Surfaces, interfaces, and screening of fractal structures

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Fractal objects strongly screen external fields; only a small "surface" portion of the object is exposed appreciably to the field. We have studied this exposed surface of several random fractals as measured by random walkers and by ballistic particles launched from outside and absorbed by the fractal. The number of absorbing sites weighted by their rate of absorption shows an apparent power-law scaling with fractal mass. For diffusion-limited aggregates, ballistically generated aggregates, and screened-growth clusters in two dimensions, this power-law relationship is for the most part in accord with mean-field predictions of previous work. This accord is poorest for the objects of lowest fractal dimensionality. We have confirmed that this scaling is different from that of the old-growth—new-growth interface studied previously. We also find that a "hierarchy" of fractal dimensions describes the external surface of diffusion-limited aggregates.

I. INTRODUCTION

Considerable interest has recently developed in the formation of fractal-like structures under nonequilibrium conditions, motivated by the realization that fractal structures occur commonly in nature, and by the discovery of a variety of simple computer models for growth and aggregation that lead to the generation of fractal objects.[1–4] Initially, the fractal geometry itself and its relationship to the growth mechanism was the major area of interest. More recently, attention has been focused on questions concerning how the aggregates grow and the kinetics of the aggregation processes.[5] Another very active area is concerned with the question of how the familiar laws of physics and chemistry are modified on fractal substrates.[6]

Since many of the unique properties of fractals are concerned with the ways in which they interact through their surfaces with an outside environment, it is important to begin to develop a better understanding of the surface properties of fractal structures. Another reason for being interested in the surfaces of fractals is that the growth of fractal structures occurs at the surface. Consequently a better understanding of the surface properties of fractals and related quantities may lead to a better understanding of how such structures are formed.

Several steps have already been taken in this direction.[7–11] (i) Meakin and Witten[7] investigated the old-growth—new-growth interface in diffusion-limited aggregation[2] and determined how the mass of the interface grows with the mass of the old growth. (ii) Coniglio and Stanley[10] have developed the idea of an unscreened perimeter to describe now the effective-surface size of a fractal depends on its diameter. In their picture, probe particles move in the vicinity of a fractal cluster of M sites, and are absorbed when they happen to touch it. Because of screening, a relatively small number M_u of sites absorb the bulk of these probe particles. This number is a direct measure of the "exposed" or "unscreened" surface. Using a mean-field argument they predict that M_u scales as the radius R to a power D_u, with

$$D_u = (D_f - 1) + (d - D_f)/D_p \ . \quad (1)$$

Here D_f is the fractal dimension of the fractal, d is the Euclidean dimension of the lattice, and D_p is the fractal dimension of the trajectory followed by probe particles used to measure M_u. One of our main objectives here is to test this idea. (iii) Plischke and Racz[8] introduced the concept of an active zone (region in which growth is occurring) in diffusion-limited aggregation and in the (nonfractal) Eden growth process.[9] (iv) Grassberger[11] has analyzed the distribution of charge in a fractal object for $d = 2$. This problem is equivalent to the distribution of sites visited by a *random-walk* trajectory.

Equation (1) emphasizes the fact that the apparent surface of a fractal depends not only on its own fractal dimension but also on the fractal dimension of the probe trajectory used to measure it. Even after the fractal dimension of the probe trajectory has been taken into account, the number of sites exposed to the probes can be defined in various ways, given the broad distribution of absorption rates. Our approach in this work is to measure several moments of this distribution. These moments usually show a common scaling behavior, and the observed scaling powers are for the most part consistent with Eq. (1). The exceptions to this behavior have some common suggestive features, as described below.

II. THE OLD-GROWTH—NEW-GROWTH INTERFACE IN DIFFUSION-LIMITED AGGREGATION AND CLUSTERS GROWN USING A SCREENED-GROWTH MODEL

The methods used to generate diffusion-limited aggregation (DLA) clusters and to measure the size of the old-growth—new-growth interface have been presented earlier.[7,12] However, it should be noted that the mass of the old-growth—new-growth interface is the number of "new"-growth sites that are nearest neighbors to one or more "old"-growth sites in the limit in which the interface is partially saturated (i.e., in the regime in which a substantial number of new-growth particles have been added, so that a finite fraction of the old-growth—new-growth contacts have been made). In the present measurements of this interface, we used $d=2$ DLA clusters that are considerably larger than those used previously (25 000 sites versus 9400 sites), and required the interface to be almost completely saturated.

