High-energy inverse free-electron-laser accelerator

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We study the inverse free-electron-laser (IFEL) accelerator and show that it can accelerate electrons to the few hundred GeV region with average acceleration rates of the order of 200 MeV/m. Several possible accelerating structures are analyzed, and the effect of synchrotron-radiation losses is studied. The longitudinal phase stability of accelerated particles is also analyzed. A Hamiltonian description, which takes into account the dissipative features of the IFEL accelerator, is introduced to study perturbations from the resonant acceleration. Adiabatic invariants are obtained and used to estimate the change of the electron phase-space density during the acceleration process.

I. INTRODUCTION

The application of high-power-laser radiation to the acceleration of particles was considered almost from the beginning of lasers. Various mechanisms and acceleration schemes were proposed for laser accelerators and their description can be found in Refs. 1 and 2.

One of the laser accelerators studied during recent years is the inverse free-electron laser (IFEL). Successful experiments with the free-electron laser[3] have shown that there is indeed transfer of energy between the laser and electron beams in the presence of the undulator magnetic field. In a free-electron laser the energy is transferred from electrons to the laser beam. In the IFEL accelerator the energy transfer is in the opposite direction, from the laser beam to electrons.

The basic principles of the IFEL accelerator, although that name was given later, were proposed by Palmer.[4] Similar systems using a longitudinal instead of a wiggler magnetic field were also proposed by Kolomensky and Lebedev[5] and later by other authors.[6] Several authors have discussed the possibility of using the IFEL to accelerate electrons to energies of the order of a few GeV (Refs. 7 and 8) and, more recently, up to the 100-GeV region.[9–11]

In an IFEL relativistic particles are moving through an undulator magnet; a plane electromagnetic wave is propagating parallel to the beam (Fig. 1). The undulator magnet produces a small transverse velocity (wiggling motion) in a direction parallel to the electric vector of the wave, so that energy can be transferred between the particle and the wave.

The acceleration rate one can achieve in an IFEL depends on the power of the laser beam. For a power density of 3.10^{15} W/cm^2 the acceleration rate for 10-GeV electrons is of the order of 0.5 GeV/m. At higher energies the rate of electron acceleration decreases because of synchrotron-radiation losses produced by the wiggling of the particles in the undulator. Acceleration of protons or heavier particles can also be made, with acceleration rates of the order of a few hundred MeV/m.

One of the major problems of laser accelerators is the confinement of a high-power-laser beam over the length of the accelerator. By focusing the beam one can easily obtain a very high-power density, but then diffraction seriously limits the acceleration length.[10] To reach energies of the order of 100 GeV one needs to keep the laser beam focused to a size of the order of 1 mm^2 over a distance of the order of 1 km.

One possibility of doing this, suggested in Ref. 12 is the confinement and propagation of the laser beam inside a hollow optical waveguide. Standard metallic waveguides show, however, very high attenuation at optical frequencies. These high losses can be drastically reduced by a proper dielectric coating of the metallic walls of the waveguide.[13] In an ideal case, without any imperfections, the losses can be reduced by several order of magnitude reaching values of the order of 10^{-5} dB/m. With losses so small it should be possible to transmit a laser beam with a power of 3×10^{15} W/cm^2 without damaging the waveguide. Having this option in mind we will simplify our analysis representing the laser radiation by a plane wave.

In Sec. II we derive the equations describing an IFEL. We will consider only the acceleration of electrons. We

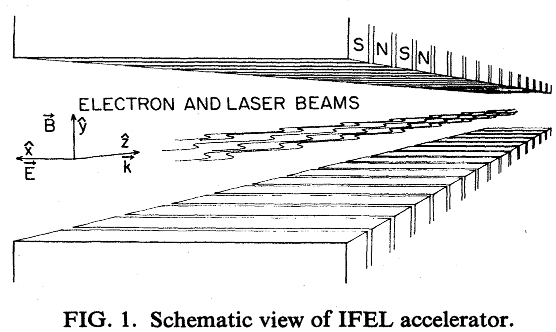

FIG. 1. Schematic view of IFEL accelerator.

also assume the transverse electron motion to be stabilized by the focusing properties of the wiggler[10] or by external focusing quadrupoles and study only the longitudinal electron motion. We consider two general types of accelerator. One, with cylindrical geometry, using a helical wiggler and circularly polarized plane electromagnetic (EM) wave and the second with planar geometry, a planar wiggler, and linearly polarized radiation. The accelerator equations are simpler for the helical wiggler case; the planar case can be scaled and the equations reduced to the circular case.

The electron state in the wiggler field in the presence of the EM wave is described by two dynamical variables: the energy γ and the relative phase ψ of the electron oscillation in the wiggler and the electromagnetic wave. They satisfy a coupled set of equations describing the electron motion along the accelerator. Defining the resonant electrons as those for which the phase remains constant, we can design the wiggler structure in such a way that the corresponding resonant energy is increasing. The acceleration can be accomplished with different wiggler structures. In Sec. III we investigate in more detail four accelerator designs in which either the wiggler period Λ_w or the wiggler magnetic field strength B_w or the wiggler parameter K are kept constant or Λ_w and B_w are varied to optimize the rate of acceleration. The synchrotron losses are an important factor in the IFEL electron accelerator and are taken into account in our analysis. In the constant-period and constant-strength accelerators the losses are growing with energy, limiting the maximum electron energy. In the constant K accelerator the losses remain smaller than the acceleration rate and there is no limiting energy. By varying both Λ_w and B_w we can optimize the driving term and the loss term and we can design an accelerator giving the maximum rate of acceleration for a given laser field strength. An analytical solution of the resonant equations for all four kinds of accelerators is found.

In Sec. IV the stability of acceleration is considered. The accelerator equations are expanded near the resonant energy and small deviations from the resonant conditions are studied. Our treatment is similar to that used in conventional accelerators,[14] or for tapered wiggler free-electron lasers.[15]

The IFEL accelerators exhibit, however, some peculiar properties; contrary to conventional accelerators, the acceleration rate is energy dependent; synchrotron-radiation losses are very important, while they are negligible in the tapered wiggler free-electron-laser case.

Because of these peculiarities it is convenient to use new dynamical variables. Transforming to new variables, $u=f(\gamma_r)\delta\gamma$, $v=g(\gamma_r)\phi$ with $\delta\gamma=\gamma-\gamma_r$, and $\phi=\psi-\psi_r$, one can find functions f and g that the system of equations for u and v becomes Hamiltonian, also in the presence of synchrotron-radiation losses. One can then use adiabatic invariants to determine the evolution of nonsynchronous electrons. When the radiative losses can be neglected, the variables permitting the Hamiltonian description are simply $\phi,\delta\gamma$. It is interesting that we can find a Hamiltonian description also in situations when the synchrotron losses are important and the system is evidently dissipative.

Using such methods we determine the rate of the decay of oscillation amplitudes for the energy and phase deviations. We illustrate our discussion of the accelerator performance and stability with numerical examples.

II. ACCELERATOR EQUATIONS

In the inverse free-electron-laser accelerator relativistic electrons are moving along a magnetic wiggler in the field of a laser EM wave also propagating along the wiggler axis, Fig. 1. These electrons can exchange energy with the EM wave. In the accelerator design one must prepare such conditions that for some electrons the increase of energy can be continuous and effective.

