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Two-photon free-free transitions in a Coulomb potential

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We report a perturbative calculation of the cross sections for two-photon free-free transitions (absorption and emission) in a Coulomb potential within the nonrelativistic dipole approximation. The matrix element was integrated analytically by the Green's-function method, and then computed numerically with great accuracy. Limiting cases are discussed (Born approximation, low and high photon-electron energy ratios). Results are presented for two-photon absorption, in the case when the initial electron momentum is perpendicular to the photon polarization vector.

Free-free transitions (FFT's) of electrons colliding with neutrals or ions play an important role in the absorption and stimulated emission of radiation from hot gases and plasmas. For the low-intensity laboratory sources used in the past, and for those of astrophysical origin, one-photon FFT's are the only ones relevant (for a review see Ref. 1). They have been extensively studied theoretically since the basic result of Sommerfeld[2] for the Coulomb potential (see also Ref. 3).

At the higher intensities now available from laser sources, multiphoton FFT's become relevant. For moderate intensities, perturbation theory to lowest nonvanishing order is still the appropriate approach for calculating the cross sections. Its results are expected to be valid over the whole range of photon energies ω, except for the limiting case of small ω (soft photons), where the well-known infrared divergences of quantum electrodynamics are bound to show up. Nonperturbative approaches, valid under limitations of their own, have also been applied to multiphoton FFT's (e.g., see Ref. 4). Thus, the theory of Kroll and Watson[5] applies precisely to the case of small ω, which cannot be encompassed by perturbation theory.

We present here the perturbative calculation of the stimulated two-photon FFT's (absorption and emission) in a Coulomb potential $V(r)=-Z/r$, within the nonrelativistic dipole approximation. This is the first perturbative FFT calculation to be done for more than one photon, which is rather surprising in view of the number of calculations existing for the related problem of two-photon ionization (e.g., see Ref. 6). Our result parallels that of Sommerfeld for one-photon bremsstrahlung. We have carried out the calculation analytically and exactly as far as possible, following a method developed earlier by one of us,[7] and have then resorted to a very accurate numerical computation.

We mention that attempts were made in the past to describe (relativistically) the closely related process of spontaneous two-photon bremsstrahlung in the Born approximation, with conflicting outcome (e.g., see Smirnov,[8] and the references therein). Also, very recently, an experiment was reported for this process.[9] On the other hand, no detailed experimental study exists for the two-photon FFT beyond the mere proof of its existence, due to Weingartshofer, Holmes, Sabbagh, and Chin.[10]

The differential cross section for the scattering of an electron with the absorption $(+)$ or emission $(-)$ of two photons from a single mode laser field can be written

$$\frac{d\sigma^{(\pm)}}{d\Omega}=\frac{p_f}{p_i}|f^{(\pm)}|^2 \ , \tag{1}$$

where, in the nonrelativistic dipole approximation, perturbation theory yields

$$f^{(\pm)}=\frac{1}{2\pi}I\omega^{-2}\langle u^{\rm in}_{\mathbf{p}_f}|(\mathbf{e}\cdot\mathbf{P})G^+(\Omega\pm)(\mathbf{e}\cdot\mathbf{P})|u^{\rm out}_{\mathbf{p}_i}\rangle \ . \tag{2}$$

Here, $\mathbf{p}_i$ and $\mathbf{p}_f$ are the initial and final momenta of the electron, I is the intensity of the laser with frequency ω. $\mathbf{e}$ is the polarization vector of the photons (we assume for simplicity linear polarization, i.e., $\mathbf{e}$ is real), $\mathbf{P}$ is the electron momentum operator, $u^{\rm in}_{\mathbf{p}_f}$ and $u^{\rm out}_{\mathbf{p}_i}$ are continuum Coulomb states corresponding to the indicated asymptotic momenta with incoming- or outgoing-spherical-wave behavior, and $G^+(\Omega)$ is the Coulomb Green's operator for energy parameter $\Omega+i\epsilon$. Conservation of energy requires

$$E_f=E_i\pm 2\omega \ , \tag{3}$$

where $E=p^2/2$. Further,

$$\Omega_\pm=E_i\pm\omega \ . \tag{4}$$

Our formulas are written in Z-scaled atomic units, i.e., momenta are measured in $Z\times$a.u., energies in $Z^2\times$Ry, and the (time-averaged) intensity I in $Z^3\times$a.u., where the a.u. of intensity is 3.51×10^{16} W/cm^2.

