^2-\omega_c^2]^{1/2}} + \frac{1}{[(\omega_0-\omega)^2-\omega_c^2]^{1/2}} \right] . \tag{6.2}$$

This implies that there is a trade off in the precision with which one can simultaneously measure $A_S'(t)$ and $A_S''(t)$. Since $A_S'(t)$ and $A_S''(t)$ are linear in the creation and annihilation operators their expectation values vanish for the vacuum state:

$$\langle 0|A_S'(t)|0\rangle = 0 \ , \qquad \langle 0|A_S''(t)|0\rangle = 0 \ . \tag{6.3}$$

The variance of these operators for the vacuum state does not vanish, and in particular,

$$\langle 0|[A_S'(t)]^2|0\rangle=\langle 0|[A_S''(t)]^2|0\rangle$$
$$=a^2\int_0^{\Delta\omega}d\omega\left[\frac{1}{[(\omega_0+\omega)^2-\omega_c^2]^{1/2}}+\frac{1}{[(\omega_0-\omega)^2-\omega_c^2]^{1/2}}\right]. \tag{6.4}$$

Since $\Delta A_S'\equiv[\langle(A_S')^2\rangle-\langle A_S'\rangle^2]^{1/2}$ one sees from (6.3), (6.4), and (6.2) that the vacuum state is a minimum uncertainty state for the field component operators. In this section some states of the radiation field, called squeezed coherent states, will be introduced which have less fluctuations in $A_S'(t)$ than the vacuum state, but because of (6.2) such states must have greater fluctuations in $A_S''(t)$ than the vacuum state has. Squeezed coherent states, also referred to as two-photon coherent states[4] and squeezed states,[18,19] can in principle be generated via a number of physical processes and devices,[4,5] among them being degenerate parametric amplifiers and degenerate four-wave mixers. Caves[20] has recently reviewed the properties of wideband squeezed states and developed a mathematical formalism to conveniently deal with them. For the purposes of this paper, the squeezed-state generator or "squeezer" will be regarded as a black box with an output port which is capable of performing the following canonical transformation[21] on incoming quanta $A_{\rm in}(\omega_0\pm\omega)$:

$$A_S(\omega_0+\omega)=G(\omega)A_{\rm in}(\omega_0+\omega)+M(\omega)A_{\rm in}^\dagger(\omega_0-\omega)\,,$$
$$A_S(\omega_0-\omega)=G(-\omega)A_{\rm in}(\omega_0-\omega)+M(-\omega)A_{\rm in}^\dagger(\omega_0+\omega)\,, \tag{6.5}$$

where $A_S(\omega_0\pm\omega)$ denotes the annihilation operator for the quanta leaving the output port of the squeezer. Since both A_S and $A_{\rm in}$ must satisfy commutation relations of the form (3.2) and (3.3), one requires

$$|G(\omega)|^2-|M(\omega)|^2=1 \tag{6.6}$$

and

$$G(\omega)M(-\omega)=G(-\omega)M(\omega)\,. \tag{6.7}$$

From these two expressions one can readily show

$$|G(\omega)|^2=|G(-\omega)|^2\,,$$
$$|M(\omega)|^2=|M(-\omega)|^2\,, \tag{6.8}$$

that is, the norms of G and M must be symmetric functions of ω. If light described by a coherent state is fed into the input port of the squeezer, the light delivered to the output port is in a squeezed coherent state. For simplicity, we will restrict ourselves to the case when the light fed into the squeezer consists only of vacuum fluctuations, that is, when the input port of the squeezer is terminated by a cold blackbody absorber. Then

$$A_{\rm in}(\omega_0\pm\omega)|0\rangle=0 \tag{6.9}$$

for all ω. The squeezed vacuum or light delivered to the output port of the squeezer will be a multiphoton state. By substituting (6.5) into (5.17) a few of the properties of the squeezed vacuum state $|{\rm sv}\rangle$ can be calculated. In particular, since $A_S'(t)$ after substituting (6.5) into (5.17) is linear in $A_{\rm in}(\omega)$ and $A_{\rm in}^\dagger(\omega)$, one has

