Phenomenological theory of the 2^3P state of ^{3}He

E. A. Hinds and J. D. Prestage
J. W. Gibbs Laboratory, Yale University, New Haven, Connecticut 06520-2159

F. M. J. Pichanick
University of Massachusetts, Amherst, Massachusetts 01003
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For the first time, a theory has been developed for the structure of the 2^3P state of ^{3}He which is consistent with all experimental data. Using our effective Hamiltonian, containing both spin-symmetric and -antisymmetric operators, we have obtained new values for several important hyperfine parameters. The three most accurate of these are the spin-symmetric constants C, D, and E corresponding, respectively, to the contact, orbital, and tensor interactions. We find $C=4283.84^{+0.02}_{-0.01}$ MHz, $D=-28.145(21)$ MHz, and $D/E=-3.9445(21)$. We have also obtained improved values for other hyperfine parameters and for the orbital g factor. Our results are shown to be in agreement with *ab initio* calculations after making some plausible corrections for isotope and relativistic shifts. We believe that this state is now well understood at the level of 20 kHz in the interaction constants.

I. INTRODUCTION

In this paper we describe a theory of the 2^3P structure in ^{3}He. We have adopted a phenomenological Hamiltonian in which the fine-structure interactions are deduced from the experimental data for ^{4}He with theoretical mass-dependent corrections and the hyperfine interactions are expressed by an effective Hamiltonian with coupling constants to be determined by experiment. The fine and hyperfine mixing of 2^1P into 2^3P have been taken into account. This approach gives the first consistent interpretation of all the experimental results.[1,2] In addition, the interaction constants are in excellent agreement with *ab initio* theoretical estimates.

This work has led to improved values both for the hyperfine parameters and for the electron orbital g factor. It has also provided new tests of the mass-dependent interactions in helium fine structure, the validity of the variational wave functions for helium, and in particular the calculated configuration mixing.

In Sec. II we describe the effective Hamiltonian used for analysis of the experiments. Section III contains a comparison with *ab initio* calculations of the hyperfine parameters. Numerical results are summarized in Sec. IV.

II. EFFECTIVE HAMILTONIAN

A. Fine structure

Our starting point is the fine structure in the 2^3P state of ^{4}He which has no hyperfine structure (see Fig. 1). This state has been the subject of intense investigation, both experimental and theoretical, from the early years of quantum mechanics to the present time. A brief historical review has been given by Lewis.[3] All the fine-structure intervals have been measured to high accuracy, the most recent measurement being that on the $J=0$—2 interval,[4] which is known to within 20 kHz. The computation of these intervals, based on the Breit interaction, has reached a similar level of accuracy.[5] The Breit Hamiltonian $H_B(\alpha^2)$ is of order α^2 Ry and describes the motion of two electrons in an external potential $(\mathbf{A},\phi)$. It may be written[6]

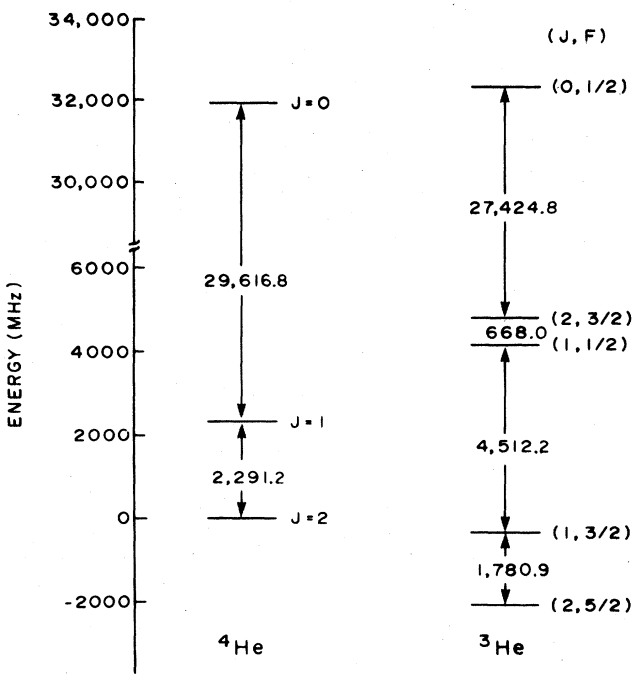

FIG. 1. Energy levels of the 2^3P states of ^{4}He and ^{3}He. The intervals are taken from Refs. 4 and 11. The intervals indicated for ^{3}He are a result of this work and are reliable to the accuracy given.

