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Mode-coupling theory of long-time tails in a classical electron gas

M. C. Marchetti*
The Rockefeller University, New York, New York 10021-6399

T. R. Kirkpatrick
Department of Physics and Astronomy and Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742
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From mode-coupling theory, we obtain the complete long-time behavior of kinetic, potential, and cross terms of the Green-Kubo integrands for the transport coefficients in a classical electron gas. The results for the kinetic parts are in agreement with those derived recently with use of kinetic theory. The differences between the long-time tails in the electron gas and those in neutral fluids are discussed in detail. The theory suggests that the frequency-dependent heat conductivity diverges as the plasma frequency is approached.

I. INTRODUCTION

In the past decade there has been considerable interest in the time dependence of the correlation functions that determine the transport coefficients in an electron gas or one-component plasma (OCP). These time correlation functions have been studied both theoretically and by computer molecular-dynamic experiments.[1]

In a recent paper[2] kinetic-theory techniques were used to calculate the long-time behavior of the velocity autocorrelation function and of the kinetic-kinetic parts of the Green-Kubo integrands for the shear viscosity η and the heat conductivity λ. Our theoretical results were restricted to moderately dense OCP and to lowest order in the plasma parameter. Furthermore, we did not calculate the potential parts of the Green-Kubo integrands for the shear viscosity and heat conductivity nor were the bulk viscosity ζ or longitudinal viscosity D_l considered.

In this paper phenomenological mode-coupling theory[3] is used to calculate the long-time behavior of the complete Green-Kubo integrands (kinetic and potential parts) for the coefficients of shear viscosity, heat conductivity, and bulk viscosity. Unlike the kinetic-theory analysis presented previously, the results of this paper are not restricted to small values of the plasma parameter.

There are two phenomenological methods that can be used to calculate the long-time tails in either a neutral fluid or in a OCP: Kadanoff-Swift mode-coupling theory[4] and nonlinear fluctuating hydrodynamic theory.[5] The mode-coupling theory of Kadanoff and Swift is elegant and straightforward but it is not as intuitive as the fluctuating nonlinear hydrodynamic approach. In this paper we use the Kadanoff-Swift theory to compute the long-time tails for the shear viscosity and the heat conductivity. This method is chosen because it allows us to separately compute the kinetic and potential parts of the Green-Kubo integrands. The separation is needed to compare the results of our kinetic-theory analysis[2] with those obtained here. We find that in general the potential contributions to the long-time tails (LTT) are of the same order in the plasma parameter as the kinetic contributions and that there can be cancellations between the various parts of the complete Green-Kubo integrands. In addition, the potential contributions can lead to qualitatively new features in the LTT. In contrast, for a fluid of particles interacting via a short-ranged potential the potential contributions to the LTT are always of higher order in the density than the kinetic ones and no such cancellation can occur.[6,7] The LTT for the shear viscosity and the heat conductivity have also been calculated using the fluctuating hydrodynamic method.[8] The results of the two calculations are in complete agreement.

For technical reasons to be discussed below, the long-time behavior of the Green-Kubo integrand for the bulk viscosity has only been evaluated using the fluctuating hydrodynamic method. Only the results are given here.

The long-time tails in a OCP differ from those in a fluid of neutral particles for two reasons. First, the equilibrium correlation functions are fundamentally different in a OCP. For instance, the static structure factor $S(k)$ vanishes with the wave number k. Secondly, there are qualitative differences in the hydrodynamic modes of the two systems. The sound modes in a neutral fluid are replaced in a OCP by plasma modes that are finite frequency modes.[9] This will be discussed in more detail below.

One of the results of this paper is particularly interesting. We find that the frequency-dependent heat conductivity diverges as the frequency approaches the plasma frequency. The experimental consequences of this will be discussed in Sec. IV.

The organization of this paper is as follows. In Sec. II we give the basic equations that are needed to calculate the LTT in a OCP. In Sec. III our results are presented and they are discussed in Sec. IV. In the Appendix we show how the mode-coupling amplitudes that appear in the Kadanoff-Swift formalism are computed for a OCP.

II. BASIC EQUATIONS

This section consists of two parts. First, the definitions of the transport coefficients and the time correlation functions of interest are given. The mode-coupling theory of Kadanoff and Swift is then used to express the long-time behavior of these correlation functions in terms of the hydrodynamic modes of the OCP.

A. Definitions

The Green-Kubo expressions for the linear transport coefficients in a OCP can be obtained by using formal projection-operator techniques.[10] The derivation is identical to the derivation for neutral fluids except the neutralizing background must be taken into account. We obtain

$$\eta=\frac{1}{k_B T}\int_0^\infty dt\, C_\eta(t)\ , \tag{2.1a}$$

$$\lambda=\frac{1}{k_B T^2}\int_0^\infty dt\, C_\lambda(t)\ , \tag{2.1b}$$

$$\tfrac{4}{3}\eta+\zeta=\rho D_l=\frac{1}{k_B T}\int_0^\infty dt\, C_l(t) =\frac{1}{k_B T}\int_0^\infty dt[\tfrac{4}{3}C_\eta(t)+C_\zeta(t)]\ , \tag{2.1c}$$

where

$$C_i(t)=\lim_{k\to 0}\lim{}_T\left[\frac{1}{\Omega}\langle j_i(\mathbf{k},t)j_i(-\mathbf{k})\rangle\right]. \tag{2.2}$$

Here $k_BT=\beta^{-1}$, where k_B is Boltzmann's constant and T is the temperature; the fluid is enclosed in a volume Ω. In Eq. (2.2) the angular brackets denote a grand canonical ensemble average and $\lim_T$ denotes the thermodynamic limit. The label i takes the values $i=\eta,\lambda,l,\zeta$ and $j_i(\mathbf{k},t)$ is the projected current at wave vector $\mathbf{k}$ and at time t with initial value $j_i(\mathbf{k})$. Due to the presence of the neutralizing background, the definition of the current $j_i(\mathbf{k})$, as well as that of the microscopic energy density $e(\mathbf{k})$, for the OCP is not straightforward. The details of the derivation can be found in Ref. 11. If the wave vector $\mathbf{k}$ is chosen to point in the x direction, i.e.., $\mathbf{k}=k\hat{\mathbf{x}}$, the resulting currents are given by

$$j_\eta(\mathbf{k})=j_{xy}(\mathbf{k})\ , \tag{2.3a}$$

$$j_\lambda(\mathbf{k})=j_x^e(\mathbf{k})-\frac{h}{\rho}j_x^\rho(\mathbf{k})\ , \tag{2.3b}$$

$$j_l(\mathbf{k})=j_{xx}(\mathbf{k})-[m\beta S(k)]^{-1}\rho(\mathbf{k}) -\frac{(\gamma-1)\rho C v}{\alpha T}T(\mathbf{k})-p\Omega\delta_{\mathbf{k},0}\ , \tag{2.3c}$$

where $\rho(\mathbf{k})$ and $T(\mathbf{k})$ are the microscopic mass density and temperature, given by

$$\rho(\mathbf{k})=m\sum_{i=1}^N e^{-i\mathbf{k}\cdot\mathbf{r}_i}-\Omega\rho\delta_{\mathbf{k},0}\ , \tag{2.4}$$

$$T(\mathbf{k})=\left[\frac{\partial T}{\partial u}\right]_\rho e(\mathbf{k})+\left[\frac{\partial T}{\partial \rho}\right]_u \rho(\mathbf{k})\ , \tag{2.5}$$

with $e(\mathbf{k})$ the microscopic energy density,

$$e(\mathbf{k})=\sum_{i=1}^N e^{-i\mathbf{k}\cdot\mathbf{r}_i}e_i(\mathbf{k})-\Omega u\delta_{\mathbf{k},0}\ . \tag{2.6a}$$

