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Spin-wave-related period doublings and chaos under transverse pumping

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The experiments of Gibson and Jeffries on ferromagnetic resonance at very high signal powers are simulated with a model involving coupling of only one spin wave to the uniform mode. Bifurcation sequences and chaos similar to the ones observed are found in the computation.

In a recent series of experiments Gibson and Jeffries[1] (GJ) have performed ferromagnetic resonance at a signal power well in excess of the second-order Suhl-instability threshold.[2] As the signal power was increased, the response went through a succession of events beginning with a limit cycle, then period doubling, chaos, and several periodic windows. Earlier, Nakamura, Ohta, and Kawasaki[3,4] worked out (in the case of parallel pumping) a similar period-doubling route to chaos by coupling two nonuniform spin-wave modes. Here we consider the case of transverse pumping used in GJ's experiment. In that case similar phenomena occur but are now due to the coupling of the uniform mode to nonuniform ones on the manifold[5] of the spin-wave spectrum degenerate with uniform mode. If the dominant one of these is singled out, then above a certain signal power, we find a limit cycle, the frequency of which is of an order higher than that observed by GJ. As the power is increased further, we also find period doublings, onset of chaos, and two periodic windows, periods 3 and 5, respectively, as observed in GJ's experiment.

The equation of motion for the magnetization is

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m}\times\mathbf{H} - \alpha\mathbf{m}\times(\mathbf{m}\times\mathbf{H}) \ . \tag{1}$$

Here, we employed Landau-Lifshitz damping. The field **H** consists of three parts: the applied field, the dipolar field, and the exchange field. **m** is expanded in spin waves in the usual manner.[5] Equating the Fourier coefficients of both sides and keeping the secular terms only,[2] we have, in reduced units,

$$\begin{aligned} \frac{dB_k}{d\tau} &= iB_0^2B_{-k}^* + i\Gamma|B_0|^2B_k - \eta B_k \ , \\ \frac{dB_0}{d\tau} &= iB_k^2B_0^* - i\omega_s - \eta B_0 \ , \end{aligned} \tag{2}$$

where B_0 and B_k are the amplitudes (with a phase factor) of the spin waves, η is the damping constant, $d\tau = 0.5\omega_m\,dt$,[6] Γ is a constant proportional to ω_m, ω_m is the magnetization (in frequency units), and ω_s is the strength of driving field. We assume $B_k = B_{-k}$ and throughout the numerical calculations assign $\eta = 0.005$ and $\Gamma = 0.5$. Equations (2) and their complex conjugates form a set of four differential equations, whose fixed points are

$$|B_0| = \omega_s/\eta \ , \quad |B_k| = 0 \ , \tag{3a}$$

$$|B_0| = \eta^{1/2}/(1-\Gamma^2)^{1/4} \ , \quad |B_k| = \left[-\eta(1-\Gamma^2)^{1/2} + \left(\frac{\omega_s^2}{\eta}(1-\Gamma^2)^{1/2} - \eta^2\Gamma^2\right)^{1/2}\right]^{1/2} \ . \tag{3b}$$

When $\omega_s < \eta(\eta/\omega_m)^{1/2}$ (Suhl threshold), (3a) is stable. When $\omega_s > \eta(\eta/\omega_m)^{1/2}$, (3a) becomes unstable and (3b) becomes stable. For $|B_k| << |B_0|$ we worked out the stability conditions for the fixed point (3b). We find

$$(\lambda+\eta)^3 - (\lambda+\eta)[(1-\Gamma^2)|\beta_0|^4 + 4\Gamma|B_k|^2|B_0|^2 - 8|B_k|^2|B_0|^2] - 4\Gamma^2\eta|B_k|^2|B_0|^2 = 0 \ , \tag{4}$$

where λ is the Lyapunov exponent. Up to $|B_k|^2 \leq 0.1|B_0|^2$ all λ's are negative. For $|B_k| \sim |B_0|$ the calculation becomes extremely tedious. So we approach it in a less precise way but with a clearer physical picture.