The equations describing the motion of electrons in the accelerator can be derived from the Lorentz equation of motion including also the force of radiation reaction,

$$m\frac{d(\gamma\mathbf{v})}{dt}=e\left[\mathbf{E}_L+\frac{\mathbf{v}}{c}\times(\mathbf{B}_L+\mathbf{B}_W)\right]+\mathbf{F}_{\mathrm{reac}}\ , \tag{1}$$

where $\mathbf{E}_L$ and $\mathbf{B}_L$ are the electric and magnetic fields of the laser radiation, $\mathbf{B}_w$ is the magnetic field of the wiggler, $\gamma=(1-\beta^2)^{-1}$, $\beta=v/c$. Because for a transverse EM wave $\mathbf{B}_L=\hat{\mathbf{k}}\times\mathbf{E}_L=\hat{\mathbf{z}}\times\mathbf{E}_L$, we get

$$m\frac{d}{dt}(\gamma\mathbf{v})=e[\mathbf{E}_L(1-\beta_z)+\hat{\mathbf{z}}(\boldsymbol{\beta}\cdot\mathbf{E}_L)+\boldsymbol{\beta}\times\mathbf{B}_w]+\mathbf{F}_{\mathrm{reac}}\ . \tag{2}$$

For relativistic electrons, assuming $\beta_T \ll \beta_z \leq 1$, and not extremely strong laser fields, the transverse motion is determined, to order $1/\gamma^2$, only by the wiggler field.[16,17] We can easily determine the transverse velocity if we neglect the reaction force and assume that the EM wave and wiggler field depend only on z. The transverse canonical momentum is then conserved

$$\mathbf{p}_T=m\gamma\mathbf{v}_T+e(\mathbf{A}_L+\mathbf{A}_w)=\mathrm{const}\ , \tag{3}$$

where $\mathbf{A}_L,\mathbf{A}_w$ are the vector potentials. We have neglected the transverse component of the reaction force. Later we give some estimate of this approximation showing its validity in the situations that we are considering.

The most important terms of the longitudinal component of Eq. (2) are those describing the change of the electron's energy. Equivalently we can use the energy component of the equation of motion,

$$mc^2\frac{d\gamma}{dt}=e\mathbf{v}_T\cdot\mathbf{E}_L-\frac{dP_{\mathrm{rad}}}{dt}\ . \tag{4}$$

The loss term is due to the synchrotron radiation by the electron oscillating the wiggler and is given by[18]

$$\frac{dP}{dt}=\frac{2}{3}\frac{e^2}{c}\gamma^6[\dot{\boldsymbol{\beta}}^2-(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}})^2]\ . \tag{5}$$

Equations (3) and (4) are a convenient and accurate starting set of equations. Using them we will derive the accelerator equations. Formally the accelerator equations are slowly varying components of these equations. In our analysis we will distinguish accelerators with helical wigglers and circularly polarized laser beam and accelerators with planar wigglers and linearly polarized EM waves.

A. Helical wiggler

The magnetic field of a helical wiggler is

$$\mathbf{B}_w=(B_w\cos(\kappa z),B_w\sin(\kappa z),0)\ , \tag{6}$$

where $\kappa=2\pi/\Lambda_w$ and Λ_w is the wiggler period. The wiggler period Λ_w as well as its strength B_w can slowly vary along the accelerator to maintain the resonant interaction of electrons with the laser radiation.

The electromagnetic wave propagating along the wiggler is described by ($\omega=kc$)

$$\mathbf{E}_L=(E_0\sin(kz-\omega t),E_0\cos(kz-\omega t),0)\ . \tag{7}$$

Using Eq. (3) we obtain for the transverse velocities

$$\begin{aligned} v_x&=c\frac{K}{\gamma}\cos(\kappa z)+c\frac{K_L}{\gamma}\cos(kz-\omega t)\ ,\\ v_y&=c\frac{K}{\gamma}\sin(\kappa z)-c\frac{K_L}{\gamma}\sin(kz-\omega t)\ , \end{aligned} \tag{8}$$

where the wiggler and EM wave parameters K, K_L are

$$K=\frac{eB_w\Lambda_w}{2\pi mc^2}=\frac{eB_w}{mc^2\kappa},\quad K_L=\frac{eE_0\lambda}{2\pi mc^2}=\frac{eE_0}{mc^2k}\ , \tag{9}$$

and having assumed that the injection of electrons is such that the velocities v_x and v_y do not have any constant term. For all cases of interest $K_L/K\ll 1$; in what follows we will use this fact whenever possible to simplify our equations. For this motion we can estimate the transverse component of the radiative reaction force. For ultrarelativistic electrons and $K\gg K_L$, we get[19]

$$\mathbf{F}_{T,\mathrm{reac}}=-\frac{2}{3}\frac{e^4}{m^2c^5}\gamma^2B_w^2\mathbf{v}_T\ . \tag{10}$$

For $\mathbf{v}_T$ given by (8) we get

$$|\,\mathbf{F}_{T,\mathrm{reac}}\,|=\frac{1}{3\pi mc^2}er_e^2\Lambda_wB_w^3\gamma\ . \tag{11}$$

Comparing this with the force created by the wiggler $|\,\mathbf{F}_w\,|=eB$ we find that the contribution of the transverse component of the force of radiative reaction can be neglected for energies

$$\gamma\ll\frac{3\pi mc^2}{r_e^2\Lambda_wB_w^2}\left[\simeq\frac{10^{20}}{[\Lambda_w(\mathrm{cm})][B_w(\mathrm{gauss})]^2}\right]. \tag{12}$$

It would be very difficult to violate this condition. The radiative reaction is, however, important for the longitudinal component of the equation of motion and the energy exchange equation.

The change of electron energy is

$$\frac{d\gamma}{dt}=\frac{e}{mc^2}\mathbf{E}\cdot\mathbf{v}-\frac{1}{mc^2}\frac{dP_{\mathrm{rad}}}{dt}\ , \tag{13a}$$

$$\frac{d\gamma}{dt}=cA\frac{K}{\gamma}\sin\psi-\tfrac{2}{3}r_ec\kappa^2\gamma^2(K^2+K_L^2+2KK_L\cos\psi)\ , \tag{13b}$$

where $A=eE_0/mc^2$ and the radius of electron $r_e=e^2/mc^2$.

The phase ψ is

$$\psi=(k+\kappa)z-\omega t \tag{14}$$

and satisfies the equation

$$\frac{d\psi}{dt}=(k+\kappa)v_z-\omega\ . \tag{15}$$

The longitudinal velocity v_z can be expressed by means of the electron energy and transverse velocity given by Eq. (8). We get

$$v_z=c\left[1-\frac{1+K^2+K_L^2+2KK_L\cos\psi}{2\gamma^2}\right]. \tag{16}$$

Changing the independent variable from the time t into the longitudinal position along the accelerator $z(d/dt=v_zd/dz\simeq cd/dz)$ we obtain the final set of equations describing the electron motion in the helical wiggler in the presence of the circularly polarized plane wave

$$\frac{d\gamma}{dz}=A\frac{K}{\gamma}\sin\psi-\tfrac{2}{3}r_e\kappa^2\gamma^2(K^2+K_L^2+2KK_L\cos\psi)\ , \tag{17}$$

$$\frac{d\psi}{dz}=\kappa-k\frac{1+K^2+K_L^2+2KK_L\cos\psi}{2\gamma^2}\ . \tag{18}$$

These are the same equations used to describe a FEL except for the presence of the synchrotron-radiation-loss term.[16] We see from Refs. 17 and 18 the relevant dynamical variables for electrons as their energies γ and phases ψ determining the relative phases of oscillations caused by the wiggler with respect to the phase of EM wave.