Here $e_i(\mathbf{k})$ is the energy of particle i, given by

$$e_i(\mathbf{k})=\tfrac{1}{2}mv_i^2+\tfrac{1}{2}\sum_{j\,(\neq i)}\frac{1}{\Omega}\sum_{\mathbf{k}'\,(\neq 0,\mathbf{k})}v(\mathbf{k},\mathbf{k}')e^{i\mathbf{k}'\cdot\mathbf{r}_{ij}}\ , \tag{2.6b}$$

where

$$v(\mathbf{k},\mathbf{k}')=v_{k'}\frac{\mathbf{k}'\cdot(\mathbf{k}'-\mathbf{k})}{|\mathbf{k}'-\mathbf{k}|^2} \tag{2.6c}$$

and u is the internal energy density. For completeness, and because it will be needed below, the microscopic momentum density is given by

$$g_\alpha(\mathbf{k})=\sum_{i=1}^N mv_{i\alpha}e^{-i\mathbf{k}\cdot\mathbf{r}_i}\ . \tag{2.7}$$

In Eqs. (2.3) the momentum, energy, and mass current are, respectively, given by

$$j_{\alpha\beta}(\mathbf{k})=\sum_{i=1}^N e^{-i\mathbf{k}\cdot\mathbf{r}_i}\left[mv_{i\alpha}v_{i\beta}+\frac{1}{2\Omega}\sum_{j\,(\neq i)}\sum_{\mathbf{k}'}W_{\alpha\beta}(\mathbf{k},\mathbf{k}')e^{i\mathbf{k}'\cdot\mathbf{r}_{ij}}\right]\ , \tag{2.7a}$$

$$j_\alpha^e(\mathbf{k})=\sum_{i=1}^N e^{-i\mathbf{k}\cdot\mathbf{r}_i}\left[e_i(\mathbf{k})v_{i\alpha}+\frac{1}{\Omega}\sum_{j\,(\neq i)}\sum_{\mathbf{k}'}W_{\alpha\beta}^e(\mathbf{k},\mathbf{k}')v_{i\beta}e^{i\mathbf{k}'\cdot\mathbf{r}_{ij}}\right]\ , \tag{2.7b}$$

$$j_\alpha^\rho(\mathbf{k})=\sum_{i=1}^N mv_{i\alpha}e^{-i\mathbf{k}\cdot\mathbf{r}_i}\ , \tag{2.7c}$$

with

$$\begin{aligned}W_{\alpha\beta}(\mathbf{k},\mathbf{k}')&=v_k\hat{k}_\alpha\hat{k}_\beta,\quad \mathbf{k}'=0,\ \mathbf{k}'=\mathbf{k}\\ &=\frac{\hat{k}_\alpha}{k}[v_{k'}k'_\beta+v_{\mathbf{k}-\mathbf{k}'}(\mathbf{k}-\mathbf{k}')_\beta],\quad \mathbf{k}'\neq 0,\ \mathbf{k}\neq\mathbf{k}'\end{aligned} \tag{2.8a}$$

and

$$\begin{aligned}W_{\alpha\beta}^e(\mathbf{k},\mathbf{k}')&=0,\quad \mathbf{k}'=0\\ &=v_k\hat{k}_\alpha\hat{k}_\beta,\quad \mathbf{k}'=\mathbf{k}\\ &=v_{k'}\frac{\hat{k}_\alpha}{k}\left[k'_\beta-\frac{\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')}{(\mathbf{k}-\mathbf{k}')^2}(\mathbf{k}-\mathbf{k}')_\beta\right],\\ &\qquad \mathbf{k}'\neq 0,\ \mathbf{k}'\neq\mathbf{k}\ .\end{aligned} \tag{2.8b}$$

Cartesian components are denoted by $\alpha,\beta=(x,y,z)$, summation convention is used for repeated Greek indices, and $\mathbf{r}_i$ and $\mathbf{v}_i$ denote the position and the velocity of the ith particle, respectively. The equilibrium mass density is $\rho=mn$, $h=(u+p)$ is the enthalpy density with p the pressure. In Eqs. (2.8) $v_{k'}=4\pi e^2/(k')^2$ is the Fourier transform of the Coulomb potential for particles of charge e. The total, i.e., volume-integrated densities and currents are the $\mathbf{k}=0$ limit of the local quantities defined above and are identical to those for neutral fluids with the potential $v(\mathbf{r}_{ij})$ everywhere replaced by

$$\frac{1}{\Omega}\sum_{\mathbf{k}'\,(\neq 0)} v_{k'}e^{i\mathbf{k}'\cdot\mathbf{r}_{ij}} .$$

The restriction in the sum, $\mathbf{k}'\neq 0$, takes into account the neutralizing background (cf. Ref. 7 and Ref. 11).

The currents j_i can in general be separated into a kinetic part j_i^k and a potential part j_i^v, with

$$j_i(\mathbf{k})=j_i^k(\mathbf{k})+j_i^v(\mathbf{k}) , \tag{2.9}$$

where the kinetic projected currents are given by

$$j_\eta^k(\mathbf{k})=\sum_{i=1}^{N} m v_{ix}v_{iy}e^{-i\mathbf{k}\cdot\mathbf{r}_i} , \tag{2.10a}$$

$$j_\lambda^k(\mathbf{k})=\sum_{i=1}^{N}\left[\frac{1}{2}mv_i^2-\frac{5}{2\beta}\right]v_{ix}e^{-i\mathbf{k}\cdot\mathbf{r}_i} , \tag{2.10b}$$

$$j_l^k(\mathbf{k})=\sum_{i=1}^{N} m\left[v_{ix}^2-\frac{v_i^2}{3}\right]e^{-i\mathbf{k}\cdot\mathbf{r}_i} . \tag{2.10c}$$

The potential parts are given by Eqs. (2.3) and (2.10). We write

$$C_i(t)=C_i^{kk}(t)+2C_i^{kv}(t)+C_i^{vv}(t) , \tag{2.11a}$$

with

$$C_i^{AB}(t)=\lim_{\mathbf{k}\to 0}\lim{}_T\frac{1}{\Omega}\langle j_i^A(\mathbf{k}it)j_i^B(-\mathbf{k})\rangle , \tag{2.11b}$$

where A and B equal k or v. Below we refer to C_i^{AB} for $A=B=k$ as the kinetic-kinetic part of $C_i(t)$, for $A=k$ and $B=v$ as the kinetic-potential part of $C_i(t)$, and for $A=B=v$ as the potential-potential part of $C_i(t)$.