It can be shown easily by using Eq. (1) that

$$\frac{d}{dt}(|B_0|^2 + |B_k|^2) = -2\omega_s\,\mathrm{Im}B_0 - 2\eta(|B_k|^2 + |B_0|^2) \ . \tag{5}$$

$|B_k|^2$ and $|B_0|^2$ can be interpreted as magnon numbers. Without damping and driving, i.e., $\omega_s = \eta = 0$, the magnon numbers are conserved, which is expected in a Hamiltonian system. In a steady state, the driving excites the system in such a way as just to compensate for the damping. Naturally, the right-hand side of Eq. (5) is zero if (3b) is used. This steady state becomes unstable when an increase (decrease) of total magnons makes the response of the system to the driving stronger (weaker). With this consideration and the fact that $\mathrm{Re}B_0 \sim 0$, we find that when $\omega_s > 4\eta|\mathrm{Im}B_0|$, the fixed point (3b) becomes unstable. Using the numbers given above, we have $\omega_{s,\text{critical}} = 1.518 \times 10^{-3}$. At that value the fixed point undergoes a Hopf bifurcation as indicated by Jeffries.[7] The frequency of the limit cycle according to our numerical calculation is $f = (1.087 \times 10^{-3})\omega_m$. In GJ's experiment, $4\pi m_s = 300$ G, so our f is equal to 5.543×10^6 Hz, one order of magnitude higher than the actually observed one. Note that in Fig. 1 the sizes of limit cycles increase significantly as the power increases and so does the fundamental frequency. A similar state of affairs was reported by Yamazaki[8] in a parallel pumping experiment. Figure 2 shows the period-3 window and its subsequent bifurcation to period 6, and a period-5