The matrix element $f^{(\pm)}$ was integrated in momentum space. Integral representations were used for the Coulomb states $u^{\rm out}_{\mathbf{p}_i}(\mathbf{q}), u^{\rm in}_{\mathbf{p}_f}(\mathbf{q})$. Thus,

$$u^{\rm out}_{\mathbf{p}}(\mathbf{q})=N\oint\left(\frac{\zeta}{\zeta-1}\right)^n\frac{d\zeta}{\{(\mathbf{q}-\mathbf{p}\zeta)^2+[\eta-ip(1-\zeta)]^2\}^2} \ , \tag{5}$$

where $n=1/ip$, N is a normalization constant, $\eta>0$ is an infinitesimal quantity, and the contour encircles the points $\zeta=0$ and $\zeta=1$ in the counterclockwise sense. Further, as known, $u^{\rm in}_{\mathbf{p}}(\mathbf{q})=[u^{\rm out}_{\mathbf{p}}(\mathbf{q})]^*$. For the Coulomb Green's

function we have used the Schwinger[11] integral representation

$$G^{+}(\mathbf{p}_1,\mathbf{p}_2;\Omega)=\frac{X^3}{2\pi^2}\frac{i\exp(i\pi\tau)}{2\sin(\pi\tau)}\int_1^{(0+)}\rho^{-\tau}\frac{d}{d\rho}\left[\frac{1-\rho^2}{\rho}\frac{1}{[X^2(\mathbf{p}_1-\mathbf{p}_2)^2+(p_1^2+X^2)(p_2^2+X^2)(1-\rho)^2/4\rho]^2}\right]d\rho \ . \quad (6)$$

Here $\tau=X^{-1}$, $X^2=-2\Omega$, $\mathrm{Re}X>0$.

By inserting Eqs. (5) and (6) into Eq. (2), we get a sixfold integral in momentum space, followed by three parameter integrals. The momentum-space integrations were carried out using formulas derived earlier,[7] and one of the parameter integrals was done by applying the residue theorem. Of the two integrals left, one can be expressed in terms of Gauss hypergeometric functions ${}_2F_1$, so that finally we are left with a single integration over a linear combination of such functions.

The matrix element $f^{(\pm)}$ can be written as

$$f^{(\pm)}=I[P^{(\pm)}+Q^{(\pm)}(\mathbf{e}\cdot\boldsymbol{\nu}_i)^2+R^{(\pm)}(\mathbf{e}\cdot\boldsymbol{\nu}_i)(\mathbf{e}\cdot\boldsymbol{\nu}_f)+S^{(\pm)}(\mathbf{e}\cdot\boldsymbol{\nu}_f)^2] \ , \quad (7)$$

where $\boldsymbol{\nu}_\alpha=\mathbf{p}_\alpha/p_\alpha$ $(\alpha=i,f)$. We give here only the amplitude P, which is typical:

$$P=C\int_0^1\rho^{-\tau+1}(1-x_0\rho)^{n_i+n_f-2}(1-x_1\rho)^{-n_i-1}\times(1-x_2\rho)^{-n_f-1}\,{}_2F_1(1+n_i,1+n_f;2;x)\,d\rho \ , \quad (8)$$

where

$$x=-\frac{16\rho p_i p_f X^2\sin^2(\theta/2)}{(p_i^2+X^2)(p_f^2+X^2)(1-x_1\rho)(1-x_2\rho)} \ . \quad (9)$$

θ is the scattering angle between $\mathbf{p}_i$ and $\mathbf{p}_f$, and C, x_0, x_1, and x_2 are functions of p_i, p_f, and Ω, but independent of θ or ρ. In the expressions of P, Q, etc., entering Eq. (7) the difference between absorption $(+)$ and emission $(-)$ lies only in the auxiliary conditions Eqs. (3) and (4) connecting E_i, E_f, and Ω.