$$H_B = H_1 + H_2 + \frac{1}{r_{12}} + B \ ,$$

where

$$H_i = \beta_i - \phi(\mathbf{r}_i) + \boldsymbol{\alpha}_i \cdot [\mathbf{p}_i + \mathbf{A}(\mathbf{r}_i)] \quad (i=1,2) \tag{1}$$

and

$$B = \frac{-1}{2r_{12}} \left[(\boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2) + \frac{(\boldsymbol{\alpha}_1 \cdot \mathbf{r}_{12})(\boldsymbol{\alpha}_2 \cdot \mathbf{r}_{12})}{r_{12}^2} \right] .$$

Here the standard notation for electron coordinates ($\mathbf{r}_i, \mathbf{r}_{ij}$), momentum ($\mathbf{p}_i$), and Dirac operators ($\boldsymbol{\alpha}_i, \beta_i$) has been used.

This Hamiltonian is an approximation to the exact Bethe-Salpeter equation and is valid only as a first-order operator.[6] When the Breit Hamiltonian is used in second order, to obtain accuracy at the level α^4 Ry, a further correction $H_{\rm DK}(\alpha^4)$ must be evaluated in first order, as discussed by Douglas and Kroll (DK).[7] Furthermore, a term $H_A(\alpha^3)$ is required to account for the anomalous magnetic moment of the electron.[8] Of course, measurements of the helium fine structure are made in the center of mass frame and the transformation to that frame yields additional terms in the effective Hamiltonian. These are the mass polarization (MP) operator,[9]

$$H_{\rm MP}(m/M) = \frac{m}{M} \mathbf{p}_1 \cdot \mathbf{p}_2 \ , \tag{2}$$

and the nuclear motion (NM) operator, derived by Douglas,[10]

$$H_{\rm NM}(\alpha^2 m/M) = -Z\alpha^2 \frac{m}{M} \sum_{1,2} \frac{\mathbf{s}_i \cdot (\mathbf{r}_i \times \mathbf{p}_N)}{r_i^3} \ , \tag{3}$$

where $\mathbf{p}_N$ is the momentum of the nucleus, and m, M are, respectively, the electronic and nuclear masses. For a calculation of the fine-structure intervals, only the spin-dependent terms need be retained and the effective Hamiltonian may be formally written, following Lewis and Serafino,[5] as

$$\begin{aligned} H_{\rm eff} = {} & H_B(\alpha^2) + H_A(\alpha^3) + H_{\rm NM}\left[\alpha^2 \frac{m}{M}\right] \\ & + H_B(\alpha^2) \rangle_{\Delta E \Delta E} \langle H_B(\alpha^2) \\ & + H_{\rm DK}(\alpha^4) + H_B(\alpha^2) \rangle_{\Delta E \Delta E} \langle H_{\rm MP}\left[\alpha^2 \frac{m}{M}\right] . \end{aligned} \tag{4}$$

The second-order terms are summed over all intermediate states with appropriate energy denominators. Using this approach, Lewis and Serafino calculated the fine structure intervals in ^{4}He and obtained results consistent with experiment but slightly less accurate.

For our purpose, only the mass-dependent terms are relevant in Eq. (4); these are the nuclear motion $H_{\rm NM}$ and the second-order mass polarization involving H_B and $H_{\rm MP}$. The total theoretical mass-dependent contributions $f(J,J')$ to the ^{4}He fine-structure intervals $^4F(J,J')$ are

$$\begin{aligned} f(0,1) &= -0.841 \pm 0.010 \text{ MHz} \ , \\ f(1,2) &= 2.984 \pm 0.024 \text{ MHz} \ , \end{aligned} \tag{5}$$

TABLE I. Effective fine-structure Hamiltonian for ^{3}He in the pure spin representation $|(SL)J\rangle$. The values of the parameters are $\Delta = 6.1431 \times 10^7$ MHz, $E_0 = 31\,908.742(23)$ MHz, $E_1' = 2296.898(9)$ MHz, $E_2 = 0$, $E_M = -17.037$ MHz.