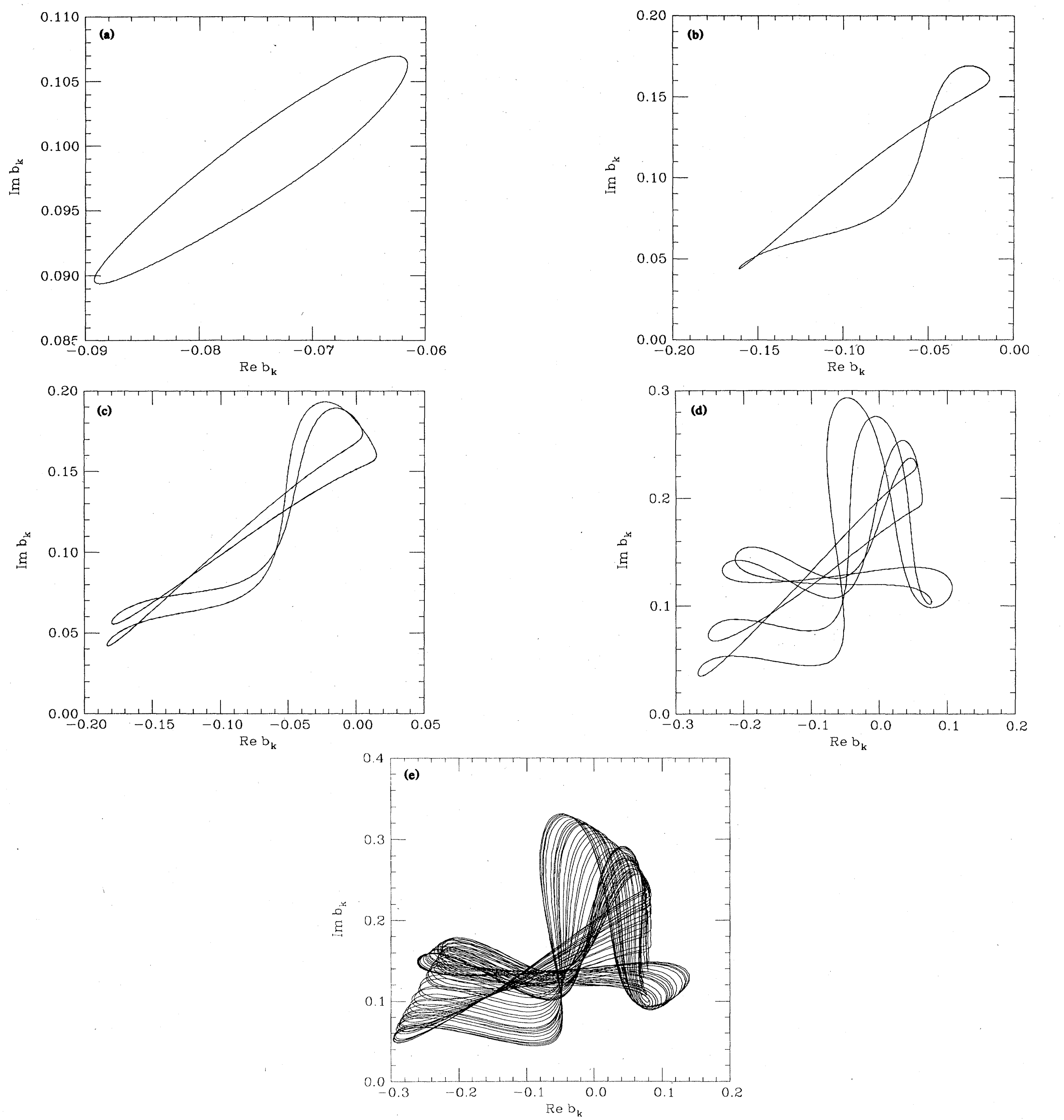

FIG. 1. (a) Projection of the first limit cycle on the Reb_k-Imb_k plane at $\omega_s = 0.00152$. The frequency $f = (1.087 \times 10^{-3})\omega_m$. (b) Distortion and expansion of (a) at $\omega_s = 0.0022$, $f = (1.47 \times 10^{-3})\omega_m$. (c) Period doubling at $\omega_s = 0.0026$, $f = (0.8196 \times 10^{-3})\omega_m$. This is more than half of (b) because the fundamental frequency increases as ω_s increases. (d) Period 4 at $\omega_s = 0.0045$, $f = (0.618 \times 10^{-3})\omega_m$. (e) Onset of chaos at $\omega_s = 0.0055$. The same period doubling and chaos in the n_0-n_k plane as well as all the corresponding power spectrums have been neglected.

window which does not bifurcate further. The symmetric pattern is due to the fact that changing the signs of both $\mathrm{Im}B_k$ and $\mathrm{Re}B_k$ leaves the equations unchanged. This causes the system to have more than one attractor. The route to chaos shown in Figs. 1 and 2 suggests a simple underlying iterative map. A set of two coupled logistic maps[9] would be a possible candidate.

In many experimental situations, surface imperfection scattering is important. The process involved is a two-magnon process[10] that conserves the total number of magnons. The argument following Eq. (5) shows that adding a term that conserves magnons will not change the picture

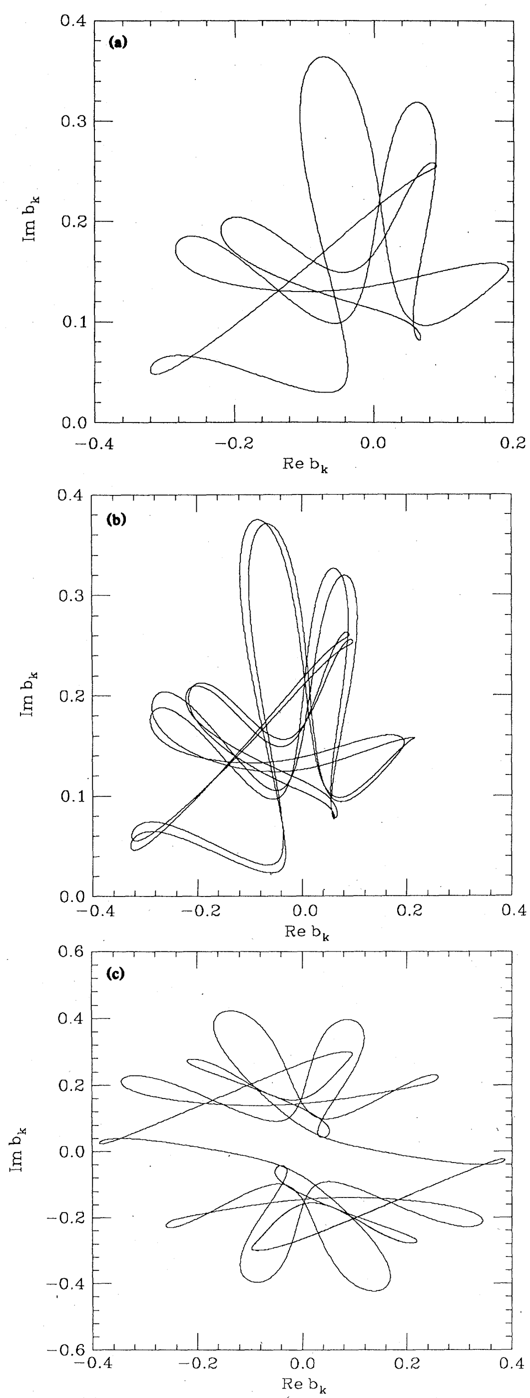

FIG. 2. (a) Period-3 window at $\omega_s=0.006\,25$, $f=(0.985\times10^{-3})\omega_m$. (b) Bifurcation to period 6 at $\omega_s=0.0065$, $f=(0.442\times10^{-3})\omega_m$. (c) Period-5 window at $\omega_s=0.009$, $f=(0.686\times10^{-3})\omega_m$. Pictures of chaotic behavior between the period-6 and period-5 windows have been neglected.