The screened-growth model is a surface-growth model in which the probability of growth at an unoccupied site adjacent to an already occupied site is determined by the independent multiplicative-screening effects of all of the occupied sites in the cluster.[13,14] For the ith interface site at position $\mathbf{r}_i$ the growth probability is given by

$$P_i = \prod_{k=1,N} e^{-A|\mathbf{r}_k - \mathbf{r}_i|^{-\epsilon}} , \quad (2)$$

where $\mathbf{r}_k$ is the position of the kth occupied lattice site and A is a constant that is set to a value of 1.0 in our simulations. This model leads to the formation of random structures with a fractal dimension D_f equal to the parameter ϵ in Eq. (2).[14,15]

The screened-growth clusters were generated on 1001×1001 square lattices. The growth was started at the center of the lattice and ended when the edges of the lattice were first reached ($D_f=1.25$, $\frac{4}{3}$, and 1.5) or the cluster had occupied 25 000 sites ($D_f=1.75$). For both the DLA and screened-growth models we find that the size or "mass" of the old-growth—new-growth interface M_i depends on the old-growth mass M according to the power-law relationship

$$M_i \sim M^{\delta} . \quad (3)$$

Table I shows the results obtained for the exponent δ. The error limits shown are only the contributions of statistical uncertainties (95% confidence limits); systematic errors may be larger than this. Seven clusters were used for each model shown in the table.

III. THE EXPOSED SURFACE OF FRACTAL AGGREGATES: PENETRATION OF PARTICLES INTO RANDOM STRUCTURES

Our approach to measuring the surface size of random structures is to probe the structure with particles following linear or random-walk trajectories. Each particle is started off at a random position outside the area occupied by the cluster and its trajectory is followed until it reaches an unoccupied surface site (an unoccupied site which is a nearest neighbor to an occupied site on the cluster). After each trajectory has been completed a record is kept of which unoccupied interface site is first contacted. A measure of the surface size can then be obtained based on the total number of times each unoccupied interface site has been "hit" after a large number of trajectories. Our definition of the surface size is based on the idea that a more penetrating probe will contact a large surface area and that this will result in a more uniform distribution of hit probabilities. For a less penetrating probe, relatively few surface sites will have a large probability of being contacted. Consequently, we have measured the "moments" μ_1, μ_2, and μ_3 defined by

$$\mu_1 = \left[\sum_i N_i\right]^2 \Big/ \sum_i N_i^2 = N_T^2 \Big/ \sum_i N_i^2 , \quad (4a)$$

$$\mu_2 = \left[N_T^3 \Big/ \sum_i N_i^3\right]^{1/2} , \quad (4b)$$

$$\mu_3 = \left[N_T^4 \Big/ \sum_i N_i^4\right]^{1/3} . \quad (4c)$$

Here N_i is the number of hits made on the ith site and N_T is the *total* number of probe particles ($\sum_i N_i$). We also calculated the *total* number of sites contacted μ.

TABLE I. Fractal dimensions and interface exponents of screened-growth and DLA models.

Model	Fractal dimension	Interface exponent
Screened growth	1.75	0.725±0.04
DLA	1.71	0.625±0.02
Screened growth	1.5	0.625±0.04
Screened growth	1.33	0.545±0.004
Screened growth	1.25	0.475±0.06

