Bremsstrahlung in high-density plasmas

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Emission and absorption coefficients of bremsstrahlung by high-temperature partially degenerate electrons are calculated for high-density plasmas where Coulomb coupling between ions is not weak. It is shown that the ion correlation substantially reduces these coefficients. The scaling property of the correlation effect with respect to the ionic charge number is analyzed.

Interactions between high-density plasmas and radiation have important effects on the behavior of high-temperature high-density matter. An example of these hot, dense materials is the compressed target of the inertial confinement fusion.

The radiation interacts with high-temperature electrons which, in dense matter, may be partially degenerate. The Coulomb interaction between electrons can usually be regarded as weak in these types of matters. Ions, on the other hand, exhibit classical behavior, and mutual Coulomb coupling becomes substantial when the charge number is not small or when the ion temperature is lower than the electron temperature. In this paper we calculate the bremsstrahlung by high-temperature partially degenerate electrons, taking the effect of ion correlations into account.

We consider a plasma composed of classical ions and partially degenerate electrons. We denote the number density, the charge, and the temperature of ions by n_i, Ze, and T_i, respectively, and those of electrons by n_e ($=Zn_i$), $-e$, and T_e. Our system is characterized by nondimensional parameters,

$$\Gamma=(4\pi n_i/3)^{1/3}(Ze)^2/k_BT_i =2.32\times10^{-2}Z^{5/3}(T_e/T_i)(n_e'/10^{24})^{1/3}/T_e' , \quad (1)$$

related to ions,

$$r_s=(3/4\pi n_e)^{1/3}/a_B=1.17(10^{24}/n_e')^{1/3} , \quad (2)$$

$$k_BT_e/E_F=0.543Z^{5/3}(r_s/\Gamma)(T_e/T_i) =2.74\times10T_e'/(n_e'/10^{24})^{2/3} , \quad (3)$$

related to electrons, and

$$T_e/T_i . \quad (4)$$

Here a_B is the Bohr radius, E_F is the Fermi energy of electrons, n_i' and n_e' are the densities in cm^{-3}, and T_i' and T_e' the temperatures in keV. We are interested in the case where

$$r_s<1 , \quad (5)$$

$$1<\Gamma<10 . \quad (6)$$

We also assume that Z is not much larger than unity and the electron temperature satisfies the condition

$$mc^2/(Z^2e^2/2a_B)>\xi=k_BT_e/(Z^2e^2/2a_B) =Z^{-2}10^3(T_e'/13.6)>1 , \quad (7)$$

where m is the electronic mass. The temperature of electrons is sufficiently high but still nonrelativistic.

In Fig. 1 we plot the relations $\Gamma=1$, $r_s=1$, 0.5, and 0.2, $k_BT_e/E_F=1$, and $\xi=1$ for $Z=1$, 2, 3, 4, 5, and 10 with $T_i/T_e=1$. The hatched area is an example of the parameter domain for $Z=4$ with $T_i=T_e$, where the conditions (5)–(7) are satisfied: Γ takes larger values when $T_i<T_e$.

Since we are interested in the radiation emitted out of plasmas, we consider photons whose frequency ω is larger than the plasma frequency $\omega_p=(4\pi n_e e^2/m)^{1/2}$,

$$\hbar\omega/E_F>\hbar\omega_p/E_F=0.941r_s^{1/2}=1.02(10^{24}/n_e')^{1/6} . \quad (8)$$

The relations $\hbar\omega_p/E_F=1$ and 0.5 are shown in Fig. 1. When ω is not very close to ω_p, we may regard the electrons as a dielectric medium with the dielectric function given by $1-\omega_p^2/\omega^2$.