$$\langle{\rm sv}|A_S'(t)|{\rm sv}\rangle=0 \tag{6.10}$$

and similarly

$$\langle{\rm sv}|A_S''(t)|{\rm sv}\rangle=0\,, \tag{6.11}$$

that is, the mean amplitude of the two quadrature components of the squeezed vacuum is zero. The mean-square fluctuation of $A_S'(t)$ about its mean value is given by

$$\langle{\rm sv}|[A_S'(t)]^2|{\rm sv}\rangle=a^2\int_0^{\Delta\omega}d\omega\left[\frac{1}{[(\omega_0+\omega)^2-\omega_c^2]^{1/2}}+\frac{1}{[(\omega_0-\omega)^2-\omega_c^2]^{1/2}}\right]$$
$$\times\left[|G(\omega)|^2+|M(\omega)|^2+4\frac{[(\omega_0-\omega)^2-\omega_c^2]^{1/4}[(\omega_0+\omega)^2-\omega_c^2]^{1/4}}{[(\omega_0+\omega)^2-\omega_c^2]^{1/2}+[(\omega_0-\omega)^2-\omega_c^2]^{1/2}}{\rm Re}[G(\omega)M(-\omega)e^{2i\phi}]\right]. \tag{6.12}$$

The quantity inside the integral of (6.12) is the power spectrum of $A_S'(t)$. Dividing this power spectrum by the vacuum power spectrum of $A_S'(t)$ given in (6.4), one can construct the normalized power spectrum

$$S(\omega)=|G(\omega)|^2+|M(\omega)|^2+4\frac{[(\omega_0-\omega)^2-\omega_c^2]^{1/4}[(\omega_0+\omega)^2-\omega_c^2]^{1/4}}{[(\omega_0+\omega)^2-\omega_c^2]^{1/2}+[(\omega_0-\omega)^2-\omega_c^2]^{1/2}}\times{\rm Re}[G(\omega)M(-\omega)e^{2i\phi}]\,. \tag{6.13}$$

When $M(\omega)=0$ the transformation (6.5) with (6.6) reduces to the identity transformation and the light delivered to the input port of the squeezer comes out of the output port unchanged. The output light then consists of vacuum fluctuations and

$$S(\omega)=1 \tag{6.14}$$

as it should for a noise-power spectrum normalized to the vacuum noise-power spectrum of $A_S'(t)$. When ω is small compared to ω_0 and ω_c the normalized power spectrum can be written in the form

$$S(\omega)=|G(\omega)|^2+|M(\omega)|^2+2\left[1-\frac{\omega^2\omega_0^2}{2(\omega_0^2-\omega_c^2)^2}+\cdots\right]\times{\rm Re}[G(\omega)M(-\omega)e^{2i\phi}]\,. \tag{6.15}$$

If the phase ϕ is chosen such that

$$\mathrm{Re}[G(\omega)M(-\omega)e^{2i\phi}] = -|G(\omega)|\,|M(\omega)| \qquad (6.16)$$

and $|G(\omega)|$ is made large, so that by Eq. (6.6) $|M(\omega)|$ can be expanded as

$$|M(\omega)| = |G(\omega)|\left[1 - \frac{1}{2|G(\omega)|^2} + \cdots\right], \qquad (6.17)$$

Eq. (6.15) can be put into the form

$$S(\omega) \simeq \frac{1}{4|G(\omega)|^2} + \frac{\omega^2\omega_0^2}{(\omega_0^2-\omega_c^2)^2}|G(\omega)|^2 . \qquad (6.18)$$

As $|G(\omega)|^2$ is made large, $S(\omega)$ first decreases and then increases. The minimum occurs when

$$|G(\omega)|^2 = \frac{\omega_0^2-\omega_c^2}{2\omega_0\omega} . \qquad (6.19)$$

The smallest value that $S(\omega)$ takes on at ω is then

$$S_{\min}(\omega) = \frac{\omega_0\omega}{(\omega_0^2-\omega_c^2)} \geq \frac{\omega}{\omega_0} . \qquad (6.20)$$

It is seen from (6.20) that the extent to which the fluctuations in $A'_S(t)$ can be reduced with squeezed vacuum radiation depends on the detector bandwidth.