IV. SURFACE SIZE OF CLUSTERS GROWN BY DIFFUSION-LIMITED AGGREGATION

Clusters in the size range of 15 000–25 000 occupied sites were grown using a lattice model for DLA.[2,11] The growth process was stopped at 17 or 18 stages during the growth (the exact number depends on the final cluster size) and 25 000 particle trajectories were used to probe the surface of the aggregate. Similar simulations were also carried out in which the surface was probed with 100 000 on-lattice linear trajectories (trajectories with random impact parameters and with a direction randomly chosen from the four equally probable directions on the lattice). The results obtained from four simulations with random-walk trajectories and five simulations with linear trajectories are shown in Figs. 1 and 2, respectively. Besides the statistics defined in Eq. (4), the total number μ of sites hit is also plotted. These results suggest power-law relationships between the quantities μ_j and the cluster mass M

$$\mu_j \sim M^{\gamma_j} \quad (j=1,2,3) . \quad (5)$$

The values obtained for γ_j from five simulations carried out using random-walk trajectories, and from six simulations carried out using linear trajectories, are shown in Table II. As before, the uncertainties shown in Table II represent the contributions of statistical uncertainties only (95% confidence limits). The systematic uncertainties are probably considerably higher. In particular, it should be noted that for the case of linear (on-lattice) trajectories we would expect that the exponents γ_1, γ_2, and γ_3 should all

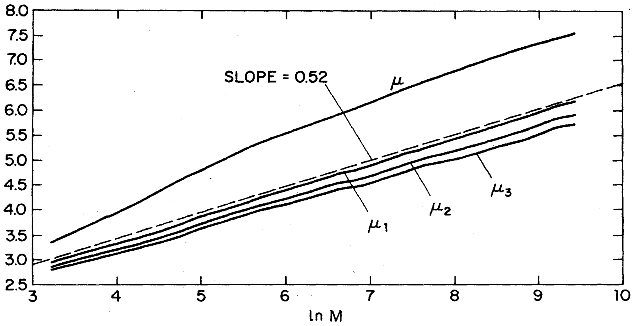

FIG. 1. Dependence of the surface sizes μ, μ_1, μ_2, and μ_3 for DLA clusters determined by 25 000 random walks on the total cluster mass M. μ is the total number of surface sites hit. The surface sizes μ_1, μ_2, and μ_3 are screened perimeter masses defined in the text.

have values equal to $1/D_f$ ($\cong 0.6$), where D_f is the fractal dimension associated with two-dimensional DLA. The results shown in Table II estimate that the systematic uncertainties are of the order of 10%. For the case of ballistic trajectories, the probability that a particular unoccupied interface site will be contacted can be found exactly. Each site at the surface of the fractal can be reached by zero, one, two, or three on-lattice ballistic trajectories. The exponents γ_1, γ_2, and γ_3 were determined in this way using five hundred 50 000-site 2D DLA clusters. For clusters in the size range $5000 \leq N \leq 50\,000$, we found $\gamma_1 = 0.634 \pm 0.004$, $\gamma_2 = 0.642 \pm 0.005$, and $\gamma_3 = 0.644 \pm 0.004$; for clusters in size range $1000 \leq N \leq 5000$, we found $\gamma_1 = 0.642 \pm 0.005$, $\gamma_2 = 0.647 \pm 0.005$, and $\gamma_3 = 0.650 \pm 0.005$. Finally, for clusters in the size range $200 \leq N \leq 1000$, we found $\gamma_1 = 0.653 \pm 0.010$, $\gamma_2 = 0.654 \pm 0.010$, and $\gamma_3 = 0.652 \pm 0.010$. These results are quite similar to those obtained using 100 000 trajectories and a much smaller number of clusters. However, the three exponents are now much more nearly equal and somewhat closer to the expected value of about 0.59. Also the trend towards smaller values for larger cluster size indicates that we are seeing the effects of finite-size corrections to the asymptotic scaling behavior. Thus we conclude that our results are consistent with the expectation that $\gamma_1 = \gamma_2 = \gamma_3 = 1/D \simeq 0.59$ in the limit of large cluster

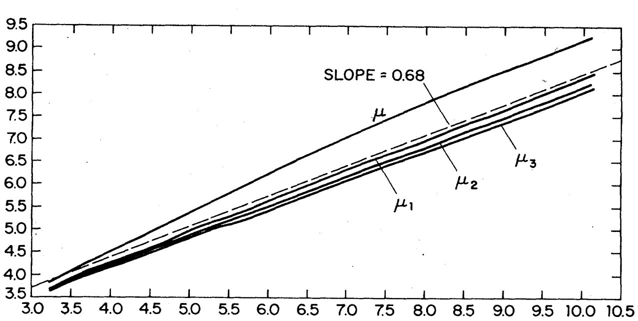

FIG. 2. This figure shows how the surface sizes μ, μ_1, μ_2, and μ_3 depend on the cluster size N for the penetration of particles following linear trajectories into DLA aggregates.