B. Long-time behavior of $C_\mu(t)$ ($\mu=\eta,\lambda,l$)

To determine the long-time behavior of $C_\mu(t)$ we use the mode-coupling theory developed by Kadanoff and Swift.[4,10] In this theory one assumes that after a short initial microscopic time the time evolution of $C_\mu(t)$ is governed by that of products of long-lived hydrodynamic excitations or modes. Thus we write

$$C_i^{AB}(t)=C_{i,0}^{AB}(t)+\delta C_i^{AB}(t) , \tag{2.12}$$

where $C_{i,0}^{AB}(t)$ is the initial short-time contribution to $C_i^{AB}(t)$ and $\delta C_i^{AB}(t)$ is the contribution due to mode-coupling effects. The latter is assumed to be given by

$$\delta C_i^{AB}(t)=\lim_{\mathbf{k}\to 0}\lim{}_T\frac{1}{2\Omega}\sum_{\mathbf{q}}\sum_{\substack{n,m\\=\rho,l,T,\epsilon_i}}\frac{\langle j_i^A(\mathbf{k})a_n(-\mathbf{q})a_m(-\mathbf{l})\rangle}{\langle a_n(\mathbf{q})a_n(-\mathbf{q})\rangle}\frac{\langle a_n(\mathbf{q},t)a_m(\mathbf{l},t)j_i^B(-\mathbf{k})\rangle}{\langle a_m(\mathbf{l})a_m(-\mathbf{l})\rangle} , \tag{2.13}$$

where $\mathbf{l}=\mathbf{k}-\mathbf{q}$. Here $a_n(\mathbf{q})$ is a vector, which components are the microscopic mass density, $\rho(\mathbf{q})$, the longitudinal momentum density, $g_l(\mathbf{q})=\hat{\mathbf{q}}\cdot\mathbf{g}(\mathbf{q})$, the temperature, $T(\mathbf{q})$, and the transverse momentum densities, $g_{\epsilon_i}(\mathbf{q})=\hat{\mathbf{q}}_\perp^i\cdot\mathbf{g}(\mathbf{q})$, for $i=1,2$. We have defined a set of orthonormal unit vectors $(\hat{\mathbf{q}},\hat{\mathbf{q}}_\perp^1,\hat{\mathbf{q}}_\perp^2)$ with $\hat{\mathbf{q}}=\mathbf{q}/|\mathbf{q}|$. For future use we also define an ensemble average by

$$(ab)=\lim{}_T\left[\frac{1}{\Omega}\langle ab\rangle\right] \tag{2.14}$$

and use

$$(\rho(\mathbf{q})\rho(-\mathbf{q}))=m\rho S(q) , \tag{2.15a}$$

$$(g_\alpha(\mathbf{q})g_\beta(-\mathbf{q}))=\delta_{\alpha\beta}\rho k_BT , \tag{2.15b}$$

$$(T(\mathbf{q})T(-\mathbf{q}))=\frac{k_BT^2}{\rho C_v}+O(q^2) , \tag{2.15c}$$

where C_v is the specific heat per unit mass at constant volume.

The time dependence of $a_N(\mathbf{q},t)$ in Eq. (2.13) is assumed to be given by the linearized hydrodynamic equations that describe long-wavelength dynamical processes in a OCP. It is convenient to define linear combinations of the $a_n(\mathbf{q})$'s that approximately diagonalize (for $q\to 0$) these hydrodynamic equations. We call the five independent linear combinations the hydrodynamic modes of the OCP: two plasma modes denoted by $\sigma=\pm$, a heat or entropy mode denoted by H, and two shear or viscous modes denoted by ϵ_i. In terms of these modes Eq. (2.13) can be written as

$$\delta C_i^{AB}(t)=\lim_{\mathbf{k}\to 0}\lim{}_T\left[\frac{1}{2\Omega}\sum_{\mathbf{q}}\sum_{\substack{\mu,\nu\\=\sigma,H,\epsilon_i}}(j_i^A(\mathbf{k})A_\mu^\dagger(\mathbf{q})A_\nu^\dagger(\mathbf{l}))(A_\mu(\mathbf{q})A_\nu(\mathbf{l})j_i^B(-\mathbf{k}))\exp\{-[\omega_\mu(q)+\omega_\nu(l)]t\}\right] . \tag{2.16}$$

Here the $A_\mu(\mathbf{q})$ are the hydrodynamic eigenmodes given, for small q, by

$$A_\sigma(\mathbf{q})=\frac{1}{\sqrt{2}}\left[\sigma\left[\frac{\omega_p}{q}+\frac{q}{\omega_p\rho\chi_T}\right]\rho(\mathbf{q})+\hat{\mathbf{q}}\cdot\mathbf{g}(\mathbf{q})+\frac{\sigma q}{\omega_p}\frac{\alpha}{\chi_T}T(\mathbf{q})\right], \quad (2.17a)$$

$$A_H(\mathbf{q})=-\frac{\gamma-1}{\alpha\rho}\rho(\mathbf{q})+T(\mathbf{q}), \quad (2.17b)$$

$$A_{\epsilon_i}(\mathbf{q})=\hat{\mathbf{q}}^i_\perp\cdot\mathbf{g}(\mathbf{q}), \quad (2.17c)$$

and the $A^\dagger_\mu(\mathbf{q})$ are the adjoint modes,

$$A^\dagger_\sigma(\mathbf{q})=\frac{\beta}{\rho\sqrt{2}}\left[\frac{\sigma\omega_p}{q}\rho^*(\mathbf{q})-\hat{\mathbf{q}}\cdot\mathbf{g}^*(\mathbf{q})+\frac{\sigma q}{\omega_p}\frac{\alpha}{\chi_T}T^*(\mathbf{q})\right], \quad (2.18a)$$

$$A^\dagger_H(\mathbf{q})=\frac{\rho C_v}{k_BT^2}\left[T^*(\mathbf{q})-\frac{\gamma-1}{\alpha\rho}\rho^*(\mathbf{q})\right], \quad (2.18b)$$

$$A^\dagger_{\epsilon_i}(\mathbf{q})=\frac{\beta}{\rho}\hat{\mathbf{q}}^i_\perp\cdot\mathbf{g}^*(\mathbf{q}). \quad (2.18c)$$