The results of this section can readily be generalized to an arbitrary squeezed coherent state produced by sending coherent radiation into the input port of the squeezer. The normalized noise-power spectrum of $A'_S(t)$ will still be given by (6.15), however the expectation value of $A'_S(t)$ will no longer be zero as it was for the vacuum state. The response of a wideband homodyne detector to wideband squeezed coherent states will now be determined.

VII. HOMODYNE DETECTION OF WIDEBAND SQUEEZED COHERENT RADIATION

Having defined the squeezed vacuum state $|\mathrm{sv}\rangle$, one can now calculate the response of a balanced homodyne detector to such a state. By substituting (6.5) into (5.11), one has

$$\langle \mathrm{sv}|I_D(t)|\mathrm{sv}\rangle = 0 , \qquad (7.1)$$

that is, the mean difference mode current generated by the balanced homodyne detector in response to an incoming squeezed vacuum state is zero. The mean-square fluctuation of the difference mode photocurrent, using (6.5) and (5.14), is given by

$$\langle \mathrm{sv}|I_D^2(t)|\mathrm{sv}\rangle = \eta A^2\left[\frac{e}{2\pi}\right]^2 \int_0^{\Delta\omega} d\omega[\,|F(\omega,\omega_0)|^2 + |F(\omega,\omega_0-\omega)|^2] \times \left[\eta[\,|G(\omega)|^2+|M(\omega)|^2] + 4\eta\,\mathrm{Re}\left[\frac{F(\omega,\omega_0)F^*(\omega,\omega_0-\omega)G(\omega)M(-\omega)e^{2i\phi}}{|F(\omega,\omega_0)|^2+|F(\omega,\omega_0-\omega)|^2}\right] + 1 - \eta\right] . \qquad (7.2)$$

The quantity inside the integral can be recognized as the power spectrum of the difference mode current. As was done in Sec. VI, it is convenient to normalize this power spectrum with respect to the power spectrum for the vacuum state which is given by (5.20). The normalized noise-power spectrum for the difference mode current $S_D(\omega)$ is given by

$$S_D(\omega) = \eta[\,|G(\omega)|^2 + |M(\omega)|^2] + 4\eta\,\mathrm{Re}\left[\frac{F(\omega,\omega_0)F^*(\omega,\omega_0-\omega)G(\omega)M(-\omega)e^{2i\phi}}{|F(\omega,\omega_0)|^2+|F(\omega,\omega_0-\omega)|^2}\right] + 1 - \eta . \qquad (7.3)$$

This quantity is readily measurable experimentally by feeding the difference mode current from the balanced homodyne detector into a spectrum analyzer and comparing the spectrum obtained for squeezed vacuum light with the base line spectrum obtained in the absence of signal light. The term $1-\eta$ in (7.3) is a noise floor below which $S_D(\omega)$ cannot be reduced. This noise comes from the losses associated with the photodetectors. This noise floor approaches zero as the efficiency of the photodetectors approaches unity. For simplicity, throughout the rest of this section a unit quantum efficiency will be assumed, then (7.3) reduces to

$$S_D(\omega) = |G(\omega)|^2 + |M(\omega)|^2 + 4\,\mathrm{Re}\left[\frac{F(\omega,\omega_0)F^*(\omega,\omega_0-\omega)G(\omega)M(-\omega)e^{2i\phi}}{|F(\omega,\omega_0)|^2+|F(\omega,\omega_0-\omega)|^2}\right] \qquad (7.4)$$