TABLE II. Surface-size exponents γ_j, obtained by probing the surface of Witten-Sander aggregates using random-walk and ballistic (linear) trajectories.

Surface-size exponent	Particle trajectory: Random walk	Linear
γ_1	0.521 ± 0.003	0.677 ± 0.016
γ_2	0.491 ± 0.001	0.660 ± 0.019
γ_3	0.473 ± 0.004	0.646 ± 0.019

sizes. The random-walk exponents are all approximately $\frac{1}{2}$; this is the prediction D_u/D_f of the mean-field argument, Eq. (1).

V. SURFACE SIZE OF SCREENED-GROWTH CLUSTERS

The surface size of clusters generated with the screened-growth model can be defined using the growth probabilities of the surface sites in a similar way to the definition of the surface size obtained by probing DLA clusters by particles following random-walk trajectories. By analogy with Eq. (4) we have

$$\mu_1 = 1 \Big/ \sum_i P_i^2 , \tag{6a}$$

$$\mu_2 = \left[1 \Big/ \sum_i P_i^3 \right]^{1/2} , \tag{6b}$$

$$\mu_3 = \left[1 \Big/ \sum_i P_i^4 \right]^{1/3} , \tag{6c}$$

where P_i is the growth probability for the ith unoccupied surface site. In this case the P_i are known exactly from Eq. (2). It should be noted that the growth probabilities used in Eq. (5) are normalized such that $\sum_i P_i = 1$.

For the case $D_f = \epsilon = 1.25$ the effective exponent γ_1 obtained from the growth probabilities is 0.155 ± 0.045 for clusters that are 1%–100% complete, 0.157 ± 0.064 for clusters that are 5%–100% complete, and 0.109 ± 0.131 for clusters that are 10%–100% complete. Similarly for $D_f = \epsilon = 1.50$; we find $\gamma_1 = 0.404 \pm 0.056$ (1%–100% complete), $\gamma_1 = 0.397 \pm 0.086$ (5%–100% complete), and $\gamma_1 = 0.392 \pm 0.100$ (10%–100% complete). For $D_f = \epsilon = 1.75$; we find $\gamma_1 = 0.562 \pm 0.046$ for clusters 1%–100% complete, $\gamma_1 = 0.576 \pm 0.053$ for clusters 10%–100% complete, and $\gamma_1 = 0.768 \pm 0.065$ for clusters

TABLE III. Surface-mass exponents obtained by probing screened-growth clusters with particles following linear trajectories. Six clusters were generated using each model to obtain these results and 100 000 trajectories were employed.

Surface-size exponent	Fractal dimension: 1.50	1.75
γ_1	0.73 ± 0.05	0.65 ± 0.01
γ_2	0.71 ± 0.05	0.63 ± 0.01
γ_3	0.70 ± 0.05	0.62 ± 0.02

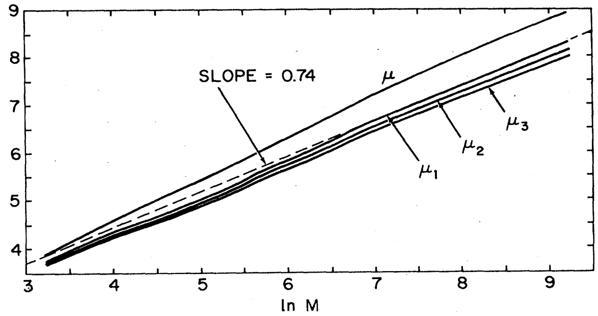

FIG. 3. Surface sizes found by using particles with a linear trajectory to probe screened-growth clusters with a fractal dimension of 1.5; μ is the total number of unoccupied sites "hit" by the trajectories and the unscreened surface lengths μ_1, μ_2, and μ_3 are defined in the text.