Here $\omega_p=(4\pi ne^2/m)^{1/2}$ is the plasma frequency, χ_T is the isothermal compressibility, α is the coefficient of thermal expansion, $\gamma=C_p/C_v$ is the ratio of specific heats, and we have used

$$\left[\frac{\partial T}{\partial u}\right]_\rho=\frac{1}{\rho C_v}, \quad (2.19a)$$

$$\left[\frac{\partial T}{\partial \rho}\right]_u=\frac{\gamma-1}{\alpha\rho}-\frac{h}{\rho^2C_v}, \quad (2.19b)$$

$$\gamma-1=\frac{\alpha^2T}{\rho C_v\chi_T}. \quad (2.19c)$$

The hydrodynamic eigenvalues $\omega_\alpha(q)$ in Eq. (2.10) are given by, for small q,

$$\omega_\sigma(q)=i\sigma\omega_p\left[1+q^2\left[\frac{c^2}{2\omega_p^2}+\frac{\gamma_p}{2\omega_p}\right]\right]+\frac{\Gamma_pq^2}{2}, \quad (2.20a)$$

$$\omega_H(q)=D_Tq^2, \quad (2.20b)$$

$$\omega_\nu(q)=\nu q^2. \quad (2.20c)$$

Here $c^2=(\partial p/\partial\rho)_s$, where s is the entropy, γ_p and Γ_p are the imaginary and real parts of the finite frequency longitudinal viscosity, respectively, $D_T=\lambda/\rho C_v$ is the thermal diffusivity for a OCP, and $\nu=\eta/\rho$ is the kinematic viscosity.

III. RESULTS

The long-time behavior of $\delta C_i(t)$ and $\delta C_i^{AB}(t)$ for $i=\eta,\lambda$ can be calculated from Eqs. (2.16) with the mode-coupling amplitudes given in the Appendix by Eqs. (A11) and (A12). The final results are

$$\delta C_\eta(t)\simeq\left[\frac{L_{\epsilon\epsilon}}{(2\nu)^{3/2}}+\frac{L_{+\epsilon}\cos(\omega_pt+\frac{3}{2}\theta_\nu)}{[(\nu+\frac{1}{2}\Gamma_p)^2+(c^2/2\omega_p+\frac{1}{2}\gamma_p)^2]^{3/4}}+\frac{L_{++}\cos(2\omega_pt+\frac{3}{2}\theta_p)}{[\Gamma_p^2+(c^2/\omega_p+\gamma_p)^2]^{3/4}}+\frac{L_{+-}}{\Gamma_p^{3/2}}\right]\frac{1}{(4\pi t)^{3/2}} \quad (3.1a)$$

and

$$\delta C_\lambda(t)\simeq\left[\frac{K_{\epsilon H}}{(\nu+D_T)^{3/2}}+\frac{K_{+H}\cos(\omega_pt+\frac{3}{2}\theta_\lambda)}{[(D_T+\frac{1}{2}\Gamma_p)^2+(c^2/2\omega_p+\frac{1}{2}\gamma_p)^2]^{3/4}}\right]\frac{1}{(4\pi t)^{3/2}}+\frac{K_{+\epsilon}\cos(\omega_pt+\frac{1}{2}\theta_\nu)}{[(\nu+\frac{1}{2}\Gamma_p)^2+(c^2/2\omega_p+\frac{1}{2}\gamma_p)^2]^{1/4}}\frac{1}{(4\pi t)^{1/2}}, \quad (3.1b)$$

where

$$\theta_\nu=\tan^{-1}\left[\frac{c^2/2\omega_p+\frac{1}{2}\gamma_p}{\nu+\frac{1}{2}\Gamma_p}\right],$$

$$\theta_\lambda=\tan^{-1}\left[\frac{c^2/2\omega_p+\frac{1}{2}\gamma_p}{D_T+\frac{1}{2}\Gamma_p}\right], \quad (3.2)$$

$$\theta_p=\tan^{-1}\left[\frac{c^2/\omega_p+\gamma_p}{\Gamma_p}\right],$$

and where

$$L_{\epsilon\epsilon}=\tfrac{7}{15}(k_BT)^2,\ L_{+\epsilon}=\tfrac{2}{5}(k_BT)^2,\quad L_{++}=\tfrac{4}{15}(k_BT)^2,\ L_{+-}=0, \quad (3.3a)$$

and

$$K_{\epsilon H}=\frac{2C_v}{4}k_B^2T^3,\quad K_{+\epsilon}=\frac{2\omega_p^2}{3\pi}(k_BT)^2,\quad K_{+H}=\frac{C_vk_B^2T^3}{3}\left[1+\frac{\partial p}{\partial u}\bigg|_\rho\right]^2. \quad (3.3b)$$

The AB parts of the correlation functions with A and B equal to k or v can be obtained in a similar way. The results can be cast into the same form as Eqs. (3.1) by attaching superscripts AB to C, L, and K. We find

$$L^{kk}_{+-}=L^{vv}_{+-}=L^{kk}_{++}=L^{vv}_{++}=L^{kv}_{++}$$
$$=L^{vk}_{++}=-L^{kv}_{+-}=-L^{vk}_{+-}=\frac{(k_BT)^2}{15}\,,$$
$$L^{kk}_{+\epsilon}=L_{+\epsilon},\ \ L^{kv}_{+\epsilon}=L^{vk}_{+\epsilon}=L^{vv}_{+\epsilon}=0\,, \tag{3.4a}$$

and

$$K^{kk}_{\epsilon H}=\frac{25k_B}{6m^2C_v}(k_BT)^3\,,$$
$$K^{kv}_{\epsilon H}=K^{vH}_{\epsilon H}=\frac{5}{3m}(k_BT)^3\left[1-\frac{5k_B}{2mC_v}\right],$$
$$K^{vv}_{\epsilon H}=\frac{2}{3}C_vk_B^2T^3\left[1-\frac{5k_B}{2mC_v}\right]^2,$$
$$K^{kk}_{+\epsilon}=K^{kv}_{+\epsilon}=K^{vk}_{+\epsilon}=0,\ \ K^{vv}_{+\epsilon}=K_{+\epsilon}\,, \tag{3.4b}$$
$$K^{kk}_{+H}=\tfrac{1}{2}K^{kk}_{\epsilon H}=\frac{25k_B}{12m^2C_v}(k_BT)^3\,,$$
$$K^{kv}_{+H}=K^{vk}_{+H}=\frac{5}{6m}(k_BT)^3\left[1+\left.\frac{\partial p}{\partial u}\right|_\rho-\frac{5k_B}{2mC_v}\right],$$
$$K^{vv}_{+H}=\frac{C_vk_B^2T^3}{3}\left[1+\left.\frac{\partial p}{\partial u}\right|_\rho-\frac{5k_B}{2mC_v}\right]^2.$$