B. Planar wiggler

In a planar undulator the magnetic field can be approximated by

$$\mathbf{B}_w=(0,B_w\sin\kappa z,0)\ . \tag{19}$$

The electric field of the EM wave is

$$\mathbf{E}_L=(E_0\sin(kz-\omega t),0,0)\ . \tag{20}$$

In this case we take $v_y=0$ and

$$v_x=c\frac{K}{\gamma}\cos(\kappa z)+c\frac{K_L}{\gamma}\cos(kz-\omega t)\ . \tag{21}$$

The longitudinal velocity v_z has fast oscillating components

$$v_z=c\left[1-\frac{2+K^2+K_L^2+2KK_L(\cos\psi_++\cos\psi_-)}{4\gamma^2}-\frac{K^2}{4\gamma^2}\cos(2\kappa z)-\frac{K_L^2}{4\gamma^2}\cos[2(kz-\omega t)]\right]. \tag{22}$$

The energy transferred between the laser beam and electron per unit length of the accelerator is

$$\left[\frac{d\gamma}{dz}\right]_T = \frac{e}{mc^3} v_x E_x = \frac{1}{2} A \left[\frac{K}{\gamma}(\sin\psi_+ + \sin\psi_-) \frac{K_L}{\gamma} \sin[2(kz-\omega t)]\right] , \tag{23}$$

where $\psi_\pm = z(k \pm \kappa) - \omega t$. The terms at twice the EM wave frequency in (22) and (23) have a very small effect on the electron dynamics and will be neglected in what follows.

Using Eq. (22) one can express the time t as a function of the distance z,

$$ct = ct_0 + z + \int_0^z dz \frac{2+K^2+K_L^2+2KK_L(\cos\psi_+ + \cos\psi_-)}{4\gamma^2} + \frac{1}{8}\frac{K^2}{\gamma^2\kappa}\sin(2\kappa z) . \tag{24}$$

The last term is an approximation for $\frac{1}{4}\int_0^z dz (K^2/\gamma^2)\cos(2\kappa z)$. This approximation is valid as changes of all parameters of the system over one wiggler period are small. Using the expansion

$$\sin(a + b\sin\phi) = \sum_{n=-\infty}^{\infty} J_n(b) \sin(a+n\phi) , \tag{25}$$

where J_n are Bessel functions, we get

$$\sin\psi_+ + \sin\psi_- = \sum_{n=-\infty}^{\infty} [J_n(G) + J_{n-1}(G)] \sin\left[\kappa z(1-2n) - k\int_0^z dz \frac{2+K^2+K_L^2+KK_L\cos\psi_+ + \cos\psi_-}{4\gamma^2} - \phi_0\right] , \tag{26}$$

where $G = kK^2/8\kappa\gamma^2$.

The accelerator can be designed in such a way that only one term of all those appearing in Eq. (26) is important because of its slow variation along the accelerator. We consider the case in which the $n=0$ term is relevant.

Including also the synchrotron loss term we can finally write the accelerator equations

$$\frac{d\gamma}{dz} = \frac{1}{2} A \frac{K}{\gamma}[J_0(G) - J_1(G)]\sin\psi - \frac{1}{3} r_e \kappa^2 \gamma^2 \{K^2 + K_L^2 + 2KK_L[J_0(G) + J_1(G)]\cos\psi\} , \tag{27}$$

$$\frac{d\psi}{dz} = \kappa - k \frac{1 + K^2/2 + K_L^2/2 + KK_L[J_0(G) - J_1(G)]\cos\psi}{2\gamma^2} . \tag{28}$$

These equations are the same as Eqs. (17) and (18) derived for a helical wiggler, except for the factor $J_0 \pm J_1$.[20] The dynamical variables for the electron are its energy γ and phase ψ. The wiggler is described by two functions: the wiggler parameter K and the wiggler period Λ_w (or $\kappa = 2\pi/\Lambda_w$). For a given wiggler both parameters are slowly varying functions of z. These functions should be specified to get an efficient acceleration.

The laser beam is described by the parameter A proportional to the field strength. In our present analysis we do not take into account the attenuation of the laser beam due to the absorption by accelerating electrons. Therefore the parameter A is a constant and not another dynamical variable as it should be in a more exact theory. Our assumption is justified for low-density electron beams and low-loss systems transporting the laser beam.

III. ACCELERATION OF RESONANT ELECTRONS IN DIFFERENT ACCELERATORS

An arbitrary choice of the two functions $K(z)$ and $\kappa(z)$ very probably will not produce much increase of the electron energy. Most probably the phase ψ will vary over a large range and the acceleration term will change signs causing cancellations of the energy exchange. To achieve a continuous acceleration we must restrict the phase variation so that the acceleration term is always positive $(0 < \psi < \pi)$.

It is customary in accelerator design to introduce a reference particle for which the phase ψ stays constant; this is also called the resonant or synchronous particle. The corresponding phase ψ_r, the resonant phase, is an important parameter of the accelerator. The rate of acceleration is largest when $\psi_r = \pi/2$, but to obtain a stable acceleration for nonresonant particles one is forced to make a choice of resonant phases giving a smaller acceleration rate.

If the phase ψ must stay constant, then Eqs. (18) or (28) imply some relation between the wiggler parameter K, the wiggler period Λ_w, and the electron energy γ_r. For the helical wiggler case the resonant condition is

$$\kappa = k\frac{1+K^2+K_L^2+2KK_L\cos\psi_r}{2\gamma_r^2} \simeq k\frac{K^2}{2\gamma_r^2} . \tag{29}$$

We can often neglect 1 and K_L as compared with K since most interesting accelerators have $K \gg 1$, $K \gg K_L$. This approximation is valid when studying the motion of the resonant particle and will be used throughout this section.

This condition removes the freedom of two arbitrary functions $K(z)$ and $\kappa(z)$ which appear in the accelerator equations. After it is imposed, only one function, $K(z)$ or $\kappa(z)$, or some combination of them, is free. We may thus write the equations for the resonant acceleration in different representations, choosing different functions as the arbitrary one. In what follows we will write explicitly the resonant accelerator equation for the cases in which (i) the wiggler period Λ_w, or (ii) the wiggler strength B_w, or (iii) the wiggler parameter $K \propto B_w\Lambda_w$ is chosen as an arbitrary function.