	2^1P_1	2^3P_0	2^3P_1	2^3P_2
2^1P_1	Δ		E_M	
2^3P_0		E_0		
2^3P_1	E_M		E_1'	
2^3P_2				E_2

and the measured intervals are

$$\begin{aligned} {}^4F(0,1) &= 29\,616.844 \pm 0.021 \text{ MHz (Ref. 4)} \ , \\ {}^4F(1,2) &= 2291.196 \pm 0.005 \text{ MHz (Ref. 11)} \ . \end{aligned} \tag{6}$$

In the absence of hyperfine structure the ^{3}He fine-structure splittings $^3F(J,J')$ are given by

$$^3F = {}^4F + (M_4/M_3 - 1)f \ , \tag{7}$$

whence, using $M_4/M_3 = 1.327\,23$,

$$\begin{aligned} {}^3F(0,1) &= 29\,616.569 \pm 0.021 \text{ MHz} \ , \\ {}^3F(1,2) &= 2292.173 \pm 0.009 \text{ MHz} \ . \end{aligned} \tag{8}$$

We define the eigenvalues of the fine-structure matrix

$$\begin{aligned} \Delta &= 6.143\ 1 \times 10^7 \text{ MHz } (2^1P_1) \ , \\ E_0 &= 31\,908.742(23) \text{ MHz } (2^3P_0) \ , \\ E_1 &= 2292.173(9) \text{ MHz } (2^3P_1) \ , \\ E_2 &= 0 \ (2^3P_2) \ . \end{aligned} \tag{9}$$

A derivation of the 2^1P_1 energy, Δ, is given in the Appendix. The eigenstates 2^3P_1 and 2^1P_1 noted in Eq. (9) are not pure spin states and since it is useful to express this effective Hamiltonian in the pure representation $|S,L;J\rangle$, we need to know the matrix element E_M of the Breit interaction between $|0,1;1\rangle$ and $|1,1;1\rangle$. This has been calculated by Araki *et al.*[12] and more recently by Drake[13] who gives the value $E_M = -0.003\,039\alpha^2 Z^4$ atomic units or $-17\,037$ MHz. Using Drake's value, we find the effective Hamiltonian given in Table I which, when diagonalized, reproduces the results given in Eq. (9).

B. Hyperfine structure

The structure of ^{3}He(2^3P) is shown in Fig. 1. For the hyperfine interactions we take the phenomenological form

$$\begin{aligned} H_{\rm hfs} = {} & C\mathbf{I}\cdot\mathbf{S} + C'\mathbf{I}\cdot\mathbf{K} + D\mathbf{I}\cdot\mathbf{L} + 2\sqrt{10}E\mathbf{I}\cdot\{\mathbf{S}C^{(2)}\}^{(1)} \\ & + 2\sqrt{10}E'\mathbf{I}\cdot\{\mathbf{K}\tilde{C}^{(2)}\}^{(1)} \ , \end{aligned} \tag{10}$$

where $\mathbf{I}$ is the nuclear spin, $\mathbf{S}$ and $\mathbf{L}$ are the total spin and orbital angular momenta, and $\mathbf{K}$ is the antisymmetric spin operator $\mathbf{s}_1 - \mathbf{s}_2$. $C^{(2)}$ and $\tilde{C}^{(2)}$ are the exchange symmetric and antisymmetric tensors $C_1^{(2)} \pm C_2^{(2)}$ where $C_i^{(2)} = (4\pi/5)^{1/2} Y_i^{(2)}(\Theta,\phi)$. The curly brackets indicate the contractions of $\mathbf{S}$ with $C^{(2)}$ and $\mathbf{K}$ with $\tilde{C}^{(2)}$ to form

vectors. This is the most general form the single-particle nonrelativistic hyperfine Hamiltonian can have. The spin-$\frac{1}{2}$ ^{3}He nucleus can only have a dipole interaction and the overall electronic tensor operator must therefore have rank 1. Angular momentum coupling theory permits only the following combinations for the ranks k_S, k_L for the spin, orbital parts of the tensor:

$$\begin{aligned} &k_S=1, k_L=0: \ \mathbf{I}\cdot\mathbf{S} \text{ and } \mathbf{I}\cdot\mathbf{K}\,, \\ &k_S=1, k_L=2: \ \mathbf{I}\cdot\{\mathbf{S}C^{(2)}\}^{(1)} \text{ and } \mathbf{I}\cdot\{\mathbf{K}C^{(2)}\}^{(1)}\,, \\ &k_S=0, k_L=1: \ \mathbf{I}\cdot\mathbf{L}\,. \end{aligned} \tag{11}$$