The cross section $d\sigma_{kp'}$ of the electric dipole emission of a photon for an electron is given by[1]

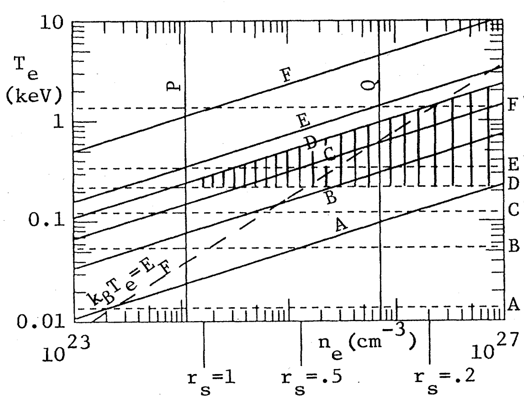

FIG. 1. Characteristic parameters of a plasma composed of electrons and ions with the charge number Z. $A-F$ and $A'-F'$ are the relations $\Gamma=1$ and $\xi=1$ for $Z=1(A,A')$, $2(B,B')$, $3(C,C')$, $4(D,D')$, $5(E,E')$, and $10(F,F')$. The relations $r_s=1$, 0.5, and 0.2, $k_BT_e/E_F=1$, and $\hbar\omega_p/E_F=1$ (P) and 0.5 (Q) are also shown.

$$d\sigma_{kp'}=\frac{(2\pi)^2}{\omega^3}\frac{m}{\hbar p}\,|\,\mathbf{e}^*\cdot\ddot{\mathbf{d}}_{fi}\,|^2\,\delta(E_i-E_f-\hbar\omega)\frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{p}'}{(2\pi)^3}\ . \tag{9}$$

Here, $\mathbf{e}$ and $\mathbf{k}$ are the polarization and the wave number of the emitted photon, $\omega=(c^2k^2+\omega_p^2)^{1/2}$, and $(\ddot{\mathbf{d}})_{fi}=(d^2\mathbf{d}/dt^2)_{fi}$ is the matrix element of the time derivative of the electric dipole moment $\mathbf{d}$ between the initial state [energy $E_i=(\hbar p)^2/2m$ and asymptotic wave number $\mathbf{p}$] and the final state [energy $E_f=(\hbar p')^2/2m$ and asymptotic wave number $\mathbf{p}'$] of the electron. Noting the condition (7), here we assume that

$$\eta=Ze^2m/\hbar^2p<1\ , \tag{10}$$

$$\eta'=Ze^2m/\hbar^2p'<1\ , \tag{11}$$

and calculate the matrix element in the Born approximation.[2] For collisions with ions distributed at $\mathbf{R}_j$, $j=1,2,\ldots$, the cross section is thus calculated as

$$\langle d\sigma_{kp'}\rangle_e=\frac{Z^2e^6}{3\pi^2mc^3\hbar p\omega^2q^2}(\omega^2-\omega_p^2)^{1/2}N_iS(q)\times\delta(E_i-E_f-\hbar\omega)d\omega\,d\mathbf{p}'\ . \tag{12}$$

Here, $\hbar\mathbf{q}=\hbar\mathbf{p}-\hbar\mathbf{p}'$ is the change of electronic momentum in the collision, and $S(q)$ is the structure factor of ions defined by

$$S(q)=\left\langle\left|\sum_j\exp(i\mathbf{q}\cdot\mathbf{R}_j)\right|^2\right\rangle_{\rm stat}\Big/N_i\ , \tag{13}$$

where $N_i=n_iV$ is the number of ions and $\langle\ \rangle_{\rm stat}$ denotes the statistical average. In (12) $\langle\ \rangle_e$ denotes that we have taken the average with respect to the polarizations and directions (solid angles) of photon.

The emission coefficient $E(\omega)d\omega$ (energy emitted per unit time, volume, solid angle, and polarization) is thus given by

$$E(\omega)d\omega=\int d\mathbf{p}\frac{2}{V(2\pi)^3}\hbar\omega\frac{\hbar p}{m}\langle d\sigma_{kp'}\rangle_e f(p)[1-f(p')]\ . \tag{14}$$

Here, $f(p)$ is the distribution function of electrons with momentum $\hbar p$,

$$f(p)=\{\exp[(\hbar^2p^2/2m-\mu)/k_BT_e]+1\}^{-1}\ , \tag{15}$$

μ being the chemical potential. Integrating with respect to $\mathbf{p}$ and $\mathbf{p}'$, we have

$$E(\omega)d\omega=\frac{2Z^2e^6}{3\pi^3c^3\hbar^3\omega}\frac{n_ik_BT_e}{\exp(\hbar\omega/k_BT_e)-1}(\omega^2-\omega_p^2)^{1/2}d\omega\times\int_0^\infty dq\,S(q)F(q)/q\ , \tag{16}$$

where

$$F(q)=\ln\left[\frac{1+\exp[\mu/k_BT_e-(\hbar^2/2mk_BT_e)(q/2-m\omega/\hbar q)^2]}{1+\exp[\mu/k_BT_e-(\hbar^2/2mk_BT_e)(q/2+m\omega/\hbar q)^2]}\right]. \tag{17}$$