We obtain for these three cases:

$$\text{(i)}\ \frac{d\gamma_r}{dz}=\left[\frac{2\lambda}{\Lambda_w(z)}\right]^{1/2}\widetilde{A}-\frac{16\pi^2}{3}r_e\frac{\lambda}{\Lambda_w^3(z)}\gamma_r^4\ , \tag{30}$$

$$\text{(ii)}\ \frac{d\gamma_r}{dz}=\left[\frac{2\Omega_w(z)}{\omega}\right]^{1/3}\frac{\widetilde{A}}{\gamma_r^{1/3}}-\tfrac{2}{3}r_e\frac{\Omega_w^2}{c^2}\gamma_r^2\ , \tag{31}$$

$$\text{(iii)}\ \frac{d\gamma_r}{dz}=\widetilde{A}\frac{K(z)}{\gamma_r}-\frac{2\pi^2}{3}r_e\frac{K^6(z)}{\lambda^2\gamma_r^2}\ , \tag{32}$$

where $\widetilde{A}=A\sin\psi_r$, and $\Omega_w=eB_w/mc$.

All these three equations are completely equivalent and can describe the same physical situations. The forms (i), (ii), and (iii) are particularly convenient for accelerators which keep (i) the wiggler period Λ_w, (ii) the wiggler strength B_w, or (iii) the wiggler parameter K as constant.

In the constant-period and constant-strength accelerators the loss terms are growing with energy. As a result, there is a maximum electron energy for any given laser power, independent of the accelerator length. This is

$$\gamma_\infty^\Lambda=\left[\frac{3\widetilde{A}}{2^{7/2}\pi^2\lambda^{1/2}r_e}\right]^{1/4}\Lambda_w^{5/8},\ \ \Lambda_w=\text{const} \tag{33}$$

$$\gamma_\infty^B=\left[\frac{3\widetilde{A}c^2}{2^{2/3}\omega^{1/3}r_e}\right]^{3/7}\frac{1}{\Omega_w^{5/7}},\ \ B_w=\text{const}\ . \tag{34}$$

The equations for the corresponding accelerators can be written as

$$\frac{d\gamma_r}{dz}=\alpha_\Lambda\left[1-\left[\frac{\gamma_r}{\gamma_\infty^\Lambda}\right]^4\right],\ \ \Lambda_w=\text{const} \tag{35}$$

$$\frac{d\gamma_r}{dz}=\frac{\alpha_B}{\gamma_r^{1/3}}\left[1-\left[\frac{\gamma_r}{\gamma_\infty^B}\right]^{7/3}\right],\ \ B_w=\text{const} \tag{36}$$

where

$$\alpha_\Lambda=\left[\frac{2\lambda}{\Lambda_w}\right]^{1/2}\widetilde{A}\ , \tag{37}$$

$$\alpha_B=\left[\frac{2\Omega_w}{\omega}\right]^{1/3}\widetilde{A}\ . \tag{38}$$

Both equations can be integrated analytically giving $z=f(\gamma_r)$, the accelerator length as a function of the electron energy.

For constant Λ_w accelerator we have

$$z=\frac{\gamma_\infty^\Lambda}{\alpha_\Lambda}\left[G_\Lambda\left[\frac{\gamma_r}{\gamma_\infty^\Lambda}\right]-G_\Lambda\left[\frac{\gamma_r(0)}{\gamma_\infty^\Lambda}\right]\right]\ , \tag{39}$$

where

$$G_A(x)=\int_0^x\frac{dx}{1-x^4}=\frac{1}{4}\ln\frac{1+x}{1-x}+\frac{1}{2}\arctan x,\ \ 0<x<1\ . \tag{40}$$

In the limiting cases $x\to 0$, or $x\to 1$, $G_\Lambda(x)$ is given by

$$\begin{aligned}&G_\Lambda(x)\simeq x\ \ \text{for } 0<x\ll 1\ ,\\ &G_\Lambda(x)\simeq-\tfrac{1}{4}\ln(1-x)\ \ \text{for } 0<1-x\ll 1\ .\end{aligned} \tag{41}$$

We have therefore

$$\gamma_r=\gamma_r(0)\alpha_\Lambda z\ \ \text{for } \gamma_r\ll\gamma_\infty^\Lambda\ , \tag{42a}$$

$$\gamma_r=\gamma_\infty-[\gamma_\infty-\gamma_r(z_0)]\exp\left[-\frac{4\alpha_\Lambda}{\gamma_\infty^\Lambda}(z-z_0)\right]\ \ \text{for } \gamma_\infty^\Lambda-\gamma_r(z_0)\ll\gamma_\infty^\Lambda\ . \tag{42b}$$

For the case $B_w=\text{const}$, we obtain

$$z=\frac{3}{\alpha_B}(\gamma_\infty^B)^{4/3}\left\{G_B\left[\left[\frac{\gamma_r}{\gamma_\infty^B}\right]^{1/3}\right]-G_B\left[\left[\frac{\gamma_r(0)}{\gamma_\infty^B}\right]^{1/3}\right]\right\}\ , \tag{43}$$

where

$$G_B(x)=\int_0^x dx\frac{x^3}{1-x^7}\ . \tag{44}$$

This integral is given explicitly in Ref. 21. In the limiting cases we have

$$\begin{aligned}&G_B(x)\simeq\tfrac{1}{4}x^4,\ \ 0<x\ll 1\\ &G_B(x)\simeq-\tfrac{1}{7}\ln(1-x),\ \ 0<1-x\ll 1\ .\end{aligned} \tag{45}$$

The corresponding energy increases are

$$\gamma_r\simeq\{[\gamma_r(0)]^{4/3}+\tfrac{4}{3}\alpha_B z\}^{3/4},\ \ \gamma\ll\gamma^B \tag{46a}$$

$$\gamma_r=\gamma_\infty^B\left\{1-3\left[1-\left[\frac{\gamma_r(z_0)}{\gamma_\infty}\right]^{1/3}\right]\times\exp\left[-\frac{7}{3}\frac{\alpha_B}{\gamma_\infty^B}(z-z_0)\right]\right\}\ ,\quad \gamma_\infty^B-\gamma_r\ll\gamma_\infty^B\ . \tag{46b}$$

The constant K accelerator has the property that the radiation losses are decreasing with energy faster than the acceleration rate. Therefore, in principle, the electron acceleration can be unlimited. Also for this accelerator we can give analytically the length as a function of energy

$$z=\frac{\delta^2}{\alpha_K}\left[G_K\left[\frac{\gamma_r}{\delta}\right]-G_K\left[\frac{\gamma_r(0)}{\delta}\right]\right]\ , \tag{47}$$

where

$$\alpha_K=\widetilde{A}K,\ \ \delta=\tfrac{2}{3}\pi^2 r_e K^5/(\lambda^2\widetilde{A})\ ,$$

and

$$G_K(x)=\tfrac{1}{2}x^2+x+\ln(x-1),\ \ 1<x\ . \tag{48}$$

In the high-energy limit the increase of energy is proportional to the square root of the accelerator length.

Considering the losses due to the synchrotron radiation the rate of acceleration can be maximized with a proper dependence of the wiggler parameters on the electron energy. Taking, for example, Eq. (30), one can notice that the right-hand side (rhs) of this equation, and the rate of acceleration, is maximum when

$$\Lambda_w = \left[\frac{32\pi^2\sqrt{\lambda} r_e}{\sqrt{2}\tilde{A}} \right]^{2/5} \gamma_r^{8/5} . \tag{49}$$

The corresponding maximum-rate accelerator is described by the equation

$$\frac{d\gamma_r}{dz} = \frac{3^{6/5}}{6} \frac{\tilde{A}^{6/5}}{r_e^{1/5} k^{2/5}} \frac{1}{\gamma_r^{4/5}} \tag{50}$$

with the solution

$$\gamma_r = \left\{ [\gamma_r(0)]^{9/5} + \frac{3^{11/5}}{10} \frac{\tilde{A}^{6/5} z}{r_e^{1/5} k^{2/5}} \right\}^{5/9} . \tag{51}$$

The properties of the accelerators we have discussed are illustrated in Figs. 2–4. In these examples the strength of the laser electric field is $E_0 = 5\times10^6$ esu$=1.5\times10^{11}$ V/m, the resonant accelerator phase is $\psi_r = 120°$, and the initial energy of the injected electrons $E(0) = 10$ GeV.