This expression is quite similar to that given in (6.13) for the normalized power spectrum of the $A'_S(t)$ component of the signal field. For a particular frequency ω, squeezing is optimized when the local oscillator phase ϕ is chosen such that

$$S_D(\omega) = |G(\omega)|^2 + |M(\omega)|^2 - 2D(\omega,\omega_0)|G(\omega)|\,|M(\omega)| , \qquad (7.5)$$

where

$$D(\omega,\omega_0) = \frac{2|F(\omega,\omega_0)|\,|F(\omega,\omega_0-\omega)|}{|F(\omega,\omega_0)|^2+|F(\omega,\omega_0-\omega)|^2} . \qquad (7.6)$$

In order to determine the frequency dependence of $D(\omega,\omega_0)$ it is useful to Taylor-series expand $F(\omega,\omega_0)$ about $\omega=0$:

$$F(\omega,\omega_0)=F(0,\omega_0)+\left.\frac{\partial F(\omega,\omega_0)}{\partial\omega}\right|_{\omega=0}\omega+\frac{1}{2}\left.\frac{\partial^2 F(\omega,\omega_0)}{\partial\omega^2}\right|_{\omega=0}\omega^2+\cdots \quad (7.7)$$

Using Eqs. (3.24), (3.12), and (3.9), one can show that the first term of the Taylor-series expansion is given by

$$F(0,\omega_0)=1 . \quad (7.8)$$

It is convenient to break the other terms of the Taylor-series expansion into their real and imaginary parts, so

$$F(\omega,\omega_0)=1+(A+iB)\omega+\frac{(C+iD)}{2}\omega^2+\cdots , \quad (7.9)$$

where

$$A=\mathrm{Re}\left[\frac{\partial F(\omega,\omega_0)}{\partial\omega}\right]_{\omega=0} . \quad (7.10)$$

Expressed in this form one can exploit the relation between $F(\omega,\omega_0)$ and $F(\omega,\omega_0-\omega)$. In particular, from (3.24) one can show

$$F^*(\omega,\omega_0-\omega)=F(-\omega,\omega_0) , \quad (7.11)$$

where use has been made of the fact that $h(\tau)$ is real:

$$h(\tau)=h^*(\tau) \quad (7.12)$$

as can be seen from (3.12) and (3.9). Using (7.11) and (7.9), being careful to keep all terms of order ω^2, one can put $D(\omega,\omega_0)$ into the form

$$D(\omega,\omega_0)=1-2A^2\omega^2+\sigma(\omega^3) . \quad (7.13)$$

Thus, in order to determine the frequency behavior of $D(\omega,\omega_0)$ to order ω^2, one need only evaluate A. From (3.24) one has

$$\mathrm{Re}\left[\frac{\partial F(\omega,\omega_0)}{\partial\omega}\right]_{\omega=0}=-\frac{\pi a^2\omega_0}{(\omega_0^2-\omega_c^2)^{1/2}}\int_{-\infty}^{\infty}d\tau\, h(\tau)\cos(\omega_0\tau)+\frac{\pi a^2}{(\omega_0^2-\omega_c^2)^{1/2}}\int_{-\infty}^{\infty}d\tau\,\tau h(\tau)\sin(\omega_0\tau) , \quad (7.14)$$

where the limits of the integration have been extended to $-\infty$ by exploiting the fact that, from (3.12), $h(\tau)$ is an even function of τ:

$$h(\tau)=h(-\tau) . \quad (7.15)$$

Using (3.12) and (3.9) the integrals in (7.14) can readily be evaluated:

$$\int_{-\infty}^{\infty}d\tau\, h(\tau)\cos(\omega_0\tau)=\frac{1}{2\pi a^2}(\omega_0^2-\omega_c^2)^{1/2} , \quad (7.16)$$

$$\int_{-\infty}^{\infty}d\tau\,\tau h(\tau)\sin(\omega_0\tau)=-\frac{1}{2\pi a^2}\frac{\omega_0}{(\omega_0^2-\omega_c^2)^{1/2}} , \quad (7.17)$$

so

$$\mathrm{Re}\left[\frac{\partial F(\omega,\omega_0)}{\partial\omega}\right]_{\omega=0}=-\frac{\omega_0}{(\omega_0^2-\omega_c^2)} . \quad (7.18)$$

