Bandwidth limits due to incoherent soliton interaction in optical-fiber communication systems

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The problem of soliton interaction in optical-fiber communication systems is investigated analytically with the assumption of incoherent soliton interaction, i.e., interaction through nonlinear intensity overlap only. This situation is found to lead to attraction forces between consecutive pulses that are weaker than those in previously investigated coherent cases. The subsequent maximum system bandwidth is correspondingly improved.

I. INTRODUCTION

The initial enthusiasm for optical communication systems based on soliton pulses has gradually waned because of fiber loss[1,2] and soliton interactions.[2,3] These tend to reduce the initial, very high, bit-rate estimates to values of the same order of magnitude as those for designs depending on zero group dispersion.[3]

Damping due to fiber loss leads to a slow but exponential increase in the soliton pulse width, which decreases the maximum possible bit rate of the system. However, it has recently been shown[4,5] that the exponential soliton pulse broadening is asymptotically replaced by a linear dispersive broadening, but with a smaller effective dispersion than the purely linear case. Furthermore, the trend towards larger wavelengths and successively lower loss could make the problem of fiber loss less severe in future soliton communication systems.

The combined field of two neighboring solitons creates an attractive potential between the pulses. The resulting pulse interaction makes the pulses oscillate around their center of mass during propagation and periodically change place with a total coalescence when the pulses meet twice during every period of oscillation. In order to keep the distance of coalescence longer than the system length, consecutive pulses must be spaced sufficiently well apart. This, however, degrades the maximum bandwidth of the optical transmission system.

Recently, the dynamics of soliton interactions has been studied numerically in several papers[2,3] and a perturbative analysis based on the inverse scattering technique has also been given.[6] However, these studies all assume perfect coherence between pulses, regarding the phases of the pulses as well as their polarization properties.

If, on the other hand, the pulse interaction occurs "incoherently" in the sense that the interaction is determined by the *intensity overlap* only, we would expect a much weaker mutual influence of closely spaced soliton pulses. Consequently, the maximum bandwidth of the corresponding soliton communication system would be expected to be larger than recent rather pessimistic estimates for the "coherent" case.[3]

In the present work we investigate the problem of soliton interaction through (incoherent) intensity overlap between pulses. Actually, this problem corresponds to the nonlinear self-interaction and mutual coupling of two circularly polarized laser pulses with opposite polarization.[7,8] Nevertheless, several factors in a realistic fiber system tend to make the interaction "incoherent" even if the polarization of the pulses at the laser source is linear in the same direction. Such factors are, e.g., the inherent depolarization properties of single-mode fibers,[9] the finite coherence time of the light source and higher-order dispersive effects.

In particular we have also investigated the effect of third-order linear dispersion on the incoherent soliton interaction. The results indicate the possibility of a weakening of the interaction strength between the solitons, in qualitative accordance with recent numerical results for the coherent case.[10]

Our analysis of the two-soliton interaction problem is based on a variational approach, which has recently been shown to be very useful in studies of one-soliton propagation problems.[5,11,12] The main result of our analysis is a second-order nonlinear oscillator equation for the distance between the interacting solitons. This equation can readily be integrated once to yield a potential function formulation, which gives a suggestive physical picture of the oscillatory soliton interaction. An approximate evaluation of the integral determining the distance of coalescence still implies exponential scaling with initial soliton separation. However, the increase is slower than in the case of coherent interaction. Finally, a comparison with previous estimates for the maximum system bandwidth indicates a significant, although not dramatic, improvement.

The influence of third-order dispersion is also investigated and is shown to lead to an increased distance of coalescence for soliton pulses which are initially "at rest" in the coordinate system moving with the pulse group velocity.

II. VARIATIONAL DESCRIPTION OF COUPLED SIMULTANEOUS NONLINEAR SCHRÖDINGER EQUATIONS

In conventional normalized units, the nonlinear Schrödinger equation for the optical wave field, ψ, can be taken as[1–12]

$$i\frac{\partial\psi}{\partial x}+\frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2}+|\psi|^2\psi=0\ , \qquad (1)$$

where x is the normalized distance of propagation and τ is the normalized reduced time.