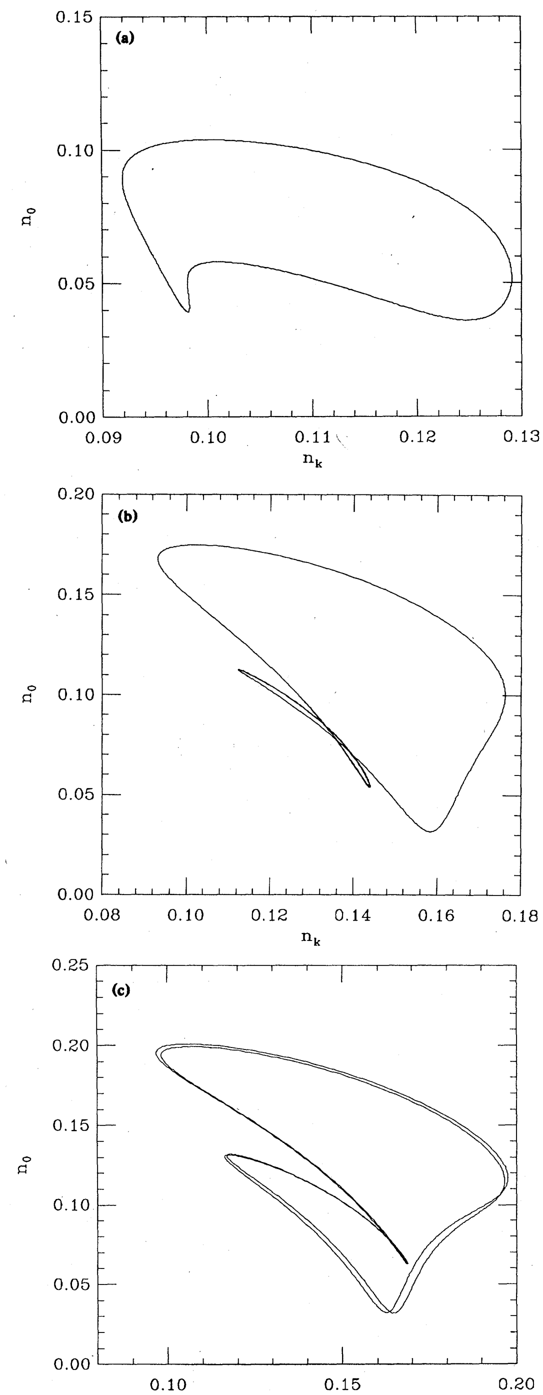

FIG. 3. (a) Projection of the limit cycle on the n_0-n_k plane at $\omega_s=0.001\,55$ in the case of adding a two-magnon scattering term, $f=(1.068\times10^{-3})\omega_m$. (b) Distortion and expansion of (a) at $\omega_s=0.0022$, $f=(1.24\times10^{-3})\omega_m$. (c) Period doubling at $\omega_s=0.0026$.

qualitatively, at least at the beginning of the instability. This is indeed true. The modified equations are[11]

$$\frac{dB_k}{d\tau} = iB_0^2 B_{-k}^* - \eta B_k + \gamma |B_0|^2 B_k + \nu_{k0} B_0 \ ,$$
$$\frac{dB_0}{d\tau} = iB_k^2 B_0^* - i\omega_s - \eta B_0 + \nu_{0k} B_k \ , \tag{6}$$

where only one spin wave has been retained in the scattering term also. Assume $\nu_{k0} = \nu_{0k} = 0.01$ [linewidth ~ 1 G (Ref. 7)]. Figure 3 shows the first few limit cycles. Their frequencies are lowered and positions shifted compared to those in Fig. 1.

We conclude with a comment on the remarkably slow decay of the excitation of 16-KHz oscillations observed by GJ when the rf power is turned off.

Although the Suhl threshold is lowest for one particular spin wave (**k** along $\hat{\mathbf{z}}$ and $\omega_k = \omega$) there are an infinite number of neighboring ones with arbitrary close thresholds. Nonetheless, as we have seen, restriction of the theory to only one such wave seems to simulate the observed bifurcation sequence quite well. This suggests that by some as yet unexplained mechanism, the affected waves concentrate into one or at most a very few waves. Suppose that a scattering mechanism is responsible for the bulk of the resonance linewidth at low signal powers, with only a small contribution coming from slow relaxation to the lattice. Then the total magnon number declines slowly; the linewidth is mainly due to the scattering from the uniform mode to the degenerate spin-wave manifold, with essentially infinite time required for backflow into the uniform mode. If for some reason there is at high signal power a sharp concentration of spin waves into a single one, then the excitation will rapidly alternate between that wave and the uniform mode, and the decay would be dominated by the weak coupling to the lattice. This would continue until the excitation level sinks below threshold; from then on the decay should be rapid.

Upon completion of most of this work, we learned of similar calculations with similar results.[12]

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