The dependence of $f^{(\pm)}$ on its variables (E_i or E_f, θ, and the orientation of $\boldsymbol{\nu}_i$ and $\boldsymbol{\nu}_f$ with respect to $\mathbf{e}$) is quite intricate. It simplifies considerably, however, in *three limiting cases:*

(1) *The Born approximation,* i.e., E_i and E_f large, such that $|n_i|, |n_f| \ll 1$. The integrands in Eq. (8) and similar ones simplify to the extent that one obtains a closed, albeit rather complicated, formula for $f^{(\pm)}$ in terms of elementary functions.[12]

(2) *High photon-electron energy ratio,* $\omega/E_i \gg 1$ (applicable to absorption only). In this limit the amplitudes P, Q, etc. become E_i- and θ-independent functions of ω, and all the angular dependence of $f^{(\pm)}$ is contained in the scalar products of Eq. (7).

(3) *Low photon-electron energy ratio,* $\omega/E_i \ll 1$, $\omega/E_f \ll 1$ (ω itself need not be small). For nonforward scattering one obtains the dominant contribution

$$f^{(\pm)}\simeq 2I\omega^{-4}(\mathbf{e}\cdot\boldsymbol{\Delta})^2 f_c(p,\Delta) \ , \quad (10)$$

where $\boldsymbol{\Delta}=\mathbf{p}_f-\mathbf{p}_i$, and

$$f_c(p,\Delta)=\frac{1}{2p^2}\frac{\Gamma(1+n)}{\Gamma(1-n)}\left(\frac{\Delta^2}{4p^2}\right)^{-1-n} \quad (11)$$

is the Coulomb scattering amplitude for the momentum magnitude $p=p_i\simeq p_f$, and momentum transfer Δ. This result is an extension to the two-photon case of the low-energy theorem of Low[13] (see also Ref. 1). This states that,

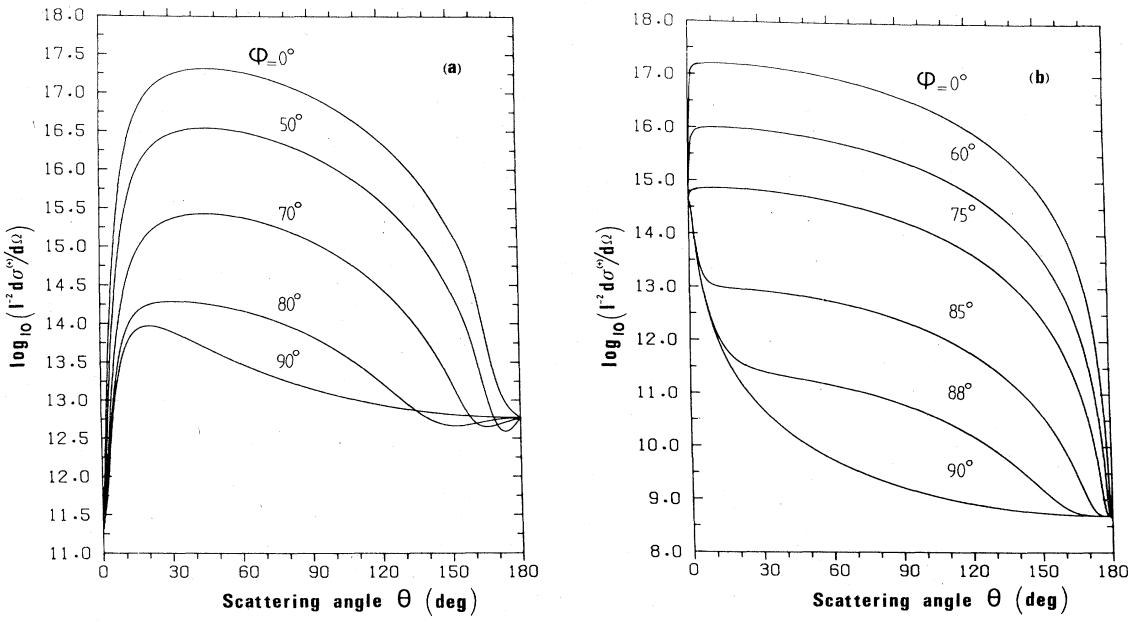

FIG. 1. Variation of $\log_{10}(I^{-2}d\sigma^{(+)}/d\Omega)$ with θ for various values of ϕ (as marked). $\omega=0.01$ Ry and (a) $E_i=0.1$ Ry, (b) $E_i=1$ Ry.