The mode-coupling amplitudes needed to evaluate the long-time tails of the correlation function for the longitudinal viscosity, $C_l(t)$, involve up to six-particle equilibrium correlation functions. For this reason we have only performed the calculation using the method of fluctuating hydrodynamics, where the explicit evaluation of the amplitudes in terms of equilibrium correlation functions is bypassed. The result is

$$\delta C_l(t)\simeq\left[\frac{N_{\epsilon\epsilon}}{(2\nu)^{3/2}}+\frac{N_{+\epsilon}\cos(\omega_pt+\frac{3}{2}\theta_\nu)}{[(\nu+\frac{1}{2}\Gamma_p)^2+(c^2/2\omega_p+\frac{1}{2}\gamma_p)^2]^{3/4}}+\frac{N_{++}\cos(2\omega_pt+\frac{3}{2}\theta_p)}{[\Gamma_p^2+(c^2/\omega_p+\gamma_p)^2]^{3/4}}+\frac{N_{+-}}{\Gamma_p^{3/2}}+\frac{N_{HH}}{(2D_T)^{3/2}}\right]\frac{1}{(4\pi t)^{3/2}} \tag{3.5a}$$

with

$$N_{\epsilon\epsilon}=\tfrac{4}{3}(k_BT)^2\left[\frac{4}{5}-\frac{\gamma-1}{\alpha T}+3\frac{\gamma-1}{2\alpha T}\right]^2,$$
$$N_{+\epsilon}=\tfrac{4}{15}(k_BT)^2\,,$$
$$N_{++}=(\tfrac{2}{15}+\tfrac{1}{4})(k_BT)^2\,, \tag{3.5b}$$
$$N_{+-}=\tfrac{1}{8}(k_BT)^2\left[2\frac{\gamma-1}{\alpha T}-1\right]^2,$$
$$N_{HH}=\left[\tfrac{1}{2}k_BT^2\rho C_v\left.\frac{\partial^2p}{\partial u^2}\right|_\rho\right]^2.$$

The long-time tails of the correlation function for the bulk viscosity, $C_\zeta(t)$, can immediately be obtained from Eqs. (2.2c), (3.1a), and (3.5a).

IV. DISCUSSION

This paper is concluded with a few remarks.

(1) We have also used the Kadanoff-Swift mode-coupling theory to calculate the long-time behavior of the tagged-particle velocity autocorrelation function, given by

$$C_D(t)=\lim{}_T\,\langle v_{1x}(t)v_{1x}\rangle\ . \tag{4.1}$$

Using the same method employed in this paper to evaluate the long-time decay of $C_i(t)$ $(i=\eta,\lambda,l,\zeta)$, we have obtained, for $t\to\infty$,

$$\delta C_D(t)\simeq\frac{2m}{3nk_BT}\left[\frac{1}{(D+\nu)^{3/2}}+\frac{1}{2}\frac{\cos(\omega_pt+\frac{3}{2}\theta_D)}{[(\nu+\frac{1}{2}\Gamma_p)^2+(c^2/2\omega_p+\frac{1}{2}\gamma_p)^2]^{3/4}}\right]\frac{1}{(4\pi t)^{3/2}} \tag{4.2a}$$

with

$$\theta_D=\tan^{-1}\left[\frac{c^2/2\omega_p+\frac{1}{2}\gamma_p}{D+\frac{1}{2}\Gamma_p}\right]. \tag{4.2b}$$

The same result has been obtained before by Gaskell.[12]

(2) We emphasize that, although phenomenological mode-coupling theory is used in this paper, the hydrodynamic modes given in Sec. II differ from those that follow from the usual phenomenological linearized hydro-

dynamic equations for a OCP. The phenomenological equations involve only zero-frequency transport coefficients. The plasma modes in a OCP are, however, finite-frequency modes.[9] As a consequence these modes involve a finite-frequency longitudinal viscosity $D_l(\omega)$, evaluated at $\omega=\omega_p$. In Eq. (2.18a) we have written $D_l(\sigma\omega_p)=i\sigma\gamma_p+\Gamma_p$. The damping of the plasma modes is therefore not simply related to the usual, zero-frequency viscosities.

The need to use finite-frequency transport coefficients to describe hydrodynamics in a OCP is not just a technical point, since the density and temperature dependence of the frequency-dependent transport coefficients is qualitatively different from that of their zero-frequency counterparts. For instance, for small plasma parameter ϵ_p, we have $\mathrm{Re}D_l(\omega_p)\sim\epsilon_p$, but $D_l(0)\sim\epsilon_p^{-1}$.

(3) In view of the remark of (2) above, the zero-frequency bulk viscosity is not a physically meaningful quantity for a OCP. All experimental techniques used to measure the bulk viscosity rely on the application of a dynamical perturbation to the system. In such experiments one therefore measures the finite-frequency bulk viscosity that enters in the damping of the plasma modes. The zero-frequency bulk viscosity could only be measured in a computer experiment by evaluating the corresponding correlation function.

(4) The results derived here for $C_i^{kk}(t)$ $(i=\eta,\lambda)$ and $C_D(t)$ using mode-coupling theory are identical to those derived previously[2] using kinetic theory if the coefficients L^{kk} and K^{kk} are replaced by their low-density values. It is interesting to note that in the long-time tails of $C_\eta(t)$ one finds $L_{+-}=0$, but $L_{+-}^{kk}\neq0$, i.e., the potential parts of the time correlation functions can cancel the kinetic part. This is to be contrasted with the case of a neutral fluid, where the potential contributions are always of higher order in the density than the kinetic-kinetic ones.

Another important difference between the long-time tails in the charged and neutral gases appears when examining the plasma parameter dependence of the various contributions to the long-time behavior of $\delta C_i(t)$, for $i=\eta,\lambda$. For small values of the plasma parameter, $\epsilon_p\ll1$, the transport coefficients on the right-hand side of Eqs. (3.1) can be evaluated using, for instance, the Landau kinetic equations. Denoting the results for $\epsilon_p\ll1$ with a subscript zero, one finds $\nu_0,D_{T0}\sim(m\beta\omega_p)^{-1}$, $\Gamma_{p0}\sim\epsilon_p/m\beta\omega_p$, and $\gamma_p\sim(m\beta\omega_p)^{-1}$. The speed of sound in this approximation is given by its ideal-gas value, $c_0^2=5/3m\beta$. It is then easy to see that all the coefficients of the long-time tails involving the coupling of two shear modes, two heat modes, a heat and a plasma mode, and a shear and a plasma mode are of $O(\epsilon_p^{3/2})$. The corresponding contributions to the transport coefficients are of order ϵ_p. Similarly, in a neutral gas at low density the ratio of the mode-coupling contributions to the transport coefficients to their Boltzmann values is of order n.[2] The coefficients of the long-time tails involving the coupling of two plasma modes behave, however, quite differently. Specifically we find $L_{++},N_{++}\sim\epsilon_p^0$ and $N_{+-}/\Gamma_p^{3/2}\sim\epsilon_p^{-3/2}$. As a consequence, the coupling of two plasma modes of opposite sign leads to a mode-coupling contribution to the longitudinal zero-frequency viscosity, $D_l=D_l(0)$, that is of the same order in the plasma parameter as the lowest-order bare contribution D_{l0}.