When electrons exhibit classical behavior, ions are not correlated [$S(q)=1$], and $\omega\gg\omega_p$, Eqs. (16) and (17) give the known result[3]

$$E(\omega)d\omega=\frac{2^{3/2}Z^2e^6}{3\pi^{3/2}m^{3/2}c^3(k_BT_e)^{1/2}}n_en_iK_0(\hbar\omega/2k_BT_e)\times\exp(-\hbar\omega/2k_BT_e)d\omega\ . \tag{18}$$

Here, $K_0(x)$ is the modified Bessel function of the 0th order.

The rate of net absorption is the difference between the rates of absorption and stimulated emission. The energy $Q(\omega)d\omega$ absorbed per unit time, volume, solid angle, and polarization is thus given by

$$Q(\omega)d\omega=\int d\mathbf{p}\frac{2}{V(2\pi)^3}\hbar\omega N_k\frac{\hbar p}{m}\langle d\sigma_{kp'}\rangle_e\times\{f(p')[1-f(p)]-f(p)[1-f(p')]\}\ . \tag{19}$$

Here, N_k is the number of photons with a polarization and wave number $\mathbf{k}$. The absorption coefficient $A(\omega)$ is related to the absorbed energy by

$$A(\omega)=\frac{Q(\omega)d\omega}{\hbar\omega N_k(\omega^2-\omega_p^2)d\omega/8\pi^3c^2}\ , \tag{20}$$

where the denominator is the photon-energy spectrum (per solid angle and polarization) multiplied by the group velocity $c(\omega^2-\omega_p^2)^{1/2}/\omega$. Similarly performing the integrals with respect to $\mathbf{p}$ and $\mathbf{p}'$, we have

$$A(\omega)=\frac{16Z^2e^6n_ik_BT_e}{3c\hbar^4\omega^2(\omega^2-\omega_p^2)^{1/2}}\int_0^\infty dq\,S(q)F(q)/q\ . \tag{21}$$

As is shown in Eqs. (16) and (21), the effect of ion correlation on these coefficients is expressed by the ratio R as

$$R=\frac{E(\omega)}{E(\omega,S(q)=1)}=\frac{A(\omega)}{A(\omega,S(q)=1)}=\frac{\int_0^\infty dq\,S(q)F(q)/q}{\int_0^\infty dq\,F(q)/q}\ . \tag{22}$$

In the above calculations we have assumed that both the initial and final states of the electron are described by the plane wave. When the final momentum of electron is too small to satisfy condition (11), the emission probability is modified[2] by a factor

$$g(\eta')=2\pi\eta'/[1-\exp(-2\pi\eta')] \ . \tag{23}$$

By further multiplying a factor

$$1/g(\eta)=[1-\exp(-2\pi\eta)]/2\pi\eta \ , \tag{24}$$

we may extend[4] the applicability of our calculation for uncorrelated ions to the cases where the latter half of condition (7) or condition (10) does not strictly hold for the initial state of the electron. The factor F included in Eqs. (16) and (21) is then modified as

$$F'(q)=\frac{\hbar^2}{2mk_BT_e}\int_0^\infty dp_\perp^2\{[f(p)-f((p^2+2m\omega/\hbar)^{1/2})]g(\eta')/g(\eta)\}_{p_\parallel=m\omega/\hbar q-q/2} \ , \tag{25}$$

where $p^2=p_\parallel^2+p_\perp^2$.

The expression of R with F and S is derived in the Born approximation and it is not, in principle, enough to replace only F by F'. The values of R with F and F', however, differ only slightly, as will be shown below, and we expect (22) with F' works as a good approximation.