50%—100% complete (i.e., clusters in the size range $12\,500 \leq N \leq 25\,000$ occupied lattice sites). To obtain these results 14 clusters were grown with a fractal dimension of 1.25, seven with a fractal dimension of 1.50, and seven with a fractal dimension of 1.75.

The surfaces of clusters generated using the screened-growth model were also examined using particles following random-walk and ballistic trajectories. Table III shows some of the results obtained by examining screened-growth clusters with fractal dimensions of 1.75 and 1.5 using linear fractal trajectories. The surface-mass exponents γ_1, γ_2, and γ_3 are somewhat larger than expected. Assuming that $\gamma_j = 1/D_f$ (0.66 for $D_f = 1.50$ and 0.57 for $D_f = 1.75$). In view of the similar results obtained for DLA clusters (see above) we attribute these deviations to both finite-size effects and the fact that 100 000 trajectories would not be sufficient to determine γ_1, γ_2, and γ_3 accurately. The effects of a finite number of trajectories is expected to be smaller for random walk than for ballistic trajectories. The reason for this is that the most frequently contacted sites contribute most to the moments of the contact probability distribution used to obtain the exponents γ_j and the contact probabilities for the most frequently contacted sites are determined the most accurately. Figure 3 shows the results obtained for the penetration of linear trajectories into screened-growth clusters having a fractal dimension of 1.50.

Similar simulations were also carried out using particles following random-walk trajectories. The results obtained using 25 000 trajectories at various stages during the growth of clusters with a fractal dimensions of 1.25, 1.50, and 1.75 are shown in Table IV. Here again the measured

TABLE IV. Surface-mass exponents γ_j describing how the surface size increases with cluster mass, obtained by probing the surfaces of clusters generated using the screened-growth model. For each value of the fractal dimension, six clusters were generated and each cluster was probed using 25 000 particles following random-walk trajectories. In most cases the measurements have recently been checked by increasing the number of trajectories tenfold to 250 000. The resulting exponents agree with those shown here, to within the indicated uncertainty.

Surface-size	Fractal dimension D_f		
exponent	1.25	1.50	1.75
γ_1	0.66 ± 0.03	0.57 ± 0.01	0.51 ± 0.01
γ_2	0.61 ± 0.02	0.53 ± 0.01	0.48 ± 0.01
γ_3	0.58 ± 0.02	0.51 ± 0.02	0.47 ± 0.01

exponents are about 0.5, in agreement with the mean-field prediction, except for the case of smallest D_f; viz., $D_f = 1.25$.

VI. THE SURFACE OF BALLISTIC AGGREGATES

Ballistic aggregates were generated using a modified version of the Vold-Sutherland model[16,17] in which both the particles and the aggregate are confined to a square lattice.[18] The growth process was stopped at various stages when the aggregate had reached a range of sizes from 25—200 000 occupied sites. This model leads to structures that are not fractals[18] but that may have a fractal surface. The surface of the aggregates was probed using either 25 000 or 50 000 particles following random-walk trajectories, or 100 000 particles following linear trajectories. Table V shows some of the results obtained from six clusters that were probed using linear trajectories. The surface-size exponents γ_j are close to but somewhat larger than the expected value of 0.5. Comparable results obtained using random-walk trajectories are shown in Table VI; similar results were obtained using either 25 000 or 50 000 probe particles. Figure 4 shows how the surface sizes μ, μ_1, μ_2, and μ_3 depend on the cluster size for the penetration of particles following random-walk trajectories into ballistically generated aggregates.

VII. DISCUSSION

Our simulation results indicate that the surface-size exponents γ_j ($j = 1-3$), which describe how the surface size measured by random-walk trajectories depends on cluster mass, are *insensitive to the fractal dimension of the structure* for a number of $d = 2$ systems with fractal dimensions in the range 1.5—2.0. This result is in good agree-

TABLE V. Surface-size exponents (γ_1, γ_2, and γ_3) for the penetration of ballistic particles in ballistically generated aggregates. Clusters in the size range $N_1 \leq N \leq N_2$ were used.

