Brief Reports

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Doubly differential cross sections for electron-impact ionization of helium

A. C. Roy*
Fachbereich Physik der Universitat Kaiserslautern, D-6750 Kaiserslautern, West Germany

T. Watanabe
Atomic Processes Laboratory, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama 351-01, Japan
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The Glauber approximation is used to calculate doubly differential cross sections (DDCS) for electron-impact ionization of helium in the incident energy range 224.58–2824.58 eV. The present calculation is done for the symmetric geometry, i.e., the two outgoing electrons have equal energies, and for a scattering angle of 45°. A comparison is made of the present DDCS with the results of absolute measurements.

In a previous paper,[1] we applied the Glauber approximation[2] (GA) to the triply differential cross sections (TDCS) for electron-impact ionization of helium in the incident energy range 224.58–2824.58 eV. In the case of a symmetric geometry, i.e., the energies of the two outgoing electrons are equal, and for a scattering angle of 45°, the coplanar TDCS obtained in the GA show a definite improvement over the first Born results and are in reasonably good agreement with experiment. In view of the success of the GA in predicting the TDCS for the He$(e,2e)$He$^+$ process in the case of symmetric geometry, we apply this method to evaluate the doubly differential cross sections (DDCS) for the same process and compare the present DDCS with the absolute measurements of van Wingerden, Kimman, van Tilburg, and de Heer.[3]

In the GA, the amplitude for ionization of He by electron impact is given by

$$F(\mathbf{q},\mathbf{k}_2)=\frac{ik}{2\pi}\int d\mathbf{b}\,d\mathbf{r}_1\,d\mathbf{r}_2\,\phi_f^*(\mathbf{r}_1,\mathbf{r}_2)\Gamma(\mathbf{b};\mathbf{r}_1,\mathbf{r}_2)\times\phi_i(\mathbf{r}_1,\mathbf{r}_2)e^{i\mathbf{q}\cdot\mathbf{b}}\ , \quad (1)$$

where

$$\Gamma(\mathbf{b};\mathbf{r}_1,\mathbf{r}_2)=1-\left[\frac{|\mathbf{b}-\mathbf{s}_1|}{b}\right]^{2i\eta}\left[\frac{|\mathbf{b}-\mathbf{s}_2|}{b}\right]^{2i\eta},$$

$\mathbf{q}=\mathbf{k}-\mathbf{k}_1$ and $\eta=1/k$. Atomic units are used throughout, unless otherwise indicated. Here $\mathbf{k}$, $\mathbf{k}_1$, $\mathbf{k}_2$ are the momenta of the incoming, scattered, and ejected electrons, respectively, and $\mathbf{q}$ represents the momentum transfer. $\mathbf{b}$, $\mathbf{s}_1$, and $\mathbf{s}_2$ are the respective projections of the position vectors of the incident particle and the two bound electrons onto the plane perpendicular to the direction of the Glauber path integration. In Eq. (1), $\mathbf{q}$, $\mathbf{b}$, $\mathbf{s}_1$, and $\mathbf{s}_2$ are all coplanar; $\phi_i(\mathbf{r}_1,\mathbf{r}_2)$ and $\phi_f(\mathbf{r}_1,\mathbf{r}_2)$ represent, respectively, the wave functions of the initial and the final states of the target. For the intitial state of He, we have chosen the following form:

$$\phi_i(\mathbf{r}_1\mathbf{r}_2)=u(\mathbf{r}_1)u(\mathbf{r}_2)\ , \quad (2)$$

where

$$u(\mathbf{r})=\lambda^{3/2}\pi^{-1/2}e^{-\lambda r}\ , \quad (3)$$

whereas for the final state of He, we have adopted the following:

$$\phi_f(\mathbf{r}_1,\mathbf{r}_2)=2^{-1/2}[\nu(\mathbf{r}_1)\chi_{\mathbf{k}_2}^-(\mathbf{r}_2)+\nu(\mathbf{r}_2)\chi_{\mathbf{k}_2}^-(\mathbf{r}_1)]\ , \quad (4)$$