In Fig. 2 we show an example of the emission coefficient for $k_BT_e=1$ keV in the case of uncorrelated ions with $Z=1$ obtained from Eq. (16) using F'. While calculations with F give the results dependent only on $\hbar\omega/k_BT_e$ and k_BT_e/E_F, those of calculations with F' depend also on k_BT_e/mc^2. Plotted are the values [without the factor $(\omega^2-\omega_p^2)^{1/2}/\omega$] normalized by (18), the classical limit in the Born approximation calculated with F: Since $F'>F$, the emission coefficient in the classical limit calculated with F' is greater than (18). We see that the degeneracy of electrons decreases the emission coefficient through the factor $f(p)[1-f(p')]$ stemming from the Pauli principle.[5]

We now look at the contribution of various values of the momentum transfer $\hbar q$ to the emission and absorption coefficients. In Fig. 3 we show some typical examples of the values of F'. The behavior of F' is similar to that of F defined by (17), where q appears through the factor

$$\exp(-\hbar^2q^2/8mk_BT_e-m\omega^2/2k_BT_eq^2) \ .$$

Thus the main contribution to the integral in (16) and (21) comes from the domain $q\sim(2m\omega/\hbar)^{1/2}$. The boundary of the contributing domain of q becomes diffuse and extends to larger wave numbers as the electrons become classical.

The characteristic scale of length of the structure factor of classical ions with intermediate or strong coupling is the mean distance between ions

$$a=(3/4\pi n_i)^{1/3}=1.919Z^{1/3}/k_F \ ,$$

where $k_F=(3\pi^2n_e)^{1/3}$ is the Fermi wave number of electrons. When we neglect the dielectric polarization of electrons, ions are regarded as the classical one-component plasma whose structure factor has been obtained by nu-

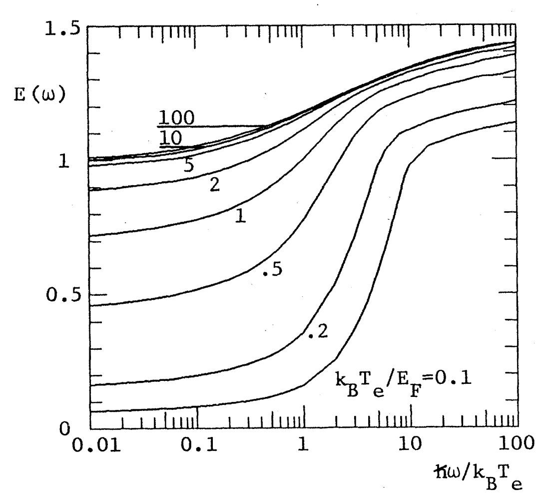

FIG. 2. Bremsstrahlung emission coefficient $E(\omega)$ of electrons with $T=1$ keV colliding against uncorrelated ions with $Z=1$. Values [without the factor $(\omega^2-\omega_p^2)^{1/2}/\omega$] normalized by the Born approximation in the classical limit, Eq. (18), are plotted.

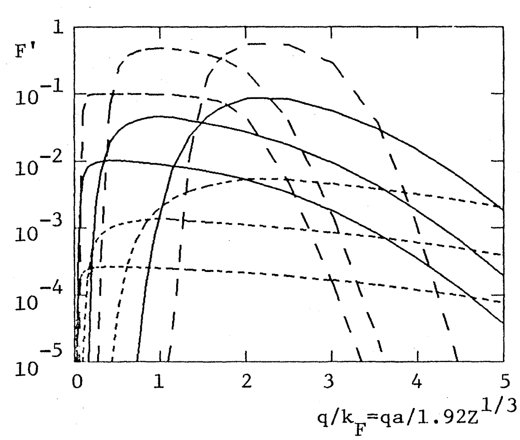

FIG. 3. Contribution of various values of the electronic momentum change $\hbar q$ to $E(\omega)$ and $A(\omega)$. Three dashed (solid, dotted) lines show, from left to right, the values of F' for $\hbar\omega/E_F=0.2$, 1, and 5, respectively, with $k_BT_e/E_F=0.2$ (1, 5), and k_F is the Fermi wave number of electrons.