Looking at the constant-period and constant-magnetic-field-strength accelerators we can observe two competing tendencies. To increase the initial rate of acceleration one should decrease the period of the wiggler or increase its strength. Then, however, the acceleration rate saturates faster and at much lower energies.

The maximum-rate accelerator optimizes both tendencies. Unfortunately, for low electron energies ($E < 50$ GeV) the required magnetic field would be too high and the wiggler period too small to consider. For higher energies ($E > 50$ GeV) the parameters of the "optimum-rate" accelerator become technically feasible. The scaling properties of the parameters for different accelerators are given in Table I.

Our discussion of different accelerators concerned up to now helical wigglers. In cases of planar wigglers, one can see from (27) that the equations contain an extra factor,

$$F = \frac{1}{2} \left[J_0 \left[\frac{kK^2}{8\kappa\gamma^2} \right] - J_1 \left[\frac{kK^2}{8\kappa\gamma^2} \right] \right] . \tag{52}$$

For a resonant electron the argument in (52) is

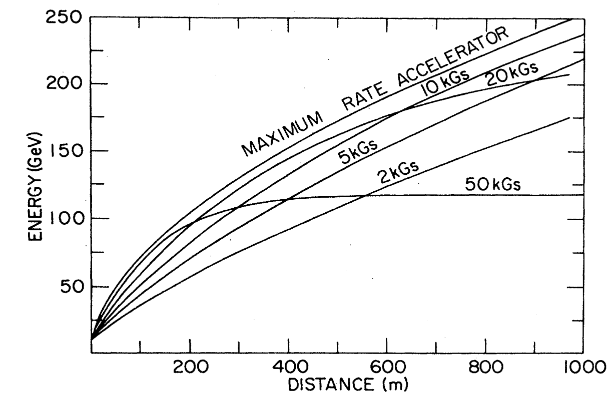

FIG. 3. Energy vs distance for a constant B_w IFEL accelerator.

$$\frac{1}{8} \frac{kK^2}{\kappa\gamma_r^2} = \frac{K^2}{2(2+K^2)} \tag{53}$$

and for large values of K the factor F tends to a constant,

$$F_0 = \tfrac{1}{2}[J_0(\tfrac{1}{2}) - J_1(\tfrac{1}{2})] = 0.35 .$$

This allows the reduction of the planar wiggler cases to the helical ones. Two other modifications are needed: the change of the resonant condition,

$$\kappa = k \frac{1+K^2/2}{2\gamma_r^2} , \tag{54}$$

and the reduction of the loss term by half. These can be effectively done by the redefinition of the wiggler parameter $K^* = K/\sqrt{2}$. With the introduction of the effective laser field strength $E_L^* = E_L\sqrt{2}\times F_0 = 0.495 E_L$, the equation for the planar wiggler case takes the identical form as the helical ones.

IV. STABILITY OF ACCELERATION

The growth of the resonant energy does not completely characterize the accelerator. In addition to the final energy of the electrons other characteristics of the accelerator

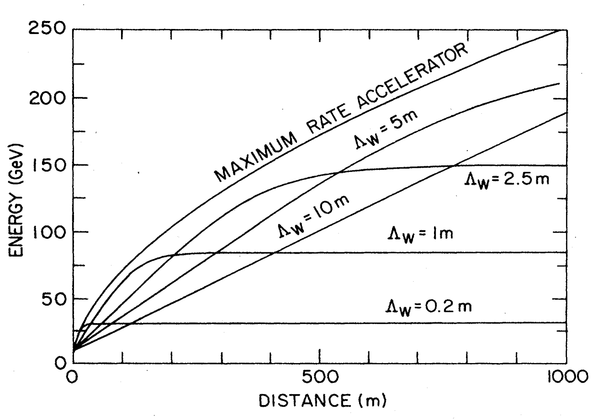

FIG. 2. Electron energy vs distance for a constant-period IFEL accelerator $E = 1.5\times10^{11}$ V/m. $\psi_r = 120°$.

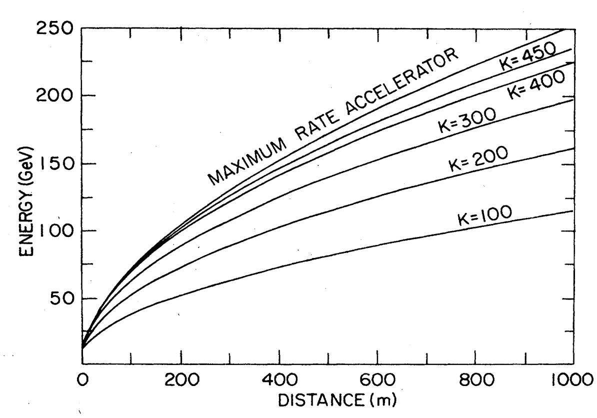

FIG. 4. Energy vs distance for a constant K IFEL accelerator.

TABLE I. Energy dependence of selected parameters in different accelerators.

Parameters	Accelerator types $\Lambda_w=\text{const}$	$B_w=\text{const}$	$K=\text{const}$	Maximum rate
Wiggler period, Λ_w	γ^0	$\gamma^{2/3}$	γ^2	$\gamma^{8/5}$
Magnetic field, B_w	γ	γ^0	γ^{-2}	$\gamma^{-7/5}$
Laser acceleration, $\frac{K}{\gamma}$	γ^0	$\gamma^{-1/3}$	γ^{-1}	$\gamma^{-4/5}$
Radiative losses	γ^4	γ^2	γ^{-2}	$\gamma^{-4/5}$
Amplitude of electron oscillations	γ^0	$\gamma^{1/3}$	γ	$\gamma^{4/5}$

may include the energy spread, initial energy acceptance, the fraction of electrons which are accelerated to those which are injected, and the emittance. Many of these properties can be investigated together with the stability of the acceleration process. We restrict our discussion to the stability of the longitudinal motion, directly related to the acceleration mechanism. Our main interest is to study what happens to those electrons which deviate from the resonant parameters.

Electrons can deviate from resonant conditions either because their energy is different or because their phase ψ is not equal to the resonant phase ψ_r. Such deviations are inevitable since electrons are injected without any control on their phases (on an optical wavelength scale) and all beams have some energy spread.