For the wave field ψ of two well-separated solitons we can write $\psi=\psi_1+\psi_2$ and assuming mutual coupling only through intensity overlap in the $|\psi|^2$ term we find the following coupled equations, cf. Refs. 6–8:

$$\begin{aligned}&i\frac{\partial\psi_1}{\partial x}+\frac{1}{2}\frac{\partial^2\psi_1}{\partial\tau^2}+(|\psi_1|^2+|\psi_2|^2)\psi_1=0\ ,\\&i\frac{\partial\psi_2}{\partial x}+\frac{1}{2}\frac{\partial^2\psi_2}{\partial\tau^2}+(|\psi_1|^2+|\psi_2|^2)\psi_2=0\ .\end{aligned} \qquad (2)$$

Equation (2) is the correct set of coupled nonlinear Schrödinger equations describing mutual interaction in addition to nonlinear self-interaction of oppositely circularly polarized laser waves,[7,8] where the vector character of the waves assures the vanishing of the phase coupling, which plays such a fundamental role in the perturbative analysis of the coherent two-soliton interaction as presented in Ref. 6.

In the absence of mutual coupling, Eq. (2) allows the well-known soliton solutions, i.e.,

$$\psi_j=2\nu_j\text{sech}[2\nu_j(\tau-\xi_j)]\exp[i\,2\mu_j(\tau-\xi_j)+i\delta_j]\ , \qquad (3)$$

where ν_j and μ_j are arbitrary constants which determine ξ_j and δ_j as $\xi_j=2\mu_j x$ and $\delta_j=2(\nu_j^2+\mu_j^2)x$.

The weak mutual interaction of the two solitons will lead to changes in the soliton propagation characteristics. Following the perturbative procedure of Refs. 6 and 11, we assume that the pulses preserve their soliton character, as described by Eq. (3), but that all parameters $(\nu_j,\mu_j,\xi_j,\delta_j)$ are allowed to vary with distance of propagation as a result of the interaction.

Instead of the approach of Ref. 6, which was based on inverse scattering technique, we will here use a variational approach and a Ritz optimization procedure to determine the parameter functions. Since this approach is analogous to the approach used in our previous works (see, e.g., Refs. 5, 11, and 12), we only give the main steps in the derivation.

We begin by expressing the coupled nonlinear Schrödinger equations, Eq. (2), as a variational problem in terms of the Langrangian L given by

$$L=L_1+L_2+L_{12}\ , \qquad (4)$$

where

$$\begin{aligned}&L_j=\frac{i}{2}\left[\psi_j\frac{\partial\psi_j^*}{\partial x}-\psi_j^*\frac{\partial\psi_j}{\partial x}\right]+\frac{1}{2}\left|\frac{\partial\psi_j}{\partial\tau}\right|^2-\tfrac{1}{2}|\psi_j|^4\ ,\\&L_{12}=-|\psi_1|^2|\psi_2|^2\ .\end{aligned} \qquad (5)$$

This implies that Eq. (2) is obtained from the variational principle

$$\delta\int\int L\left[\psi_j,\frac{\partial\psi_j}{\partial x},\frac{\partial\psi_j}{\partial\tau},\cdots\right]dx\,d\tau=0\ . \qquad (6)$$

The variation of the characteristic soliton parameters can be determined by inserting the ansatz, Eq. (3), into Eq. (6) and performing the integration over τ. This yields the reduced variational problem:

$$\delta\int\langle L\rangle dx=0\ . \qquad (7)$$

The averaged Langrangian $\langle L\rangle$ is

$$\langle L\rangle=\sum_{j=1,2}\left[4\nu_j\left[-2\mu_j\frac{d\xi_j}{dx}+\frac{d\delta_j}{dx}\right]+8\nu_j\left[-\frac{\nu_j^2}{3}+\mu_j^2\right]\right]-32\nu_1^2\nu_2^2\int_{-\infty}^{+\infty}\text{sech}^2z_1\text{sech}^2z_2dx\equiv\sum_{j=1,2}\mathcal{L}_j+\mathcal{L}_{12}\ , \qquad (8)$$

where $z_j=2\nu_j(\tau-\xi_j)$.