Exponent	$N_1 = 25$ $N_2 = 200\,000$	$N_1 = 300$ $N_2 = 200\,000$	$N_1 = 3000$ $N_2 = 200\,000$	$N_1 = 20\,000$ $N_2 = 200\,000$
γ_1	0.531 ± 0.005	0.523 ± 0.004	0.513 ± 0.005	0.507 ± 0.007
γ_2	0.531 ± 0.004	0.522 ± 0.004	0.513 ± 0.005	0.506 ± 0.007
γ_3	0.530 ± 0.004	0.521 ± 0.004	0.511 ± 0.005	0.503 ± 0.007

TABLE VI. Surface-mass exponents (γ_1, γ_2, and γ_3) for the penetration of particles following random walks into aggregates grown using linear trajectories. Aggregates in the size range $N_1 < N < N_2$ were used.

Exponent	$N_1=20$ $N_2=100\,000$	$N_1=100$ $N_2=100\,000$	$N_1=700$ $N_2=100\,000$	$N_1=20$ $N_2=700$
γ_1	0.478±0.006	0.479±0.003	0.480±0.005	0.475±0.020
γ_2	0.462±0.005	0.464±0.003	0.469±0.006	0.457±0.015
γ_3	0.450±0.004	0.454±0.004	0.461±0.007	0.444±0.019

ment with the mean-field estimate of Coniglio and Stanley,[10] Eq. (1). According to Eq. (1), the dimension of the unscreened interface D_u is equal to $D_f/2$ for the case $d=2$ and $D_p=2$. This means that the exponent describing how the mass of the unscreened perimeter increases with increasing cluster size will have a value of 0.5 for the penetration of particles following random-walk trajectories into any fractal embedded in a two-dimensional space or lattice. This result is in good agreement with the results given in Tables II, IV, and VI which indicate that $\gamma_j \cong 0.5$ for all of the structures with $D_f \geq 1.5$ which we have examined. The case of the screened-growth cluster with $D_f = 1.25$ is an obvious exception to this pattern. Its measured γ values differ from both the mean-field prediction and from one another. Since the three measured γ's are not equal, we suspect that the asymptotic regime has not been reached in this lowest-dimension case. The factors governing the approach to the asymptotic regime clearly require more study.

For the case $D_p=1$ (ballistic trajectories), Eq. (1) and simple geometric arguments both lead to the conclusion that $\gamma_j = 1/D_f$ for all n. In this case our simulation results are not in such good agreement with these theoretical predictions. However, our more extensive studies using large numbers of DLA clusters indicate that this is mainly a result of finite-size corrections and that our results are consistent with the idea that $\gamma_j = 1/D_f$ in the limit $j \to \infty$. For some cases simulations were carried out with a smaller number of particle trajectories and similar results were obtained. It is possible that a *very* much larger number of trajectories is needed. However, the results obtained for DLA clusters using exact contact probabilities (equivalent to an infinite number of trajectories)

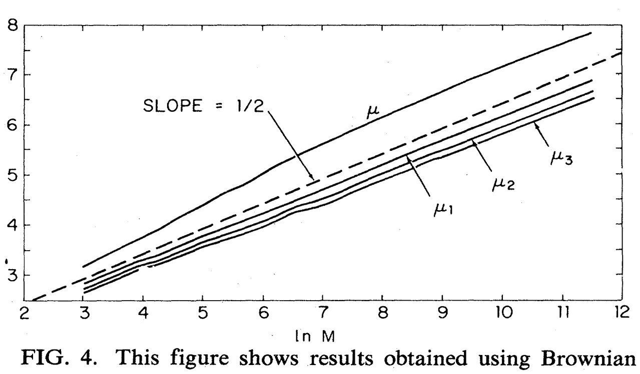

FIG. 4. This figure shows results obtained using Brownian probes to measure μ, μ_1, μ_2, and μ_3 for lattice $2d$ ballistic aggregates.

indicate that the uncertainties introduced by using only 100 000 trajectories are relatively small.

These simulations were carried out using the largest DLA, screened-growth and ballistic aggregation clusters that could be grown at the time the work was carried out. Very recently[19] a new algorithm for growing DLA clusters has been developed that will enable us to generate many ($\cong 10^3$), large ($\cong 10^5$), particle clusters. It is hoped that his will enable us to test Eq. (1) more rigorously. These algorithms were in fact used to obtain the more extensive results for DLA clusters with ballistic trajectories reported above.