$H_{\rm hfs}$ is therefore an appropriate effective Hamiltonian whose matrix elements can be evaluated using nonrelativistic wave functions. Higher-order relativistic corrections can be embodied in appropriate adjustments of the coupling constants C, C', D, E, and E' as shown by Sandars and Beck.[14]

The spin-antisymmetric terms characterized by C' and E' have no matrix elements diagonal in S and therefore make no first-order contribution to the 2^3P structure. The second-order terms involving C' are large enough to be included in the analysis of existing data but the E' terms are negligible and we do not consider them further.

TABLE II. Effective Hamiltonian for the 2^3P states of ^{3}He including the relevant 2^1P interactions. The values of the fine-structure parameters Δ, E_0, E_1', and E_M are given in Table I. The hyperfine parameters C, C', D, and E are discussed in the text and ϵ is an abbreviation for $E/5$.

				2^1P		2^3P								
	M_l			$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
		M_S		0	0	0	1	1	1	0	1	0	1	0	-1
			M_L	1	1	0	1	1	0	1	0	1	-1	0	1
	$\frac{1}{2}$	0	1	$\Delta+\frac{D}{2}$				$\frac{-C'}{\sqrt{2}}-\sqrt{2}\epsilon$	$\frac{-E_M}{\sqrt{2}}+3\epsilon$	$\frac{E_M}{\sqrt{2}}+\frac{C'}{2}-2\epsilon$					
2^1P	$-\frac{1}{2}$	0	1		$\Delta-\frac{D}{2}$	$\frac{D}{\sqrt{2}}$					$\frac{-E_M}{\sqrt{2}}-3\epsilon$	$\frac{E_M}{\sqrt{2}}-\frac{C'}{2}+2\epsilon$	$6\sqrt{2}\epsilon$	$-3\sqrt{2}\epsilon$	$\frac{C'}{\sqrt{2}}+\sqrt{2}\epsilon$
	$\frac{1}{2}$	0	0		$\frac{D}{\sqrt{2}}$	Δ					$\frac{-C'}{\sqrt{2}}+\sqrt{8}\epsilon$	$-3\sqrt{2}\epsilon$	$\frac{-E_M}{\sqrt{2}}-3\epsilon$	$\frac{C'}{2}+4\epsilon$	$\frac{E_M}{\sqrt{2}}-3\epsilon$
	$\frac{1}{2}$	1	1				$\frac{C}{2}+\frac{D}{2}+2\epsilon$								
	$-\frac{1}{2}$	1	1	$\frac{-C'}{\sqrt{2}}-\sqrt{2}\epsilon$				$\frac{-C}{2}-\frac{D}{2}-2\epsilon$	$\frac{D}{\sqrt{2}}+3\sqrt{2}\epsilon$	$\frac{C}{\sqrt{2}}-\sqrt{2}\epsilon$					
	$\frac{1}{2}$	1	0	$\frac{-E_M}{\sqrt{2}}+3\epsilon$				$\frac{D}{\sqrt{2}}+3\sqrt{2}\epsilon$	$\frac{E_1'}{2}+\frac{C}{2}-4\epsilon$	$\frac{-E_1'}{2}+3\epsilon$					
	$\frac{1}{2}$	0	1	$\frac{E_M}{\sqrt{2}}+\frac{C'}{2}-2\epsilon$				$\frac{C}{\sqrt{2}}-\sqrt{2}\epsilon$	$\frac{-E_1'}{2}+3\epsilon$	$\frac{E_1'}{2}+\frac{D}{2}$					
2^3P	$-\frac{1}{2}$	1	0		$\frac{-E_M}{\sqrt{2}}-3\epsilon$	$\frac{-C'}{\sqrt{2}}+\sqrt{8}\epsilon$					$\frac{E_1'}{2}-\frac{C}{2}+4\epsilon$	$\frac{-E_1'}{2}-3\epsilon$	$\frac{D}{\sqrt{2}}-3\sqrt{2}\epsilon$	$\frac{C}{\sqrt{2}}+2\sqrt{2}\epsilon$	0
	$-\frac{1}{2}$	0	1		$\frac{E_M}{\sqrt{2}}-\frac{C'}{2}+2\epsilon$	$-3\sqrt{2}\epsilon$					$\frac{-E_1'}{2}-3\epsilon$	$\frac{E_1'}{2}-\frac{D}{2}$	$6\sqrt{2}\epsilon$	$\frac{D}{\sqrt{2}}$	$\frac{C}{\sqrt{2}}-\sqrt{2}\epsilon$
	$\frac{1}{2}$	1	-1		$6\sqrt{2}\epsilon$	$\frac{-E_M}{\sqrt{2}}-3\epsilon$					$\frac{D}{\sqrt{2}}-3\sqrt{2}\epsilon$	$6\sqrt{2}\epsilon$	$\frac{E_0}{3}+\frac{E_1'}{2}+\frac{C}{2}-\frac{D}{2}+2\epsilon$	$\frac{-E_0}{3}-3\epsilon$	$\frac{E_0}{3}-\frac{E_1'}{2}$
	$\frac{1}{2}$	0	0		$-3\sqrt{2}\epsilon$	$\frac{C'}{2}+4\epsilon$					$\frac{C}{\sqrt{2}}+2\sqrt{2}\epsilon$	$\frac{D}{\sqrt{2}}$	$\frac{-E_0}{3}-3\epsilon$	$\frac{E_0}{3}$	$\frac{-E_0}{3}+3\epsilon$
	$\frac{1}{2}$	-1	1		$\frac{C'}{\sqrt{2}}+\sqrt{2}\epsilon$	$\frac{E_M}{\sqrt{2}}-3\epsilon$					0	$\frac{C}{\sqrt{2}}-\sqrt{2}\epsilon$	$\frac{E_0}{3}-\frac{E_1'}{2}$	$\frac{-E_0}{3}+3\epsilon$	$\frac{E_0}{3}+\frac{E_1'}{2}-\frac{C}{2}+\frac{D}{2}-2\epsilon$