The coupling mechanisms that involve two plasma modes of the same sign or a plasma mode and a purely diffusive mode lead to oscillating long-time tails of the form $t^{-3/2}\cos(\omega_pt+\frac{3}{2}\theta)$ or $t^{-1/2}\cos(\omega_pt+\frac{1}{2}\theta_v)$ for the thermal conductivity. In an actual experiment one analyzes the behavior of the correlation functions by looking at their time average over an observation time T, with $T\gg\omega_p^{-1}$. The time average of the oscillating tails of the form $t^{-3/2}\cos(\omega_pt+\frac{3}{2}\theta)$ is then found to delay one power of the time faster than the average of the purely decaying $t^{-3/2}$ tail. The Green-Kubo integrand for the thermal conductivity has a long-time contribution proportional to $t^{-1/2}\cos(\omega_pt+\frac{1}{2}\theta_v)$. The time average of this oscillating tail leads to the same long-time decay as the time average of the $t^{-3/2}$ tail. The "true" asymptotic behavior of the correlation functions is then governed by the purely decaying $\sim t^{-3/2}$ tail and by the oscillating tail $\sim t^{-1/2}\cos(\omega_pt+\frac{1}{2}\theta_v)$ for the thermal conductivity.

(5) It is well known in the theory of neutral fluids that the same mode-coupling mechanisms that govern the long-time behavior of the Green-Kubo integrands also lead to nonanalyticities in the small-k expansion of the hydrodynamic dispersion relations.[10]

In a OCP the purely decaying, $\sim t^{-3/2}$, long-time tails of the correlation functions lead to nonanalyticities in the small-q expansions of the heat and viscous hydrodynamic dispersion relations. From symmetry arguments one expects that, for small q, the hydrodynamic dispersion relations have an analytic expansion in powers of q^2. Consequently, the first nonvanishing corrections to the dispersion relations given in Eqs. (2.20b) and (2.20c) should be of order q^4. One can, however, show that the mode-coupling effects discussed here lead to corrections to Eqs. (2.20b) and (2.20c) of $O(q^3)$.

The dispersion relation for the finite-frequency plasma modes is evaluated by expanding about $k\sim0$ and $\omega\sim\omega_p$. Nonanalyticities in such an expansion can arise from the oscillating long-time tails of the Green-Kubo integrands. The result is that the corrections to Eq. (2.20a) are also of $O(q^3)$. Such behavior has been observed in computer simulations of a OCP.[1]

(6) The oscillating tail for the correlations function for the thermal conductivity contains an oscillating tail of the form $t^{-1/2}\cos(\omega_pt+\theta_v)$. This will lead to a divergence in the zero-wave-number finite-frequency thermal conductivity $\lambda(\omega)$ for $\omega\sim\pm\omega_p$, i.e., $\lambda(\omega)\sim(\omega\pm\omega_p)^{-1/2}$. It is not clear how to directly measure a finite-frequency heat conductivity. We note, however, that this singularity is another mechanism that can lead to terms of $O(q^3)$ in the hydrodynamic dispersion relation for the plasma modes.

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APPENDIX

In this appendix we briefly indicate how the equilibrium correlation functions determining the mode-coupling amplitudes can be evaluated for a OCP and list the results.

In general, we have to evaluate correlation functions of the form $\langle a_n(\mathbf{k})a_{n'}(-\mathbf{k})\rangle$ and $\langle j_i(-\mathbf{k})a_n(\mathbf{q})a_m(\mathbf{l})\rangle$, with $a_n(\mathbf{k})$ the densities defined in Eqs. (2.4)–(2.7) and $j_i(\mathbf{k})$ the currents, in the limit of $\mathbf{k}\rightarrow 0$ and $\mathbf{q}\rightarrow 0$.

Due to the vanishing of the static structure factor as $\mathbf{k}\rightarrow 0$ the fluctuation formula relating the compressibility to density fluctuations has to be modified for a OCP and reads

$$\lim_{k\rightarrow 0}\left[\frac{1}{S(k)}-\frac{1}{k^2\lambda_D^2}\right]^{-1}=\chi_T/\chi_T^0 , \tag{A1}$$

with $\chi_T^0=\beta/n$ the ideal-gas compressibility. This has been discussed by Baus.[13] As a consequence, we cannot use here the technique employed for neutral fluids that consists in taking the limit $\mathbf{k}\rightarrow 0$ and $\mathbf{q}\rightarrow 0$ from the outset and then evaluating the correlation functions by relating them to certain thermodynamic derivatives.[7] We will instead proceed as follows. The correlation functions of interest are first expressed exactly in terms of the configurational two-, three-, etc., particle distribution functions, for nonzero values of $\mathbf{k}$ and $\mathbf{q}$. Such expressions may involve distribution functions of order as high as the six-particle distribution function. Certain exact properties of the distribution functions for $\mathbf{k}\rightarrow 0$ and $\mathbf{q}\rightarrow 0$ are then used to obtain the desired result. To illustrate our method we will display here a few details for the calculation of one of the correlation functions of interest. We will need the $\mathbf{k}\rightarrow 0$ limit of the functions $W_{\alpha\beta}$ and $W_{\alpha\beta}^e$ determining the potential part of $j_{\alpha\beta}(\mathbf{k})$ and $j_\alpha^e(\mathbf{k})$, respectively, defined in Eqs. (2.8). These are given by

$$\lim_{\mathbf{k}\rightarrow 0} W_{\alpha\beta}(\mathbf{k},\mathbf{k}')=\begin{cases}0, & \mathbf{k}'=0\\ v_{k'}\hat{k}_\alpha[\hat{k}_\beta-2\hat{k}'_\beta(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')], & \mathbf{k}'\neq 0\end{cases} \tag{A2a}$$

and

$$\lim_{\mathbf{k}\rightarrow 0} W_{\alpha\beta}^e(\mathbf{k},\mathbf{k}')=\begin{cases}0, & \mathbf{k}'=0\\ v_{k'}\hat{k}_\alpha[\hat{k}_\beta-\hat{k}'_\beta(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')], & \mathbf{k}'\neq 0.\end{cases} \tag{A2b}$$

Using Eqs. (A2) it is easy to see that in the limit $\mathbf{k}=0$ the momentum and energy currents are identical to those for neutral fluid, except that the potential $v(r_{ij})$ is everywhere replaced with

$$\frac{1}{\Omega}\sum_{\mathbf{k}'(\neq 0)} v_{k'}e^{i\mathbf{k}'\cdot\mathbf{r}_{ij}} .$$