Hence to order ω^2 one has

$$D(\omega,\omega_0)=1-\frac{2\omega_0^2\omega^2}{(\omega_0^2-\omega_c^2)^2}+\cdots . \quad (7.19)$$

Equation (7.5) can now be written in the form

$$S_D(\omega)=|G(\omega)|^2+|M(\omega)|^2-2\left[1-\frac{2\omega_0^2\omega^2}{(\omega_0^2-\omega_c^2)^2}+\cdots\right]|G(\omega)|\,|M(\omega)| . \quad (7.20)$$

Comparing this expression with that of Eq. (6.15) when (6.16) holds, one sees that the expressions are very similar, except for a numerical factor of 4 difference between the coefficients in the ω^2 term in the large parentheses. Maximum squeezing now occurs when the gain is set at

$$|G(\omega)|^2=\frac{\omega_0^2-\omega_c^2}{4\omega\omega_0} . \quad (7.21)$$

The minimum value that $S_D(\omega)$ takes on is then

$$S_D^{\min}(\omega)=\frac{2\omega_0\omega}{\omega_0^2-\omega_c^2}\geq\frac{2\omega}{\omega_0} . \quad (7.22)$$

Comparing this with Eq. (6.20) one sees that a balanced homodyne detector's ultimate performance is a factor of 2 worse than that of the ideal measuring device for measuring one of the amplitude components of a radiation field. It has been shown in the preceding paper[21] that the ideal measuring device for measuring an amplitude component of the radiation field can be realized by replacing the photoemissive detectors in the balanced homodyne detector with bolometers or other devices that measure the operator $E^{(-)}(t)E^{(+)}(t)$, that is, the instantaneous power. For such a device Eq. (6.13) will hold. One could ask how the results of this paper might have changed had one followed the usual practice of neglecting the rapidly oscillating terms in the transition amplitude for photoemission, that is, if instead of starting with Eq. (3.4) one had started with

$$P(t)=\int_{-\infty}^{t}dt'\int_{-\infty}^{t}dt''\,k(t'-t'')A^{(-)}(t')A^{(+)}(t'') . \quad (7.23)$$

In this case the term proportional to ω^2 in the large parentheses of (7.20) would have been absent. For fast photodiodes operating at optical frequencies ω/ω_0 will be of the order of 10^{-5}, hence the ω^2 term will be too small to be of any consequence. If one uses SIS junctions[17] to perform homodyne detection at microwave frequencies (tens of GHz) and uses a 1-GHz bandwidth for the output, then ω/ω_0 will be of the order of 10^{-1} and hence the ω^2 term will be observable. Although the effects neglected by using (7.23) rather than (3.4) are generally very small, these effects must be included if one wishes to determine the ultimate performance possible for systems employing squeezed coherent states and homodyne detectors.

VIII. CONCLUSIONS

A photoemission rate operator appropriate for a unit-quantum-efficiency photoemissive detector has been constructed. This operator has been generalized for the case when the photoemissive detector has less than unit quantum efficiency. The resulting operator was used to discuss wideband balanced homodyne detection. The noise-power spectrum for the difference mode current of the balanced homodyne detector was computed for the case when the incoming signal consisted of a wideband squeezed coherent state. These calculations were carried out without making local approximations to the sensitivity function and without neglecting rapidly oscillating terms in the photoemission amplitude. In real photodetectors the absorbing atoms are distributed over a finite thickness. The electromagnetic field amplitude will exponentially decay as it penetrates this medium and the absorbing atoms will emit vacuum fluctuations. It is not clear to what extent these processes will modify (3.26). Hence it may be useful to extend the analysis of this paper to more realistic models of photodetectors following the lines of approach recently taken by Bondurant.[16]

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