We have found it most convenient to compute the matrix of $H_{\rm hfs}$ in the $|I,M_I;S,M_S;L,M_L\rangle$ representation. To this we have added the effective fine-structure Hamiltonian of Table I after transforming it to the same representation. The result, given in Table II, is the Hamiltonian used in our analysis of experimental data with C, C', D, and E treated as fitting parameters.

III. THEORETICAL ESTIMATES OF HYPERFINE PARAMETERS

A. Introduction

We take as our starting point the single-particle hyperfine Hamiltonian $H_{\rm hfs}$, given to order $(m/M)\alpha^2$ Ry by Bethe and Salpeter,[6]

$$H_{\rm hfs}=-2\mu_0\sum_{i=1,2}\left[\frac{8\pi}{3}(\mathbf{s}_i\cdot\boldsymbol{\mu})\delta(\mathbf{r}_i)-\frac{\mathbf{l}_i\cdot\boldsymbol{\mu}}{r_i^3}+\frac{1}{r_i^3}\left[\mathbf{s}_i\cdot\boldsymbol{\mu}-\frac{3(\mathbf{s}_i\cdot\mathbf{r}_i)(\boldsymbol{\mu}\cdot\mathbf{r}_i)}{r_i^2}\right]\right], \tag{12}$$

where μ_0 is the Bohr magneton and $\boldsymbol{\mu}$ is the nuclear-magnetic moment. The first term corresponds to the C and C' terms of our phenomenological Hamiltonian [Eq. (10)] while the second and third terms correspond to the phenomenological D and E terms, respectively. These operators will be used to estimate the hyperfine parameters. The leading electrodynamic corrections are of relative order α, due to the anomalous magnetic moment of the electron, and $(Z\alpha)^2$ from a variety of relativistic effects. There are also corrections due to nuclear size and structure, which are important in the contact terms, and to the finite nuclear mass, which leads to the usual reduced mass and mass polarization corrections of order m/M.

In estimating the hyperfine parameters below we will neglect either $(Z\alpha)^2$ terms or m/M terms, both of which are approximately 200 ppm. The associated uncertainties will be of order 900, 6, and 1 kHz in C, D, and E, respectively. A phenomenological argument based on the known hyperfine structure of He^+ will be used to reduce the uncertainty in the contact terms.