We consider now the following correlation function:

$$\langle j_\alpha^e(\mathbf{k})\rho(-\mathbf{q})g_\beta(-\mathbf{l})\rangle=\delta_{\alpha\beta}\frac{5p}{2\beta^2}S(q) + \frac{np}{\beta\Omega}\sum_{\mathbf{k}'} W_{\alpha\beta}^e(\mathbf{k},\mathbf{k}')[g_2(k')+g_2(|\mathbf{q}-\mathbf{k}'|)+ng_3(\mathbf{q}-\mathbf{k}',-\mathbf{q})-n\Omega\delta_{\mathbf{q},0}g_2(k')] . \tag{A3}$$

Here we have introduced the Fourier transform of the configurational part of the two- and three-particle distribution functions, defined by

$$n^2g_2(k)=\int d\mathbf{r}_{12}\lim\nolimits_T\left[\frac{1}{\Omega}\langle N(N-1)e^{-i\mathbf{k}\cdot\mathbf{r}_{12}}\rangle\right] , \tag{A4a}$$

$$n^3g_3(\mathbf{k},\mathbf{q})=\int d\mathbf{r}_{12}\int d\mathbf{r}_{32}\lim\nolimits_T\left[\frac{1}{\Omega}\langle N(N-1)(N-2)e^{-i\mathbf{k}\cdot\mathbf{r}_{12}}e^{-i\mathbf{q}\cdot\mathbf{r}_{32}}\rangle\right] . \tag{A4b}$$

It is convenient to express g_2 and g_3 in terms of cluster functions, defined by

$$g_2(k)=h(k)+\Omega\delta_{\mathbf{k},0} , \tag{A5a}$$

$$g_3(\mathbf{k},\mathbf{q})=\Omega^2\delta_{\mathbf{k},0}\delta_{\mathbf{q},0}+\Omega\delta_{\mathbf{k},0}h(q)+\Omega\delta_{\mathbf{q},0}h(k)+\Omega\delta_{\mathbf{k},-\mathbf{q}}h(k)+H_3(\mathbf{k},\mathbf{q}) , \tag{A5b}$$

where $h(k)$ is the Fourier transform of the usual pair-correlation function, related to $S(k)$ by

$$S(k)=1+nh(k) . \tag{A5c}$$

Substituting Eqs. (A5) into Eq. (A3) we obtain

$$\langle j_\alpha^e(\mathbf{k})\rho(-\mathbf{q})g_\beta(-\mathbf{l})\rangle=\delta_{\alpha\beta}\frac{5p}{2\beta^2}S(q)+\frac{np}{\beta\Omega}\sum_{\mathbf{k}'} W_{\alpha\beta}^e(\mathbf{k},\mathbf{k}')S(q)[\Omega\delta_{\mathbf{k}',0}+\Omega\delta_{\mathbf{k}',\mathbf{q}}+h(k')+h(|\mathbf{q}-\mathbf{k}'|)+H_3(\mathbf{q}-\mathbf{k}',\mathbf{q})] . \tag{A6a}$$

To evaluate the second term on the right-hand side of Eq. (A6a) we use the convolution approximation[14] for H_3, denoted by H_3^c, which is exact for small wave numbers for a OCP. This leads to

$$h(k')+h(|\mathbf{q}-\mathbf{k}'|)+H_3^c(\mathbf{q}-\mathbf{k}',\mathbf{q}) = S(q)h(k')+h(|\mathbf{q}-\mathbf{k}'|)S(\mathbf{k}') . \quad \text{(A6b)}$$

Combining Eqs. (A6) and using that $S(q)\sim q^2$ for small q, we obtain

$$\lim_{\mathbf{k}\to 0}(j_\alpha^e(\mathbf{k})\rho(-\mathbf{q})g_\beta(-1)) = \frac{n\rho}{\beta}v_q S(q)\hat{k}_\alpha[\hat{k}_\beta-\hat{q}_\beta(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})]+O(q^2) . \quad \text{(A7)}$$

The coefficient of the term of order q^2 in the expansion of $S(q)$ is given exactly by that obtained from the Debye-Hückel value of $S(q)$, i.e.,

$$S(q)=q^2\lambda_D^2+O(q^4) . \quad \text{(A8)}$$

Equation (A8) is exact in all orders in the plasma parameter ϵ_p and is a consequence of the perfect screening or Stillinger-Lovett sum rule.[15]

To conclude, we first list the results for the correlation functions that are needed to evaluate the projected currents and the mode-coupling amplitudes. They are

$$\lim_{\mathbf{k}\to 0}(j_\alpha^e(\mathbf{k})g_\beta(-\mathbf{k}))=\delta_{\alpha\beta}\frac{h}{\beta} , \quad \text{(A9a)}$$

$$\lim_{\mathbf{k}\to 0}\hat{k}_\beta(j_{\alpha\beta}(\mathbf{k})\rho(-\mathbf{k}))=\hat{k}_\alpha\frac{\rho}{\beta} , \quad \text{(A9b)}$$

$$\lim_{\mathbf{k}\to 0}\hat{k}_\beta(j_{\alpha\beta}(\mathbf{k})e(-\mathbf{k}))=\lim_{\mathbf{k}\to 0}\hat{k}_\beta(g_\alpha(\mathbf{k})j_\beta^e(-\mathbf{k}))=\hat{k}_\alpha\frac{h}{\beta} , \quad \text{(A9c)}$$

$$\lim_{\mathbf{k}\to 0}\hat{k}_\beta(j_{\alpha\beta}(\mathbf{k})T(-\mathbf{k}))=\hat{k}_\alpha\frac{\gamma-1}{\alpha}k_B T , \quad \text{(A9d)}$$

and

$$\lim_{\mathbf{k}\to 0}\hat{k}_\beta(j_{\alpha\beta}(\mathbf{k})\rho(-\mathbf{q})\rho(-1)) = \frac{m\rho}{\beta}\left[\hat{k}_\alpha S(q)-\hat{q}_\alpha(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})\frac{\partial S(q)}{\partial q}\right] , \quad \text{(A10a)}$$

$$\lim_{\mathbf{k}\to 0}([j_{\alpha\beta}(\mathbf{k})-p\Omega\delta_{\mathbf{k},0}\delta_{\alpha\beta}]g_\gamma(-\mathbf{q})g_\epsilon(-1)) = \rho/\beta^2(\delta_{\alpha\beta}\delta_{\gamma\epsilon}+\delta_{\alpha\gamma}\delta_{\beta\epsilon}+\delta_{\alpha\epsilon}\delta_{\beta\gamma}) , \quad \text{(A10b)}$$

$$\lim_{\mathbf{k}\to 0}\hat{k}_\alpha(j_\alpha^e(\mathbf{k})\rho(-\mathbf{q})g_\beta(-1)) = \frac{n\rho}{\beta}\frac{vq}{\beta^2}S(q)[\hat{k}_\beta-\hat{q}_\beta(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})]+O(q^2) , \quad \text{(A10c)}$$

$$\lim_{\mathbf{k}\to 0}(j_\lambda(\mathbf{k})T(-\mathbf{q})g_\beta(-1)) = \hat{k}_\beta k_B^2T^3\left[1-\left.\frac{\partial p}{\partial u}\right|_\rho\right]+O(q^2) , \quad \text{(A10d)}$$

$$\lim_{\mathbf{k}\to 0}(g_\alpha(\mathbf{k})\rho(-\mathbf{q})g_\beta(-1))=\frac{m\rho}{\beta}\delta_{\alpha\beta}S(q) , \quad \text{(A10e)}$$

$$\lim_{\mathbf{k}\to 0}(g_\alpha(\mathbf{k})e(-\mathbf{q})g_\beta(1))=\frac{\rho}{\beta^2}\delta_{\alpha\beta}+O(q^2) . \quad \text{(A10f)}$$