TABLE I. Effect of ion correlation on emission and absorption coefficients for $\Gamma=2$ and $Z=1$. The ratio of these coefficients to those without ion correlation is shown.

	k_BT_e/E_F									
$\hbar\omega/E_F$	0.1	0.2	0.5	1	2	5	10	20	50	100
0.01	0.128	0.126	0.124	0.134	0.155	0.188	0.213	0.235	0.260	0.276
0.02	0.145	0.142	0.141	0.152	0.174	0.209	0.234	0.256	0.281	0.297
0.05	0.175	0.172	0.171	0.184	0.207	0.244	0.269	0.291	0.314	0.329
0.1	0.210	0.207	0.207	0.219	0.243	0.278	0.303	0.324	0.346	0.359
0.2	0.258	0.256	0.257	0.270	0.293	0.325	0.347	0.365	0.384	0.395
0.5	0.363	0.363	0.368	0.379	0.366	0.417	0.430	0.440	0.449	0.454
1	0.508	0.508	0.512	0.515	0.517	0.519	0.519	0.518	0.516	0.513
2	0.717	0.715	0.709	0.698	0.683	0.659	0.639	0.621	0.601	0.588
5	0.949	0.948	0.941	0.929	0.908	0.868	0.832	0.794	0.747	0.715
10	1.00	1.00	1.00	0.997	0.990	0.973	0.950	0.918	0.868	0.829
20	1.00	1.00	1.00	1.00	1.00	1.00	0.997	0.987	0.961	0.931
50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.995
100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

merical experiments.[6] Due to the long-range nature of the Coulomb interaction, $S(q)$ is proportional to q^2 for small values of q and approaches to unity for $qa \gg 1$. For $\Gamma \lesssim 10$, the oscillation of $S(q)$ around unity is small and $S(q)$ is substantially smaller than unity when $qa \lesssim 3$. If the integrals in (16) and (21) are effectively determined by the contribution from the domain where $S(q)$ is small, the ratio R is then reduced compared with the case of $S(q)=1$ or uncorrelated ions. Since the main contribution to the integral in (16) and (21) comes from the domain characterized by $q \sim (2m\omega/\hbar)^{1/2}$, the effect of ion correlation becomes important when $(2m\omega/\hbar)^{1/2}a \lesssim 3$ or $\hbar\omega/E_F \lesssim 2.4Z^{-2/3}$.

When

$$\hbar\omega/k_BT_e \sim 1$$

and

$$\hbar\omega/E_F = (\hbar\omega/k_BT_e)(k_BT_e/E_F) \gg 1$$

in the classical case, the integration with respect to q extends to very large values, $\sim (mk_BT_e)^{1/2}/\hbar$, corresponding to the thermal de Broglie wavelength. The effect of the ion correlation in the domain of small wave numbers $\sim 1/a$ therefore has a very small effect on the ratio R. For low frequencies such that $\hbar\omega/k_BT_e \ll 1$, however, R is affected by the ion correlation even in the classical case.

In Tables I and II and Figs. 4(a), 4(b), 5(a), and 5(b), we show the values of the ratio R for $\Gamma=2$ and 10 and several cases of electron degeneracy calculated with the structure factor of the one-component plasma.[6] We see that the ion correlation has enough effect to reduce the emission and absorption coefficients in most cases. Small enhancement for the case of $\Gamma=10$ reflects the small overshooting of $S(q)$ beyond unity. These values of R are calculated with F' and we have set $k_BT_e=1$ keV in the additional parameter k_BT_e/mc^2 of F'. The factors (23) and (24), however, have a small effect on the ratio R, and calculations with F give almost the same values: In fact, tabulated values are applicable with errors less than 2% in the domain $0.1 < k_BT_e < 10$ keV.