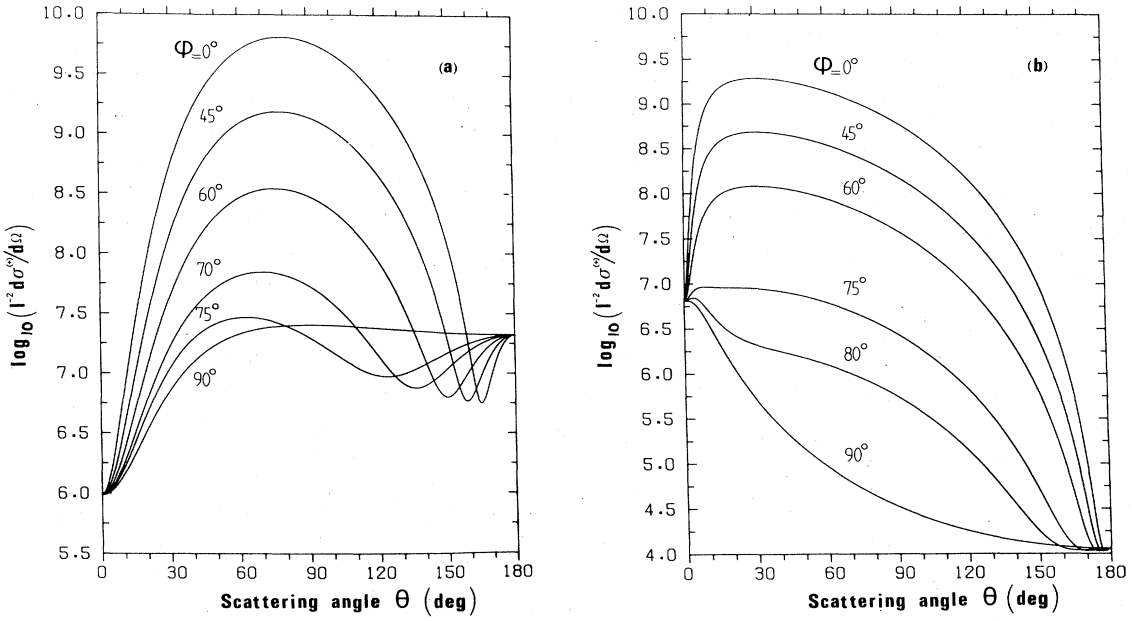

FIG. 2. Same as for Fig. 1, but with $\omega = 0.1$ Ry and (a) $E_i = 0.1$ Ry, (b) $E_i = 1$ Ry.

in the soft photon limit, the dominant term of the one-photon bremsstrahlung amplitude becomes proportional to the elastic scattering amplitude. At the same time Eq. (10) represents the weak-intensity limit of the Kroll and Watson result for two-photon FFT's. Note, however, that Low's result (as well as that of Kroll and Watson[5]) was derived for a short-range interaction, whereas we are dealing here with a Coulomb potential as a cause for the acceleration of the electron. This shows up in our case in that the low-energy Eq. (10) *does not hold for forward scattering* ($\Delta = |p_f - p_i|$). Indeed, by choosing, for example, $\mathbf{e}\cdot\boldsymbol{\Delta} = 0$, $f^{(\pm)}$ from Eq. (10) vanishes, whereas the exact $f^{(\pm)}$ from Eq. (7) does not.

We have computed the angular dependence of the cross sections for absorption and emission at various values of E_i and ω. The relative accuracy of the computation is better than 10^{-5}. We have checked that our numerical results go over smoothly into the analytical results for the three limiting cases considered above.