In our earlier work on the size of the old-growth—new-growth interface we found[7] that the exponent δ that describes how the interface mass M_i depends on the old-growth mass M ($M_i \sim M^\delta$) had a value of 0.603 ± 0.021 (approximately equal to the value $1/D_f$, which might be expected from a geometrical argument). Our new results for DLA ($\delta = 0.625 \pm 0.02$) are in quite good agreement with our earlier results. The results obtained in Table I from the screened-growth model clearly demonstrate that δ is not even approximately equal to $1/D_f$ in this case.

The results of this exploratory study raise many questions which are important for future work to resolve. The first of these questions is the appearance of two scaling powers, γ and δ, describing the exposed or "unscreened" surface of a fractal. A third power also appears implicitly in this work. This is the dependence of the total number μ of sites hit on the mass of the cluster and on the number of trajectories. To understand these different power laws on a common basis requires a knowledge of the distribution of absorption rates over the fractal. This distribution should be measured directly. This distribution is of theoretical interest as well, since it may represent[11] a "fractal measure," exhibiting an indefinite number of scaling powers. Indeed, we have calculated the higher moments μ_j and their corresponding exponents γ_j for a sequence of values of j up to $j=8$. We find an entire hierarchy of fractal dimensions, monotonically depending on j, and, apparently, tending toward a limiting value as $j \to \infty$. This result is not altogether surprising, since we know that there is *some* dependence upon j: For two extreme values of j, $j=-1$, and $j=\infty$, the exponents differ by more than a factor of 2—with $\gamma(-1)=1$ and $\gamma(\infty)=1-1/D_f$. The first result follows immediately from the definition (6) and the fact that the total surface in DLA scales with exponent D_f just like the total mass. The second follows from the recent theorem (F. Leyvraz, private communication) that $P_{\max}$ (the maximum value of all the P_i) scales with cluster mass to the exponent $(1-1/D_f)$. This prediction is confirmed by our calculations for $j=1,8$: our hierarchy or "spectrum" of surface-fractal dimensions, apparently tending toward the value $1-1/D_f$ as $j\rightarrow\infty$. Why, then, does the mean-field approach of Coniglio and Stanley[10] give but a single surface exponent? Presumably because it does not distinguish the different values of the P_i. Rather, the picture in Ref. 10 is that $P_i=0$ for the "screened" sites well inside the invaginations, while $P_i=P_{\max}$ for the "unscreened" sites that are exposed on the tips. Let us think of the DLA model of dielectric breakdown,[20] so that the cluster is grounded and every perimeter site is at some potential not equal to zero due to some external electric field. Then the perimeter sites near the unscreened tips are much "hotter" than in the deeply invaginated region of the perimeter. However, there is a continuum of "temperatures", not just *two* temperatures ("hot" and "cold"). An analogous situation arises in the voltage distribution that arises when a current is passed through a percolation cluster near the percolation threshold. The singly connected, or "red," bonds carry all the current and hence have the unit voltage drop across each. The remaining multiply-connected, or "blue," bonds are less "hot" as they form large blobs that share the current. Recently, the voltage distribution of the incipient infinite cluster was considered, and an infinitely hierarchy of exponents was found.[21] We believe that our result for the surface case is entirely analogous. Our finding is also somewhat reminiscent of the hierarchy of fractal dimensions that has very recently been found for the harmonic measure.[22] In fact, our exponents γ_j can be used to obtain the fractal dimensions D_j that describe the DLA surface. Substituting the definition $M\sim L^{D_f}$ into Eq. (5), we find $D_j=\gamma_j D_f$. A deeper understanding of the unifying features of these observations is being sought at the present time.

Our notions of exposed surface should apply equally to any fractal object. It would thus be valuable to extend the present studies to regular, hierarchical fractals and to "equilibrium" fractals such as self-avoiding walks and percolation clusters.

Note added: After this work was submitted for publication, a method was proposed for actually calculating the self-screening of an aggregate.[23]

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