In doing the stability analysis one must go back to the pair of equations describing the evolution of electrons, either Eqs. (17) and (18) or (27) and (28). Using the equations for a helical wiggler one can get the following system of equations for the energy and phase deviations:

$$\delta\gamma=\gamma-\gamma_r,\quad \phi=\psi-\psi_r\ , \tag{55}$$

$$\frac{d\phi}{dz}=\frac{2\pi}{\Lambda_w}\left[1-\frac{1+K^2+K_L^2+2KK_L\cos(\phi+\psi_r)}{K_T^2(1+\delta\gamma/\gamma_r)^2}\right], \tag{56}$$

$$\frac{d\delta\gamma}{dz}=\widetilde{A}\frac{K}{\gamma_r}\left[\frac{\sin(\phi+\psi_r)}{\sin\psi_r}\frac{1}{1+\delta\gamma/\gamma_r}-1+\frac{\mu K^5}{6\gamma_r}\left[1-(1+\delta\gamma/\gamma_r)^2\frac{K^2+K_L^2+2KK_L\cos(\phi+\psi_r)}{K_T^2-1}\right]\right], \tag{57}$$

with the resonant energy defined by

$$\frac{d\gamma_r}{dz}=\widetilde{A}\frac{K}{\gamma_r}\left[1-\tfrac{1}{6}\mu\frac{K^5}{\gamma_r}\right], \tag{58}$$

and

$$\mu=r_ek^2/\widetilde{A},\quad K_T^2=1+K^2+K_L^2+2KK_L\cos\psi_r\ .$$

In these equations $\delta\gamma$ and ϕ appears in a nonlinear fashion. It is important not to neglect the terms $2KK_L\cos(\phi+\psi_r)$ in the rhs of (56) and (57) since they contribute to determine the evolution of $\delta\gamma$ and ϕ.

In this form the system is general. Particular examples of accelerators we have considered can be obtained specifying the dependence of parameter K on the resonant energy. These are given in Table II.

In this system the electron position along the accelerator z is an independent variable. The resonant energy γ_r, which in the first two equations appears as a parametric function, is completely determined by (58) and is uncoupled from the others. Because of this structure and also because in interesting cases K is given as a function of γ_r, it is convenient to choose the resonant energy γ_r as the independent variable. There is one-to-one correspondence between γ_r and z as the energy grows monotonically with distance.

Applying this change of variables we get a system of two equations,

$$\frac{d\phi}{d\gamma_r}=\frac{kK_T^2/K}{2\widetilde{A}\gamma_r\left[1-\frac{1}{6}\mu\frac{K^5}{\gamma_r}\right]}\left[1-\frac{1+K^2+K_L^2+2KK_L\cos(\phi+\psi_r)}{K_T^2(1+\delta\gamma/\gamma_r)^2}\right], \tag{59}$$

$$\frac{d\delta\gamma}{d\gamma_r}=\frac{1}{1-\frac{1}{6}\mu\frac{K^5}{\gamma_r}}\left[\frac{\sin(\phi+\psi_r)}{(1+\delta\gamma/\gamma_r)\sin\psi_r}-1+\frac{\mu K^5}{6\gamma_r}\left[1-(1+\delta\gamma/\gamma_r)^2\frac{K^2+K_L^2+2KK_L\cos(\phi+\psi_r)}{K_T^2-1}\right]\right]. \tag{60}$$

TABLE II. Energy dependence of wiggler parameter K in different accelerators.

Accelerator type	
Constant K	$K=K_0$
Constant Λ_w	$K=\left[\frac{2\lambda}{\Lambda_w}\right]^{1/2}\gamma_r$
Constant B_w	$K=\left[\frac{2\Omega_w}{\omega}\right]^{1/3}\gamma_r^{2/3}$
Maximal	$K=\gamma_r^{1/5}/\mu^{1/5}$

The right-hand side of Eqs. (59) and (60) depends on ϕ and $\delta\gamma$. The system then appears as a nonconservative system and the volume of the phase-space span by $\delta\gamma$ and ϕ is not conserved. In fact the phase-space area is decreasing in the evolution. That is a consequence of the fact that the divergence of the velocity vector field $(d\phi/d\gamma_r, d\delta\gamma/d\gamma_r)$ is negative

$$\frac{\partial}{\partial\phi}\dot{\phi}+\frac{\partial}{\partial\delta\gamma}\dot{\delta\gamma}=-\frac{1}{\gamma_r}\frac{\frac{1}{3}\mu K^5}{\gamma_r-\frac{1}{6}\mu K^5}<0\ . \tag{61}$$

The dissipative features of this set are perhaps of no surprise as we are taking into account the radiative damping. It is worth stressing, however, that the system is not dissipative in the absence of radiation losses. In fact putting $\mu=0$, corresponding to zero radiation losses, will make the divergence (61), equal to zero.

We can find a conservative and Hamiltonian system which is equivalent to Eqs. (59) and (60) by a transformation of variables $u=a(\gamma_r)\delta\gamma$, $v=b(\gamma_r)\phi$. The functions $a(\gamma_r), b(\gamma_r)$ can be chosen in such a way that the pair of variables (u,v) will conserve the associated phase-space area and the evolution can be described by a certain Hamiltonian. The simplest illustration of this procedure can be given when the radiation losses are neglected.

No radiation losses. In that case Eqs. (59) and (60) reduce to

$$\frac{d\phi}{d\gamma_r}=\frac{kK_T^2}{2\tilde{A}\gamma_r K}\left[1-\frac{1+K^2+K_L^2+2KK_L\cos(\phi+\psi_r)}{(1+\delta\gamma/\gamma_r)^2K_T^2}\right], \tag{62}$$

$$\frac{d\delta\gamma}{d\gamma_r}=\frac{\sin(\phi+\psi_r)}{(1+\delta\gamma/\gamma_r)\sin\psi_r}-1\ . \tag{63}$$

These can be obtained from the Hamiltonian

$$H=\frac{kK_T^2}{2\tilde{A}K}\left[-1+\frac{\delta\gamma}{\gamma_r}+\frac{1+K^2+K_L^2+2KK_L\cos(\phi+\psi_r)}{(1+\delta\gamma/\gamma_r)K_T^2}\right]+\phi\ . \tag{64}$$

Equations (62) and (63) can be linearized and written as

$$\frac{d\phi}{d\gamma_r}=\frac{k}{\tilde{A}}\frac{K_T^2}{K}\frac{\delta\gamma}{\gamma_r^2}+\frac{\phi}{\gamma_r}\ , \tag{65}$$

$$\frac{d\delta\gamma}{d\gamma_r}=(\cot\psi_r)\phi-\frac{\delta\gamma}{\gamma_r}\ . \tag{66}$$

This is a phase-space area conserved system when ϕ and $\delta\gamma$ are considered to be the phase variables.

Expanding (64) up to second order in $\delta\gamma$ we get

$$H=H(\phi,\delta\gamma,\gamma_r)=\frac{1}{2M}(\delta\gamma)^2+V(\phi,\delta\gamma,\gamma_r)\ , \tag{67}$$

where the mass M is

$$M=\frac{\tilde{A}}{k}\frac{\gamma_r^2K}{K_T^2}\sim\frac{\tilde{A}\gamma_r^2}{kK} \tag{68}$$

and the potential

$$V(\phi,\gamma_r)=\frac{1}{\sin\psi_r}\{[\cos(\phi+\psi_r)-\cos\psi_r]\times(1-\delta\gamma/\gamma_r)+\phi\sin\psi_r\} \tag{69}$$

is a well-known potential binding the electrons in an accelerating bucket.[15] The potential is shown in Fig. 5, neglecting $\delta\gamma/\gamma$ with respect to 1.