B. Estimate of C

The first term in Eq. (12) with the substitution $\boldsymbol{\mu}=-g_I\mu_0\mathbf{I}$, yields

$$C=-\frac{8\pi}{3}g_I\mu_0^2\langle 2^3P\,|\,\delta(\mathbf{r}_1)+\delta(\mathbf{r}_2)\,|\,2^3P\rangle\ , \tag{13}$$

$$C'=-\frac{8\pi}{3}g_I\mu_0^2\langle 2^1P\,|\,\delta(\mathbf{r}_1)-\delta(\mathbf{r}_2)\,|\,2^3P\rangle\ . \tag{14}$$

The radial integral $\langle 2^3P\,|\,\delta(\mathbf{r}_1)\,|\,2^3P\rangle$ has been calculated by Schiff, Lifson, Pekeris, and Rabinowitz[15] using variational methods to solve the nonrelativistic Schrödinger equation with infinitely heavy nucleus. Their result was confirmed by Accad, Pekeris, and Schiff[16] who give the value 1.258 860 atomic units. Using Williams's value,[17] $g_I=[2.317\,4824(7)]\times10^{-3}$, we arrive at a first estimate $C=-4281.723$ MHz. In a crude attempt to account for relativistic and radiative effects and for the nuclear mass and volume, we consider the ground state of $^3He^+$. There the simple Schrödinger theory gives a hyperfine splitting of 8661.260 MHz, while the interval measured by Fortson, Major, and Dehmelt[18] [8665.649 90(50) MHz] is larger by 507 ppm. We use the argument, due originally to Hambro,[19] that the contact interaction in the 2^3P state of neutral helium is dominated by the same $1s$ electron and must be subject to essentially the same correction factor. Of course this correction can only possibly be valid to the extent that C for $^3He^+$ is the same as C for the 2^3P state of ^3He, namely to about 1%. Thus our best theoretical estimate is

$$C=-4283.890(20)\ \text{MHz}\ . \tag{15}$$

C. Estimate of C'

The sophisticated wave functions of Schiff *et al.*[15] and Accad *et al.*[16] have not been used to calculate the integral $\langle 2^1P\,|\,\delta(\mathbf{r}_1)\,|\,2^3P\rangle$ and we therefore turn to the relatively simple wave functions of Araki, Ohta, and Mano,[12] who give

$$|\,2P(0,\mathbf{r}_2)\rangle=\left[\frac{1}{8\pi}\right]^{1/2}[a_0N(K,\mu)e^{-\mu r_2}+a_2N(K,\mu_2)e^{-\mu_2 r_2}]r_2Y_m^1\ , \tag{16}$$

where

$$\sqrt{3}N(K,\mu)=4\sqrt{K^3\mu^5}$$

and

$$K=1.991\,186\ ,$$
$$\mu=0.554\,575\ ,$$
$$\mu_2=1.975\,000\ \text{ for } 2^3P\ ,$$
$$a_0=0.990\,273\ ,$$
$$a_2=0.013\,874\ ,$$
$$K=2.003\,024\ ,$$
$$\mu=0.482\,363\ ,$$
$$\mu_2=1.437\,000\ \text{ for } 2^1P\ ,$$
$$a_0=1.000\,566\ ,$$
$$a_2=-0.000\,407\ .$$

These wave functions give $C'/C=1.004$. In order to investigate the accuracy of this result we calculated the diagonal integrals C and C_{1P} (the same parameter in the 2^1P state) using Eq. (16) and found that they were in agreement with those quoted by Schiff *et al.* within 1%. We also found it useful to consider the ratio

$$\frac{(C')^2}{CC_{1P}}=\frac{[\langle 2^1P(0,\mathbf{r}_2)\,|\,2^3P(0,\mathbf{r}_2)\rangle]^2}{\langle 2^1P(0,\mathbf{r}_2)\,|\,2^1P(0,\mathbf{r}_2)\rangle\langle 2^3P(0,\mathbf{r}_2)\,|\,2^3P(0,\mathbf{r}_2)\rangle}\ . \tag{17}$$

According to the Schwartz inequality, this ratio is less than or equal to unity and hence

$$\left|\frac{C'}{C}\right| \leq \sqrt{C_{1P}/C} = 1.006 , \tag{18}$$

where C_{1P}/C is taken from Schiff *et al.* Our conclusion is therefore that

$$\frac{C'}{C} = 1.004^{+0.002}_{-0.01} \tag{19}$$

in which higher-order corrections are expected to be negligible.