where

$$\chi_{k_2}^-(\mathbf{r})=(2\pi)^{-3/2}e^{\gamma\pi/2}\Gamma(1+i\gamma)e^{i\mathbf{k}_2\cdot\mathbf{r}}\times{}_1F_1[-i\gamma,1,-i(k_2r+\mathbf{k}_2\cdot\mathbf{r})]\ , \quad (5)$$

with

$$\nu(\mathbf{r})=(\lambda')^{3/2}\pi^{-1/2}e^{-\lambda' r}\ , \quad (6)$$

and $\gamma=Z/k_2$. In the present formalism, we have chosen $Z=\lambda$ so that $\phi_i(\mathbf{r}_1,\mathbf{r}_2)$ and $\phi_f(\mathbf{r}_1,\mathbf{r}_2)$ are orthogonal to each other.

The triply differential cross section is given by

$$\frac{d^3\sigma}{d\hat{\mathbf{k}}_1 d\hat{\mathbf{k}}_2 dE_2}=\frac{k_1k_2}{k}|F(\mathbf{q},\mathbf{k}_2)|^2\ . \quad (7)$$

The doubly differential cross section is obtained by integrating the TDCS over the solid angle for the ejected electron:

$$\frac{d^2\sigma}{d\hat{\mathbf{k}}_1 dE_2}=\frac{k_1k_2}{k}\int d\hat{\mathbf{k}}_2|F(\mathbf{q},\mathbf{k}_2)|^2\ . \quad (8)$$

The present calculation was performed using the technique of Roy, Das, and Sil[1,4] that reduces the eight-dimensional Glauber amplitude for the He$(e,2e)$He$^+$ process to a two-dimensional integral. Two different choices of wave functions for the ground state of He have been made. Choice 1 involves the adoption of the single-parameter

TABLE I. Doubly differential cross sections $d^2\sigma/d\hat{k}_1 dE_2$ in units of 10^{-6} a_0^2 sr^{-1} eV^{-1} in the GA for electron-impact ionization of He for various incident energies E with $E_1=E_2$ and $\theta_1=45°$.

	GA		
$E-\epsilon$ (eV)[a]	I[b]	II[c]	Experiment[d]
200	317	335	417
300	145	152	195
400	79.8	83.5	107
500	49.4	51.6	65.8
600	33.1	34.5	41.9
800	17.3	18.0	21.0
1000	10.4	10.7	12.5
1500	3.99	4.13	4.84
2000	2.01	2.07	2.36
2800	0.888	0.918	0.989

[a] ϵ denotes the binding energy and is defined by $E-E_1-E_2$.
[b] Present Glauber cross sections calculated with the choice-1 wave function for the ground state of He.
[c] Present Glauber cross sections calculated with the choice-2 wave function for the ground state of He.
[d] Reference 3.

product type wave function with the screening parameter $\lambda=1.6875$,[5] whereas choice 2 concerns the use of a similar single-parameter product type wave function with the screening parameter $\lambda=1.618$.[6]

Table I presents our GA results for the DDCS along with the corresponding experimental data[3] for the ionization of He by electron-impact in the incident energy range 224.58–2824.58 eV for $E_1=E_2$ and $\theta_1=45°$. We see that the GA cross sections obtained with the choice-1 wave function are always smaller than those obtained with the choice-2 wave function. At the incident energy of 2824.58 eV the cross section predicted by choice 2 differs from that by choice 1 by 3% while at the incident energy of 224.58 eV they differ by about 5%. This means that with the decrease in incident energy the difference of cross sections predicted by the two choices does not alter substantially. In addition, we notice that in the present case of symmetric geometry the GA cross sections are in reasonably good agreement with experiment. As expected, the agreement decreases at lower energies.

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*Permanent address: Department of Physics, University of Kalyani, Kalyani 741235, West Bengal, India.

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