Let us now consider the scaling of R with respect to Z. Here we neglect the weak (k_BT_e/mc^2) dependence of R calculated by F' and analyze the scaling of R based on F. Rewriting the numerator of (22) as

TABLE II. Same as Table I for $\Gamma=10$.

	k_BT_e/E_F									
$\hbar\omega/E_F$	0.1	0.2	0.5	1	2	5	10	20	50	100
0.01	0.070	0.068	0.072	0.085	0.108	0.145	0.173	0.198	0.227	0.245
0.02	0.079	0.089	0.081	0.096	0.121	0.161	0.190	0.216	0.245	0.263
0.05	0.095	0.094	0.099	0.116	0.144	0.188	0.219	0.246	0.274	0.292
0.1	0.114	0.113	0.119	0.138	0.169	0.214	0.246	0.273	0.302	0.319
0.2	0.140	0.140	0.148	0.170	0.203	0.251	0.282	0.307	0.335	0.350
0.5	0.199	0.201	0.217	0.243	0.277	0.321	0.349	0.371	0.391	0.403
1	0.296	0.306	0.324	0.348	0.375	0.406	0.424	0.438	0.450	0.456
2	0.535	0.535	0.538	0.542	0.544	0.541	0.537	0.532	0.527	0.524
5	1.01	1.00	0.978	0.941	0.890	0.816	0.765	0.721	0.674	0.647
10	1.03	1.03	1.03	1.03	1.02	0.987	0.941	0.887	0.817	0.771
20	0.994	0.994	0.994	0.995	0.997	1.01	1.01	0.994	0.950	0.903
50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

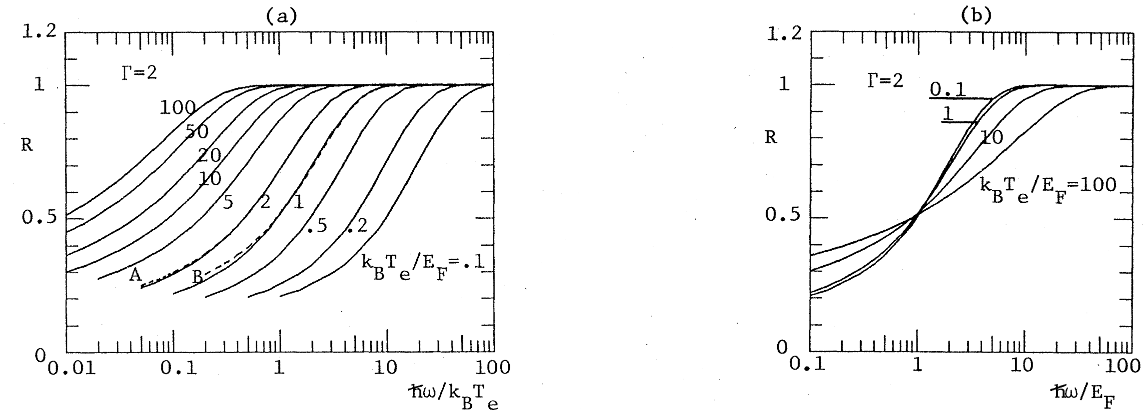

FIG. 4. (a) and (b) Effect of ion correlation on emission and absorption coefficients for $\Gamma=2$ and $Z=1$. The ratio of these coefficients to those without ion correlations is plotted. Dotted lines show the values for $Z=2^{3/2}$ with $k_BT_e/E_F=1$ (A) and 0.5 (B), which, when the scaling (30) exactly holds, are equal to those for $Z=1$ with $k_BT_e/E_F=2$ and 1.

$$\int_0^\infty dq\, S(q)F(q)/q=\int_0^\infty dq' S(q';\Gamma)F(q',\hbar\omega/k_BT_e;Z^{2/3}k_BT_e/E_F,\mu/k_BT_e)/q' \ , \tag{26}$$

where $q'=qa$ and

$$F(q',\hbar\omega/k_BT_e;\tau,\mu')=\ln\left[\frac{1+\exp\{\mu'-(4/9\pi)^{2/3}(4\tau)^{-1}[q'-(9\pi/4)^{2/3}\tau(\hbar\omega/k_BT_e)/q']^2\}}{1+\exp\{\mu'-(4/9\pi)^{2/3}(4\tau)^{-1}[q'+(9\pi/4)^{2/3}\tau(\hbar\omega/k_BT_e)/q']^2\}}\right] , \tag{27}$$

we have

$$R=R(\hbar\omega/k_BT_e;\Gamma,Z^{2/3}k_BT_e/E_F,\mu/k_BT_e) \ . \tag{28}$$

We have computed the values of R for $Z\leq 5$ with various cases of other parameters. From these results, we have found that for