III. VARIATIONAL EQUATIONS

We can now proceed to derive the variational equations with respect to the parameter functions for the reduced variational principle Eq. (7) with $\langle L\rangle$ given by Eq. (8). Since $\mathcal{L}_{12}=\mathcal{L}_{12}(\nu_j,\xi_j)$ we obtain

$$\frac{\delta\langle L\rangle}{\delta\delta_j}=0\rightarrow\frac{d\nu_j}{dx}=0\ , \qquad (9a)$$

$$\frac{\delta\langle L\rangle}{\delta\xi_j}=0\rightarrow\frac{d}{dx}(-2\nu_j\mu_j)=\frac{\partial\mathcal{L}_{12}}{\partial\xi_j}\ , \qquad (9b)$$

$$\frac{\delta\langle L\rangle}{\delta\mu_j}=0\rightarrow-2\nu_j\frac{d\xi_j}{dx}+4\nu_j\mu_j=0\ , \qquad (9c)$$

$$\frac{\delta\langle L\rangle}{\delta\nu_j}=0\rightarrow-\left[2\mu_j\frac{d\xi_j}{dx}-\frac{d\delta_j}{dx}\right]+(-2\nu_j^2+2\mu_j^2)+\frac{\partial\mathcal{L}_{12}}{\partial\nu_j}=0\ . \qquad (9d)$$

We note that in the absence of mutual interaction Eqs. (9a)—(9d) reduce to the correct one-soliton relations for the respective solitons, i.e., ν_j and μ_j are constants, $\xi_j = 2\mu_j x$ and $\delta_j = 2(\nu_j^2 + \mu_j^2)x$.

However, the coupling of the solitons will lead to important modifications of the single soliton relations, although from Eq. (9a) we infer that ν_j is still a constant of motion. Equation (9d) can be regarded as an equation determining the phases δ_j once the other parameter functions are known. From Eq. (9c) we have

$$\frac{d\xi_j}{dx} = 2\mu_j \; , \tag{10}$$

which together with Eq. (9b), viz.,

$$\frac{d\mu_j}{dx} = -\frac{1}{2\nu_j}\frac{\partial \mathscr{L}_{12}}{\partial \xi_j} = \frac{\partial \mathscr{L}_{12}}{\partial z_j} \tag{11}$$

implies that we can derive a second-order equation for the distance between the solitons, i.e., $\Delta = \xi_1 - \xi_2 > 0$

$$\frac{d^2\Delta}{dx^2} = 2\frac{d}{dx}(\mu_1 - \mu_2) = 2\left[\frac{\partial \mathscr{L}_{12}}{\partial z_1} - \frac{\partial \mathscr{L}_{12}}{\partial z_2}\right] . \tag{12}$$

IV. POTENTIAL FUNCTION FORMULATION

Equation (12) can be made self-contained if we consider the same limit as in Ref. 6, i.e., two almost equal and widely spaced solitons. This implies that we can approximate Eq. (12) as

$$\frac{d^2y}{dx^2} \simeq -2(16\nu^2)^2 y e^{-y} \; , \tag{13}$$

where $y = 4\nu\Delta$ and $\nu = (\nu_1 + \nu_2)/2$.

Assuming the solitons to be initially at rest, i.e.,

$$\frac{dy}{dx}(0) = 0 \quad \text{at} \quad y(0) = y_0 \; , \tag{14}$$

Eq. (13) can be integrated once to yield a suggestive potential function formulation

$$\frac{1}{2}\left[\frac{dy}{dx}\right]^2 + \pi(y) = 0 \; . \tag{15}$$

The potential function, $\pi(y)$, is given by

$$\pi(y) = A^2[(y_0+1)e^{-y_0} - (y+1)e^{-y}] \; , \tag{16}$$

where $A = 16\sqrt{2}\nu^2$. A qualitative plot of $\pi(y)$ is given in Fig. 1. A mechanical analogy with a particle moving in the potential field $\pi(y)$ suggests that the particle, originally at rest at the point $(y_0, 0)$ in the phase plane, will move down the potential slope towards $y = 0$. At the point of coalescence ($y = 0$), the particle will meet the other "soliton particle" moving symmetrically but in the opposite direction. The particles will then periodically change places and the period of oscillation L_p is defined by

$$L_p = \sqrt{2}\int_0^{y_0} \frac{dy}{\sqrt{-\pi(y)}} \; . \tag{17}$$

Expanding the potential function around its zero point y_0 we find to lowest order

$$L_p \simeq \frac{1}{2(2\nu)^2}\exp(y_0/2) \; . \tag{18}$$

Keeping one more term in the expansion around y_0 we find the more complicated expression

$$L_p \simeq \frac{1}{2\sqrt{2}(2\nu)^2}\frac{e^{y_0/2}}{\sqrt{y_0}}\left[\frac{y_0+1}{y_0-1}\right]^{1/2} \times \ln\left[\frac{y_0^2}{y_0-1} + y_0\left[\frac{y_0+1}{y_0-1}\right]^{1/2}\right] . \tag{19}$$