D. Estimate of D

To order $(m/M)\alpha^2$ Ry, the second term in Eq. (12) corresponds to the term $D\mathbf{I}\cdot\mathbf{L}$ in Eq. (10), with

$$D = -2g_I\mu_0^2\left\langle 2^3P \left| \frac{(l_1)_z}{r_1^3} + \frac{(l_2)_z}{r_2^3} \right| 2^3P \right\rangle \tag{20}$$

evaluated for $M_L = +1$. This integral has been calculated by Schwartz[20] using a 439-term variational solution to the Schrödinger equation and his result, which was confirmed by Hambro, gives $D = -28.1430$ MHz. A transformation to the center-of-mass frame yields the simple two-body reduced mass correction[6] $(1-3m/M)$ plus an additional mass polarization correction of relative order m/M due to the electron correlation. The latter has not been calculated and neither has the order $(Z\alpha)^2$ correction. Thus we estimate that

$$D = -28.128(8) \text{ MHz } (300 \text{ ppm}) . \tag{21}$$

E. Estimate of E

To order $(m/M)\alpha^2$ Ry the last term in Eq. (12) corresponds to the E and E' terms in Eq. (10). In particular, the value of E is given by

$$E = -\tfrac{5}{2}g_I\mu_0^2\left\langle 2^3P \left| \frac{(C_1)_0^{(1)}}{r_1^3} + \frac{(C_2)_0^{(1)}}{r_2^3} \right| 2^3P \right\rangle \tag{22}$$

with $M_L = +1$, which has been calculated by Hambro[19] using Schwartz's wave function. After correcting for the two-body reduced mass with the factor $(1-3m/M)$ and for the anomalous magnetic moment of the electron with the factor $(1+\alpha/2\pi)$, he finds

$$E = 7.130(20) \text{ MHz} , \tag{23}$$

in which the uncertainty is associated with the extrapolation of a series and not with the physical approximations discussed in Sec. III A.

It is interesting to note that to lowest order, the configuration (s,p) in a purely central potential has $D/E = -4$, whereas Eqs. (21) and (23) give

$$D/E = -3.945(11) . \tag{24}$$

Only 10% of the deviation from -4 is due to the anomalous moment of the electron. The rest is due to polarization of the $1s$ and $2p$ orbitals by the noncentral part of the electron-electron interaction.

IV. NUMERICAL RESULTS

Prior to this work there were three publications on the measurement of the ^{3}He(2^3P) hyperfine structure. The first was a preliminary report on level crossing data[2] by German, who was subsequently unable to find an internally consistent interpretation of the results.[2] The second was by Johnson and Pichanick[21] giving preliminary values for the hyperfine parameters based on atomic-beam magnetic resonance (ABMR) data. These parameters were inconsistent with theory. Next, Freeman *et al.*[22] reported hyperfine parameters obtained from laser spectroscopic measurements. These were much less accurate than earlier results but were consistent with theory. However, the energy levels were inconsistent with the parameters given in the same article and with the intervals given in Ref. 2.

From the available data it should have been possible to

TABLE III. Summary of data on the 2^3P states of ^{3}He.

	Experiment		
Parameter	ABMR	Level crossing	Theory
C	$-4283.84^{+0.02}_{-0.01}$ MHz[a]		$-4283.89(2)$ MHz[e]
C'/C		1.010(16)[c]	$1.004^{+0.002}_{-0.01}$ [f]
D	$-28.060(60)$ MHz[a]	$-28.145(21)$[c]	$-28.128(8)$ MHz[f]
D/E	$-3.950(10)$[a]	$-3.944\,5(21)$[c]	$-3.945(11)$[g]
g_S'	2.002 243 2(22)[b]		2.002 239 9[h]
g_L'	0.999 825 0(90)[b]	0.999 828 6(20)[c] 0.999 826 5(30)[d]	0.999 833 0(4)[h]

[a]Reference 1.
[b]Reference 11.
[c]Reference 2.
[d]Reference 23.
[e]Equation (15) and Ref. 19.
[f]Equations (19) and (21).
[g]Equation (24) and Ref. 19.
[h]Reference 24, where g_L has been mass corrected for ^{3}He.

obtain values of the hyperfine parameters accurate to order 10 kHz, and yet there were discrepancies of order 1 MHz. As a result of this work we know now that the main difficulty lay in the treatment of the fine-structure interactions including the $^1P_1$-$^3P_1$ off-diagonal interaction. Also, German did not have the correct value for the orbital g factor of the 2^3P state.