$$\Gamma<10,\ \ k_BT_e/E_F>0.5,\ \ \hbar\omega/E_F>0.5 \ , \tag{29}$$

the values of R approximately satisfy the scaling

$$R=R(\hbar\omega/k_BT_e;\Gamma,Z^{2/3}k_BT_e/E_F) \ , \tag{30}$$

or, in other words, the μ/k_BT_e dependence of R can be neglected. The error in the scaling (30) increases with the increase of Z or the decrease of $\hbar\omega/k_BT_e$ or k_BT_e/E_F. For $Z=5$, the maximum deviation is about 1% for $\Gamma=2$ and 3% for $\Gamma=10$. As an illustration, we plot the values of R for $Z=2^{3/2}$ in Figs. 4(a) and 5(a): With $\hbar\omega/k_BT_e$ and Γ fixed, R for $Z=2^{3/2}$ and $k_BT_e/E_F=\tau$ is approximately equal to R for $Z=1$ and $k_BT_e/E_F=2\tau$. The cases of $\tau=1$ and 0.5 are shown and we see that the scaling (30) is satisfied accurately.

Thus far, we have neglected the effect of electronic po-

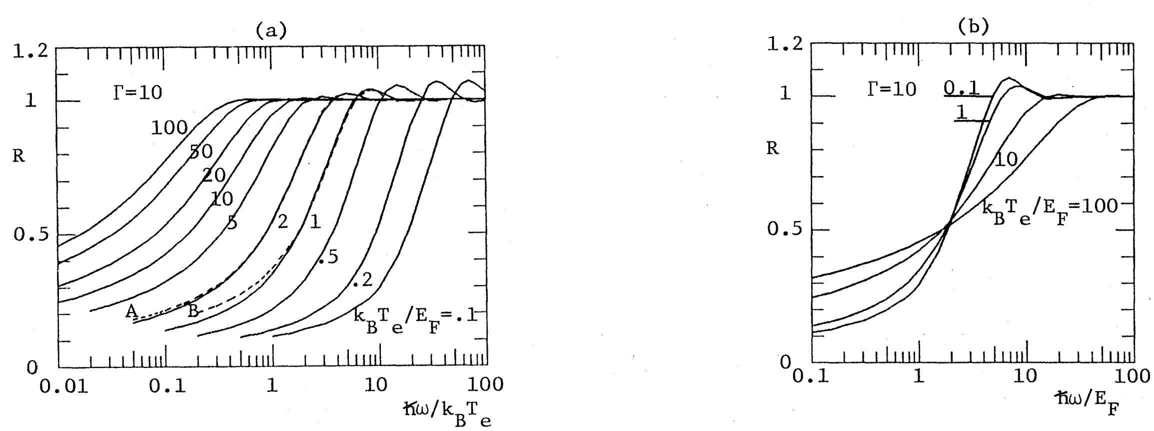

FIG. 5. (a) and (b) Same as Fig. 4 for $\Gamma=10$.

larization on the structure factor of ions: Interactions between ions are screened and the structure factor becomes finite at the long-wavelength limit. This effect has been analyzed by numerical experiments,[7] and it has been shown that $S(0)\sim 0.3$ for $\Gamma=2$, $r_s=1$, and $k_B T_e/E_F=0.272$, and that $S(0)\sim 0.1$ for $\Gamma=10$, $r_s=1$, and $k_B T_e/E_F=0.0543$. The polarizability of electrons decreases with the decrease of r_s or the increase of $k_B T_e/E_F$. Since we are considering the case where $r_s<1$ and, for the same values of r_s and Γ, the values of $k_B T_e/E_F$ increase with the increase of Z or the decrease of T_i/T_e, we may use the structure factor of the one-component plasma as the first approximation.

The effect of ion correlation on the scattering of an electron by ions and the rate of bremsstrahlung have already been analyzed in previous works.[8,9] In these studies, Coulomb coupling between ions has been assumed to be weak and the Debye-Hückel (or the random-phase) approximation has been used. In the cases considered in this paper, however, one cannot apply the Debye-Hückel approximation. We have calculated the bremsstrahlung emission and absorption from partially degenerate electrons on the basis of the accurate structure factor of intermediately coupled ions and have discussed the dependence of the rates on the charge number.

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