V. BANDWIDTH LIMITS

A good semiempirical fit for L_p given by Eq. (19) and valid in the range $\nu\Delta(0) \leq 10$ is

$$L_p \simeq 0.4\frac{1}{(2\nu)^2}\exp(y_0/2) \; . \tag{20}$$

The expressions for the distance of coalescence as given by Eqs. (18) and (19) should be compared with the oscillation period in the case of coherent interactions,[6] viz.,

$$L_p \simeq \frac{\pi}{2(2\nu)^2}\exp\left[\frac{y_0}{4}\right] . \tag{21}$$

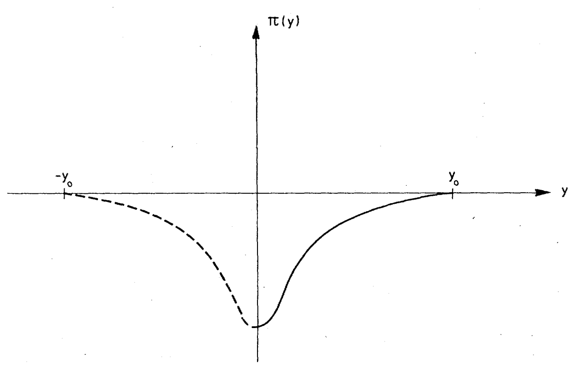

FIG. 1. Qualitative plot of the potential function.

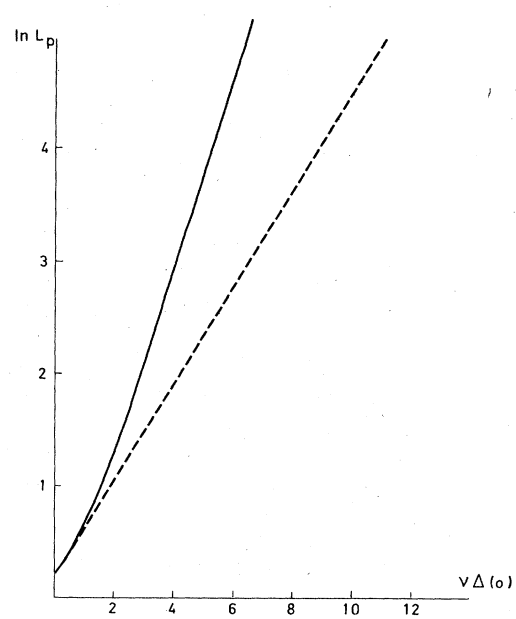

FIG. 2. Period of oscillation as function of normalized initial separation for coherent (— — —) and incoherent (——) interactions.

The weaker interaction in the incoherent case makes the oscillation period increase much more rapidly than in the case of coherent interactions, cf. Fig. 2.

This will have important consequences for the maximum system bandwidth. If we require that the normalized system length L_s should be some fraction α of the distance of coalescence L_c ($L_c \simeq L_p/2$, for initially well-spaced solitons), the maximum normalized bandwidth B_0 is given by[2]

$$B_0 = \frac{1}{1+\Delta_c} , \tag{22}$$

where

$$\Delta_c = \begin{cases} \ln \dfrac{4L_s}{\alpha\pi} & \text{coherent} \\ \frac{1}{2}\ln \dfrac{L_s}{0.2\alpha} & \text{incoherent} . \end{cases} \tag{23}$$

In Fig. 3 we have compared the maximum system bandwidth for coherent and incoherent interactions showing a significant improvement for the incoherent case.

VI. INFLUENCE OF THIRD-ORDER DISPERSION

Several higher-order effects are likely to be of importance for the interaction dynamics. In Ref. 2 damping was shown to enhance the mutual interaction in the sense that the distance of coalescence decreased. On the other hand, Ref. 10 numerically investigated the effect of third-order dispersion and found that it tended to weaken

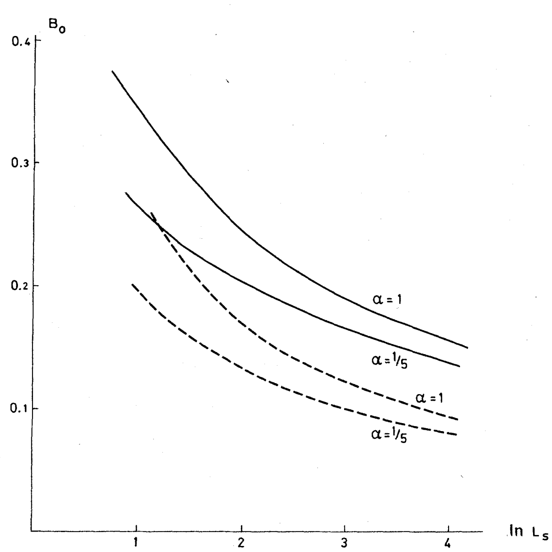

FIG. 3. Normalized maximum bandwidth B_0 as function of system length for two values of α($\alpha=1$ and $\alpha=\frac{1}{5}$) for coherent (— — —) and incoherent (——) interactions.