Recently we published accurate values for C, D, and E based on the earlier measurements of Johnson and Pichanick combined with new data.[1] The analysis we have described here was used to deduce those parameters from the data and to resolve the apparent disagreement with theory. Since then we have analyzed the level crossing data. We find that the $\Delta m=2$ level crossing results are internally consistent and our analysis has yielded new information on the Zeeman g factors as well as the hyperfine parameters. We have also calculated the energy levels in zero field and the results are shown in Fig. 1.

The overall consistency between different experiments provides confirmation that our phenomenological Hamiltonian is correct at the 20 kHz level and that we now have reliable values for the hyperfine parameters. It is therefore interesting to compare them with *ab initio* estimates.

A convenient set of hyperfine parameters for comparison with theory is C, C'/C, D, and D/E. In the Yale atomic-beam magnetic resonance (ABMR) experiments[1] on ^{3}He, four transitions were observed giving enough degrees of freedom for independent determinations of C, D, and D/E. The results of that fit are given in Table III under "ABMR." The two g factors shown in the same column are the results of an earlier Yale ABMR experiment[11] on ^{4}He. An isotope correction has been made to g_L' and is negligible for g_S'. In the level crossing experiments of German[2] and Sands, 13 $\Delta m=2$ level crossings were measured, providing enough degrees of freedom for independent determination of all six parameters. We have made such a fit[25] and find values for the parameters that are all consistent with the ABMR results but with relatively large error bars. The best results are obtained by taking the values of C and g_S' from the ABMR data and using the level crossing data to determine C'/C, D, D/E, and g_L'. These results are shown in Table III under "level-crossing" and all four are more accurate than any previously published numbers. We also show the best previous value for g_L', obtained by Lhuillier *et al.*[23] from level crossing measurements on ^{4}He and mass corrected for ^{3}He.

This analysis has shown that the experiments are consistent internally and with each other. In addition, we find agreement with theory at a high level of accuracy which tests many subtle aspects of the wave functions including the core polarization, the mass dependence, and a variety of radial integrals. One of the more remarkable conclusions is that the large relativistic correction of C, based on the model of a weakly perturbed hydrogenic $1s$ electron, appears to be valid to 1%. We conclude that the many confusing aspects of this subject have been resolved and that at the level of accuracy indicated, the 2^3P hyperfine interactions are now well understood.

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APPENDIX: 2^1P_1-2^3P_2 SPLITTING IN ^{3}He

Recent spectroscopic information on ^{4}He has been summarized by Martin.[26] He quotes experimental values for the ionization energies

$$I(2^1P_1)=27\,175.775\ \text{cm}^{-1}\ ,$$
$$I(2^3P_1)=29\,223.829\ \text{cm}^{-1}\ , \tag{A1}$$

and theoretical values for the specific mass shifts

$$\epsilon_m(2^1P_1)=1.385\ \text{cm}^{-1}\ ,$$
$$\epsilon_m(2^3P_1)=-1.943\ \text{cm}^{-1}\ . \tag{A2}$$

On the basis of these values we separate out the normal and specific mass shifts; thus,

$$I(2^1P_1)=27\,180.886-3.726-1.385\ \text{cm}^{-1}\ ,$$
$$I(2^3P_1)=29\,225.892-4.007+1.943\ \text{cm}^{-1}\ , \tag{A3}$$

using the electron- to alpha-particle mass ratio[22] $m_e/M_\alpha=1.370\,9337\times10^{-4}$.

After scaling both mass shifts by M_4/M_3, we obtain the value 2049.050 cm^{-1} for the 2^1P_1-2^3P_1 interval in ^{3}He. This result is then added to the 2^3P_1-2^3P_2 interval $^3F(1,2)$ given in Eq. (8) to yield a value Δ for the 2^1P_1-2^3P_2 splitting in ^{3}He.

$$\Delta=6.1431\times10^7\ \text{MHz}\ . \tag{A4}$$

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