We can easily explain some features of our system evolution by considering the case when the "mass" and potential "height" is independent of the energy. The electron motion takes one of two forms. One corresponds to an unbound and unlimited motion outside the potential wells, the second to a bounded and periodic oscillation inside the potential wells (Fig. 6). Only the second case is of interest from the point of view of accelerators. Only these oscillating electrons are accelerated to high energy. In this approximation these two motions are completely separated and particles can switch from one to the other only by an external interaction (e.g., by a collision with an atom).

When one takes into account the change of the mass and "potential" with growing γ_r, then the evolution is more complicated. One can use the adiabatic invariant[22]

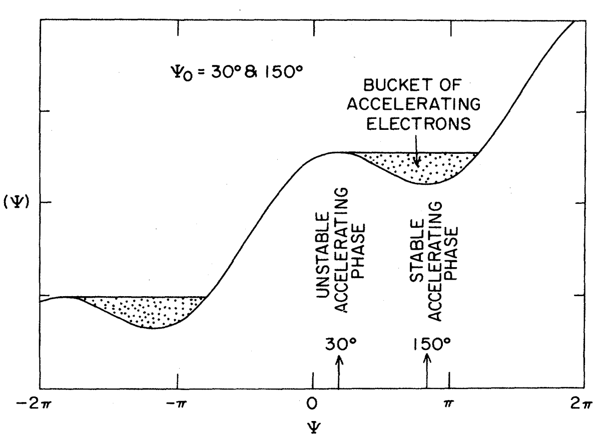

FIG. 5. Binding potential $V(\phi)$ for accelerating electrons.

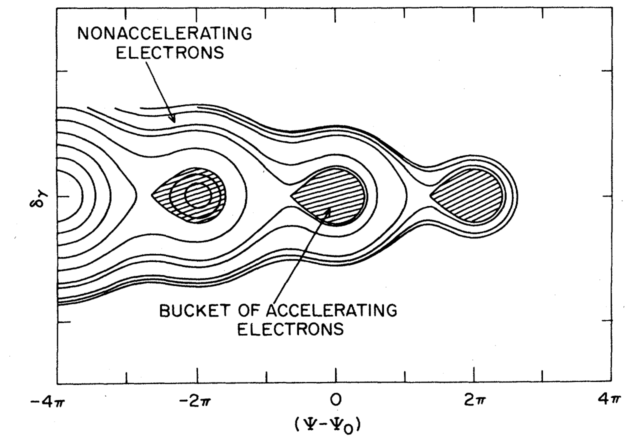

FIG. 6. Phase diagram in Hamiltonian approximation with constant parameters.

$$J=\oint u\,d\phi=\oint (2M)^{1/2}[H-V(\phi)]d\phi\ , \tag{70}$$

to determine what happens to the oscillation amplitude as γ_r grows.

Near the top of potential wells the evolution is complicated as the period of oscillation becomes longer and the change of resonant energy in one oscillation period can be large so that the adiabatic invariance is no longer preserved. In this case the phase space does not split into completely separated acceleration buckets and unbounded trajectories.

Restricting further to small oscillations near the bottom of the potential well described by (65) and (66), we can use the variables

$$u=\gamma_r\delta\gamma\ , \tag{71}$$

$$v=\phi/\gamma_r\ , \tag{72}$$

to rewrite the Hamiltonian as

$$H=\frac{u^2}{2M\gamma_r^2}-\tfrac{1}{2}(\cot\psi_r)\gamma_r^2 v^2\ . \tag{73}$$

For bounded oscillations the potential must be positive having its minimum at $\phi=0$. This requires that $\cot\psi_r<0$ or $\pi/2<\psi_r<\pi$.

Calculating the adiabatic invariant (70) we can establish that

$$(M)^{1/2}H\propto\frac{\gamma_r}{K^{1/2}}H=\text{const}=C_0\ , \tag{74}$$

$$C_0=\frac{1}{2}\frac{k}{\widetilde{A}}\frac{K^{1/2}}{\gamma_r^3}u^2-\tfrac{1}{2}(\cot\psi_r)\frac{\gamma_r^3}{K^{1/2}}v^2\ . \tag{75}$$

For C_0 to be constant the amplitudes of v and u oscillations v_a and u_a must satisfy

$$v_a=\text{const}\frac{K^{1/4}}{\gamma_r^{3/2}}\ , \tag{76}$$

$$u_a=\text{const}\frac{\gamma_r^{3/2}}{K^{1/4}}\ . \tag{77}$$

TABLE III. Phase and energy oscillation amplitude vs resonant energy, when radiative losses are neglected.

Accelerator type	$\phi_a(\gamma_r)$	$\delta\gamma_a(\gamma_r)$	$\gamma_r(z)$
Constant Λ_w	$\gamma_r^{-1/4}$	$\gamma_r^{+1/4}$	z
Constant B_w	$\gamma_r^{-1/3}$	$\gamma_r^{+1/3}$	$z^{3/4}$
Constant K	$\gamma_r^{-1/2}$	$\gamma_r^{1/2}$	$z^{1/2}$

The product of $u_a\times v_a$ is constant and any decay of oscillations in v brings amplifications of oscillations in u. Returning to the energy deviation $\delta\gamma$ and ϕ it would change as

$$\delta\gamma_a=\gamma_r^{1/2}/K^{1/4},\ \ \phi_a=K^{1/4}/\gamma_r^{1/2}\ . \tag{78}$$

This equation indicates that for all laser accelerators in which K is increasing with the energy, the energy spread is decreasing. Table III gives the scaling of the ϕ and $\delta\gamma$ oscillation amplitude in various accelerators.

A more detailed description of these oscillations can be obtained by solving Eqs. (65) and (66). That can be done either numerically or analytically, using the WKB method. This method gives

$$\phi=A_0\frac{K^{1/4}}{\gamma_r^{1/2}}\cos\left[[-(k/\widetilde{A})\cot\psi_r]^{1/2}\int_{\gamma_r(0)}^{\gamma_r}d\gamma\frac{K^{1/2}}{\gamma}+f_0\right]. \tag{79}$$

A_0 and f_0 are the constants of integration which must be determined from the initial data. Notice that the amplitude of these oscillations has the same behavior as established before using the adiabatic invariant.

Now we will generalize this procedure to describe also accelerators in which one cannot neglect radiative losses. It is interesting that for those situations we can still obtain a Hamiltonian description and effectively use the adiabatic invariants.

Hamiltonian description with radiative losses. The origin of the dissipation in the system described by Eqs. (59) and (60) is the presence of terms proportional to μ in their rhs. The transformation

$$u=a(\gamma_r)\delta\gamma,\ \ v=b(\gamma_r)\phi\ , \tag{80}$$

where

$$a(\gamma_r)=\exp\left[\int^{\gamma_r}\frac{d\gamma}{\gamma}\frac{\gamma+\frac{1}{3}\mu K^5}{\gamma-\frac{1}{6}\mu K^5}\right], \tag{81}$$

$$b(\gamma_r)=\exp\left[-\int^{\gamma_r}\frac{d\gamma}{\gamma-\frac{1}{6}\mu K^5}\right], \tag{82}$$

allows us to rewrite Eqs. (59) and (60) in the form

$$\frac{dv}{d\gamma_r}=f(\gamma_r)u\ , \tag{83}$$

$$\frac{du}{d\gamma_r}=-g(\gamma_r)v\ . \tag{84}$$

TABLE IV. Transformation function $a(\gamma_r)$ to obtain accelerator equations in Hamiltonian form.