the mutual interaction even to the point of reversing the sign of the interaction force, making it repulsive rather than attractive.

We will here concentrate on the effects of third-order dispersion on the two-soliton incoherent interaction. The generalized nonlinear Schrödinger becomes, cf. Ref. 10

$$i\frac{\partial\psi}{\partial x} + \frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2} + |\psi|^2\psi - i\beta\frac{\partial^3\psi}{\partial\tau^3} = 0 , \tag{24}$$

where β denotes normalized third-order dispersion.

This dispersive term gives rise to additional terms ΔL_j in the Lagrangians L_j, viz.,

$$\Delta L_j = -\frac{i\beta}{2}\left[\frac{\partial\psi_j^*}{\partial\tau}\frac{\partial^2\psi_j}{\partial\tau^2} - \frac{\partial\psi_j}{\partial\tau}\frac{\partial^2\psi_j^*}{\partial\tau^2}\right] , \tag{25}$$

with the following change in the averaged Lagrangian

$$\Delta\mathscr{L}_j = 32\beta\mu_j\nu_j(\nu_j^2+\mu_j^2) . \tag{26}$$

Consequently, only the variational equations for ν_j and μ_j are changed, in particular, cf. Eq. (9c),

$$-2\nu_j\frac{d\xi_j}{dx} + 4\nu_j\mu_j + 8\beta\nu_j(\nu_j^2+3\mu_j^2) = 0 . \tag{27}$$

Thus we obtain for Δ [$\mu = (\mu_1+\mu_2)/2$],

$$\frac{d\Delta}{dx} = 2(\mu_1-\mu_2)(1+12\beta\mu) + 4\beta(\nu_1^2-\nu_2^2) . \tag{28}$$

Since

$$\frac{\partial\mathscr{L}_{12}}{\partial z_1} = -\frac{\partial\mathscr{L}_{12}}{\partial z_2} , \tag{29}$$

we infer, cf. Eq. (9b), that μ is a constant of the motion and consequently we obtain, cf. Eq. (13)

$$\frac{d^2y}{dx^2} = -2(1+12\beta\mu)(16\nu^2)^2 y e^{-y} . \tag{30}$$

This again (assuming the solitons to be initially at rest) implies the potential formulation given by Eqs. (15) and (16), however, with

$$A^2 \to A_{\text{eff}}^2 = A^2(1+12\beta\mu) . \tag{31}$$

The condition that the solitons initially are at rest requires, cf. Eq. (27)

$$\mu_j + 2\beta(\nu_j^2+3\mu_j^2) = 0 , \tag{32}$$

which perturbatively yields

$$\mu_j \simeq -2\beta\nu_j^2 \tag{33}$$

and consequently,

$$A_{\text{eff}}^2 \simeq A^2[1-12\beta^2(\nu_1^2+\nu_2^2)] . \tag{34}$$

Two consequences of Eq. (34) are in qualitative agreement with the numerical results of Ref. 10.

(i) The third-order dispersion effect decreases the strength of the mutual interaction making L_p longer.

(ii) For sufficiently strong coefficient β the interaction may even become repulsive.

A more detailed investigation of the effect of third order-dispersion on the coherent two-soliton interaction dynamics is well under way, but will be presented elsewhere.

VII. CONCLUSION

We have investigated two-soliton coupling through incoherent intensity overlap. This form of interaction is (as expected) weaker than the coherent interaction, which has previously been investigated. The requirement on consecutive soliton spacing in order to avoid pulse coalescence is thus relaxed and the maximum system bandwidth correspondingly enhanced.

Incoherent soliton pulse interaction may result from the polarization characteristics of the pulses and possibly also decorrelating mechanisms like depolarization and finite laser source coherence time.

Third-order dispersion is found to affect the coupling strength and leads to a degradation of the attraction forces between the pulses (at least for pulses initially "at rest").

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