For simplicity we shall illustrate here only the case of *ab-*

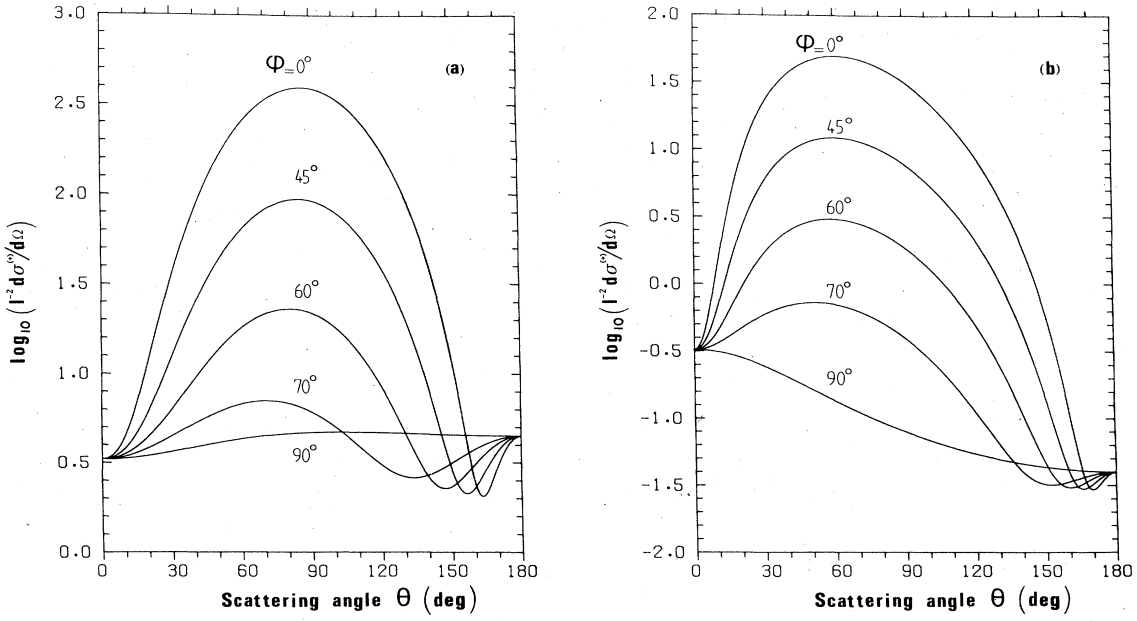

FIG. 3. Same as for Fig. 1, but with $\omega = 1$ Ry and (a) $E_i = 0.1$ Ry, (b) $E_i = 1$ Ry.

sorption for the *special geometry* $\nu_i \cdot \mathbf{e} = 0$. The $0z$ axis was set along ν_i, the $0x$ axis along $\mathbf{e}$, and the polar angles θ and ϕ of ν_f were defined in the usual way. For symmetry reasons it is sufficient to consider the cross section for $0 \leq \phi \leq \pi/2$. Thus

$$f^{(+)} = P^{(+)}(\theta) + S^{(+)}(\theta) \sin^2\theta \cos^2\phi \quad . \tag{12}$$

The differential cross section Eq. (1) is displayed for a number of cases in Figs. 1–3 as function of θ at given ω, E_i and ϕ. As is apparent, the cross section can change by many orders of magnitude depending mainly on the value of ω. Roughly speaking the smaller ω is, the larger the cross section. For given ω and E_i, all curves for different ϕ start from the same value at $\theta = 0$, and end at the same value for $\theta = 180°$, which is a consequence of Eq. (12). The curve for $\phi = 90°$ represents the θ dependence of $|P^{(+)}(\theta)|^2$. In most cases the θ dependence of the cross section has a maximum for an angle smaller than 90°.

Some of the cases considered can be understood in terms of the low-energy approximation Eq. (10). This is particularly true for Fig. 1(b), where $\omega/E_i = 0.01 << 1$, and to a lesser extent for Figs. 1(a) and 2(b). The agreement with Eq. (10) extends over the whole range of angles θ, except close to $\theta = 0°$ and $\theta = 180°$, or for values of ϕ close to 90°. The latter are cases in which Eq. (10) gives a vanishing result, whereas the exact expression Eq. (12) does not. The large forward-backward asymmetry with respect to θ is due to the Coulomb scattering amplitude f_c in Eq. (10).

On the other hand, Figs. 2(a), 3(a), and 3(b) represent situations lying entirely out of the realm of the approximate analytical formulas derived for the limiting cases (1)–(3) mentioned above, and in which only the numerical computation can yield an adequate description.

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