Finally, the mode-coupling amplitudes that are needed to evaluate Eq. (2.16) are listed. Those involving the shear current are

$$(j_\eta(\mathbf{k})A_{\epsilon_i}^\dagger(\mathbf{q})A_{\epsilon_j}^\dagger(1))=(j_\eta^k(\mathbf{k})A_{\epsilon_i}^\dagger(\mathbf{q})A_{\epsilon_j}^\dagger(1)) = \frac{1}{\rho}(\hat{q}_{\perp x}^i\hat{q}_{\perp y}^j+\hat{q}_{\perp y}^i\hat{q}_{\perp x}^j) , \quad \text{(A11a)}$$

$$(A_{\epsilon_i}(\mathbf{q})A_{\epsilon_j}(1)j_\eta^*(\mathbf{k}))\equiv(A_{\epsilon_i}(\mathbf{q})A_{\epsilon_j}(1)j_\eta^{k*}(\mathbf{k})) = \frac{\rho}{\beta^2}(\hat{q}_{\perp x}^i\hat{q}_{\perp y}^j+\hat{q}_{\perp y}^i\hat{q}_{\perp x}^j) , \quad \text{(A11b)}$$

$$(j_\eta(\mathbf{k})A_\sigma^\dagger(\mathbf{q})A_{\sigma'}^\dagger(1))=\frac{\beta^2}{\rho^2}(A_\sigma(\mathbf{q})A_{\sigma'}(1)j_\eta^*(\mathbf{k})) = -\frac{1}{2\rho}(1+\sigma\sigma')\hat{q}_x\hat{q}_y , \quad \text{(A11c)}$$

$$(j_\eta^k(\mathbf{k})A_\sigma^\dagger(\mathbf{q})A_{\sigma'}^\dagger(1))=\frac{\beta^2}{\rho^2}(A_\sigma(\mathbf{q})A_{\sigma'}(1)j_\eta^{k*}(\mathbf{k})) = \frac{1}{2\rho}\hat{q}_x\hat{q}_y , \quad \text{(A11d)}$$

$$(j_\eta(\mathbf{k})A_\sigma^\dagger(\mathbf{q})A_{\epsilon_i}^\dagger(1))=(j_\eta^k(\mathbf{k})A_\sigma^\dagger(\mathbf{q})A_{\epsilon_i}^\dagger(1)) = \frac{-1}{\rho\sqrt{2}}(\hat{q}_{\perp x}^i\hat{q}_y+\hat{q}_{\perp y}^i\hat{q}_x) , \quad \text{(A11e)}$$

$$(A_\sigma(\mathbf{q})A_{\epsilon_i}(1)j_\eta^*(\mathbf{k}))=(A_\sigma(\mathbf{q})A_{\epsilon_i}(1)j_\eta^{k*}(\mathbf{k})) = \frac{\rho}{\beta^2\sqrt{2}}(\hat{q}_x\hat{q}_{\perp y}^i+\hat{q}_y\hat{q}_{\perp x}^i) . \quad \text{(A11f)}$$

All other mode-coupling amplitudes involving j_η vanish as $k\to 0$ and $q\to 0$.

The amplitudes involving the heat conductivity current j_λ are obtained from Eqs. (A10) and are given by

$$(j_\lambda(\mathbf{k})A_{\epsilon_i}^\dagger(\mathbf{q})A_H^\dagger(1))=\frac{C_v}{k_B^2T^3}(A_{\epsilon_i}(\mathbf{q})A_H(1)j_\lambda^*(\mathbf{k})) = C_v\hat{q}_{\perp x}^i , \quad \text{(A12a)}$$

$$(j_\lambda^k(\mathbf{k})A_{\epsilon_i}^\dagger(\mathbf{q})A_H^\dagger(1))=\frac{C_v}{k_B^2T^3}(A_{\epsilon_i}(\mathbf{q})A_H(1)j_\lambda^{k*}(\mathbf{k})) = \frac{5k_\beta}{2m}\hat{q}_{\perp x}^i , \quad \text{(A12b)}$$

$$(j_\lambda(\mathbf{k})A_\sigma^\dagger(\mathbf{q})A_H^\dagger(1))=\frac{C_v}{k_B^2T^3}(A_\sigma(\mathbf{q})A_H(1)j_\lambda^*(\mathbf{k})) = \frac{1}{\sqrt{2}}C_v\left[1+\left.\frac{\partial p}{\partial u}\right|_\rho\right]\hat{q}_x , \quad \text{(A12c)}$$

$$(j_\lambda^k(\mathbf{k})A_\sigma^\dagger(\mathbf{q})A_H^\dagger(1))=\frac{C_v}{k_B^2T^3}(A_\sigma(\mathbf{q})A_H(1)j_\lambda^{k*}(\mathbf{k})) = \frac{1}{\sqrt{2}}\frac{5k_\beta}{2m}\hat{q}_x , \quad \text{(A12d)}$$

$$\langle j_\lambda(\mathbf{k})A^\dagger_{\epsilon_i}(\mathbf{q})A^\dagger_\sigma(\mathbf{l})\rangle=\frac{\beta^2}{\rho^2}\langle A_{\epsilon_i}(\mathbf{q})A_\sigma(\mathbf{l})j^*_\lambda(\mathbf{k})\rangle =\frac{1}{\sqrt{2}}\frac{\sigma\omega_p}{q\rho}\hat{q}^{\,i}_{\perp x}\;, \tag{A12e}$$

$$\langle j^k_\lambda(\mathbf{k})A^\dagger_{\epsilon_i}(\mathbf{q})A^\dagger_\sigma(\mathbf{l})\rangle=\frac{\beta^2}{\rho^2}\langle A_{\epsilon_i}(\mathbf{q})A_\sigma(\mathbf{l})j^{k*}_\lambda(\mathbf{k})\rangle =0\;. \tag{A12f}$$

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*Present address: Department of Physics, City College of the City University of New York, New York, N.Y. 10031.

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