Accelerator type	$a(\gamma_r)$	$b(\gamma_r)$
Constant K $(\gamma_r > \delta)$	$\frac{1}{\gamma_r^2}(\gamma_r - \delta)^3$	$1/(\gamma_r - \delta)$
Constant Λ_w $(\gamma_r < \gamma_\infty^\Lambda)$	$\gamma_r \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^\Lambda}\right]^4\right]^{-3/4}$	$\gamma_r^{-1} \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^\Lambda}\right]^4\right]^{1/4}$
Constant B_w $(\gamma_r < \gamma_\infty^B)$	$\gamma_r \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^B}\right]^{7/3}\right]^{-9/7}$	$\gamma_r^{-1} \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^B}\right]^{7/3}\right]^{3/7}$
Maximum rate	$\gamma_r^{8/5}$	$\gamma_r^{-6/5}$

In this form the linearized equations can be derived from the Hamiltonian

$$H = \tfrac{1}{2} f(\gamma_r) u^2 + \tfrac{1}{2} g(\gamma_r) v^2 , \tag{85}$$

where

$$f(\gamma_r) = \frac{\gamma_r}{M(\gamma_r - \frac{1}{6}\mu K^5)} \frac{b(\gamma_r)}{a(\gamma_r)} , \tag{86}$$

$$g(\gamma_r) = -\frac{\gamma_r \cot\psi_r + \frac{1}{3}\mu K^4 K_L \sin\psi_r}{\gamma_r - \frac{1}{6}\mu K^5} \frac{a(\gamma_r)}{b(\gamma_r)} . \tag{87}$$

The functions $a(\gamma_r)$ and $b(\gamma_r)$, given by Eqs. (81) and (82), can be explicitly calculated for all the accelerators we have been considering and are given in Table IV.

For the definitions of γ_∞^Λ, γ_∞^B, and δ, see Eqs. (33), (34), and (47) of Sec. III. Putting $\gamma_\infty^{\Lambda,B} = \infty$ or $\delta = 0$ we again get the equations for the no-radiative-losses case which we considered before.

In the general case, for small oscillations near the potential minima, we can find as before the adiabatic invariant

$$\frac{f^2 + g v^2}{(fg)^{1/2}} = \text{const} . \tag{88}$$

The oscillation amplitudes satisfy the scaling law

$$\phi_a = \text{const} \frac{K^{1/4}}{\gamma_r^{1/2} a^{1/2}(\gamma_r) b^{1/2}(\gamma_r)} , \tag{89}$$

$$\delta\gamma = \text{const} \frac{\gamma_r^{1/2}}{K^{1/4} a^{1/2}(\gamma_r) b^{1/2}(\gamma_r)} , \tag{90}$$

and for the product $\phi(\delta\gamma)$ we have

$$\phi_a \delta\gamma_a = \frac{\text{const}}{a(\gamma_r) b(\gamma_r)} . \tag{91}$$

Table V gives the explicit behavior of ϕ_a and $\delta\gamma_a$ as a function of energy in various accelerators.

We can also establish the dependence of these amplitudes on z, the distance along the accelerator. Table III gives this dependence for the case when the radiative losses can be neglected. When the radiative losses are large, near the saturation energy in constant-period and constant-magnetic-strength accelerators, the asymptotic dependence of the energy on z is given by Eqs. (42b) and (46b). Using these expressions we get in both accelerators

$$\phi_a \propto \delta\gamma_a \propto e^{-(\alpha/\gamma_\infty)z} . \tag{92}$$

For the maximum-rate accelerator we get

$$\phi_a \propto z^{-29/36}, \quad \delta\gamma_a \propto z^{-11/36} . \tag{93}$$

V. CONCLUSIONS

We discussed some features of the inverse free-electron accelerator. The most important is the high rate of energy increase. With a power density of the laser light of 3×10^{15} W/cm^2, the rate of energy increase, for low ener-

TABLE V. Phase and energy oscillation amplitude vs resonant energy, with radiative losses included.

Accelerator type	ϕ_a	$\delta\gamma_a$
Constant K $(\gamma_r > \delta)$	$\frac{\gamma_r^{1/2}}{(\gamma - \delta)}$	$\frac{\gamma_r^{3/2}}{(\gamma_r - \delta)}$
Constant Λ_w $(\gamma_r < \gamma_\infty^\Lambda)$	$\frac{1}{\gamma_r^{1/4}} \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^\Lambda}\right]^4\right]^{1/4}$	$\gamma_r^{1/4} \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^\Lambda}\right]^4\right]^{1/4}$
Constant B_w $(\gamma_r < \gamma_\infty^B)$	$\gamma_r^{1/3} \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^B}\right]^{7/3}\right]^{9/14}$	$\frac{1}{\gamma_r^{1/3}} \left[1 - \left[\frac{\gamma_r}{\gamma_\infty^B}\right]^{7/3}\right]^{3/7}$
Maximum rate	$\frac{1}{\gamma_r^{13/20}}$	$\frac{1}{\gamma_r^{1/4}}$

gies, is 400 MeV/m. For higher energies ($E > 100$ GeV) it is reduced due to the synchrotron losses, but still the energy increase in 1 km can exceed 200 GeV, giving the average acceleration rate of 200 MeV/m.

An interesting property of this accelerator is that it does not distinguish between electrons and positrons. The accelerator equations depend on e^2. Therefore the acceleration of electrons and positrons occurs at the same resonant phase. This can allow accelerating overlapping electron and positron bunches, also avoiding space-charge effects.

During the acceleration the phase-space volume measured in the energy and phase variables δ_γ, ϕ is decreasing, due to the radiation losses. In consequence, the IFEL accelerator should offer a smaller phase-space area than linear accelerators, where loses are negligible.

To provide the stability, the resonant phase must be chosen to give $\sin\psi_r < 1$. As electrons gain energy the phase spread of electron bunches shrinks. This allows changing the resonant phase to values giving a higher rate of acceleration. Therefore optimizing the structure of the accelerator one should consider accelerators with a slowly varying resonant phase. Such accelerators should give not only higher final energies but also increase the acceptance efficiency. The energy spread of the beam could be further reduced.

In our discussion we have used some simplifying assumptions. Although we proposed to use a low-loss waveguide to propagate the laser beam, our analysis was done for the case of a plane wave. The presence of the waveguide would bring some new features to the accelerator. One is the spacial distribution of the field intensity in the transverse direction. The second is a modification of the phase velocity of the waveguide wave. That modification is very small if the cross section of the waveguide is of the order of centimeters and the wavelength of radiation of the order of microns.

To complete the accelerator design, the transverse stability of the motion must be studied. One stabilizing force is due to the longitudinal magnetic field B_z present in the undulator, which produces a transverse focusing force.[10] For better transverse stabilization, focusing systems similar to those used in other accelerators could be needed.

The effect of quantum fluctuations, and how they balance with the radiation damping studied here, has also to be studied to have a full description of the final beam emittance and energy spread.

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