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Filamentation instability of beat waves in a hot magnetized plasma

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A large-amplitude longitudinal electrostatic electron plasma wave excited at the beat frequency of two laser beams is shown to be strongly unstable against the filamentation instability for arbitrary short-wavelength perturbations in the presence of an external transverse magnetic field. The Vlasov equation in terms of guiding-center coordinates has been employed to obtain the nonlinear response of electrons in the plasma. It is concluded that a strong static magnetic field efficiently reduces the growth rate of the filamentation instabilty of the excited electron plasma wave.

I. INTRODUCTION

In recent years there has been considerable interest in the excitation of a large-amplitude longitudinal electron plasma wave at the beat frequency of two parallel intense laser beams shone on a plasma in the application to laser-plasma beat-wave accelerators.[1–5] The coherent large electrostatic field propagating with phase velocity close to the velocity of light will trap and accelerate the plasma particles to high energy in large flux. However, the excited electron plasma wave may couple with low-frequency perturbations present in the hot plasma to create various possible instabilities. The filamentation instability is one of such undesirable instabilities which prevents the excited electron plasma wave from accelerating particles uniformly.

Recently, Katsouleas and Dawson[5] have shown that the use of an external static magnetic field transverse to the excited electron plasma waves eliminates the upper limit of the maximum energy gain of plasma particles in laser-plasma accelerators. The various possible instabilities due to the excited large-amplitude electron plasma wave may be drastically affected by this magnetic field. In this paper we have investigated the effect of an external transverse magnetic field on the filamentation instability of the excited electron plasma wave at the difference frequency of two uniform laser beams in a hot collisionless plasma.

In Sec. II we have solved the nonlinear Vlasov equation expressed in terms of the guiding-center coordinates to obtain the nonlinear response of electrons at the low-frequency ion-acoustic perturbation and two high-frequency scattered electron plasma waves in the presence of a transverse external static magnetic field in a hot, collisionless and homogeneous plasma. Using Poisson's equation we have derived the nonlinear dispersion relation for the low-frequency perturbation mode. The dispersion relation is then solved to obtain the growth rates of the filamentation instability of the excited electron plasma wave at the difference frequency of two laser beams in Sec. III. Finally, a brief discussion of the results is presented in Sec. IV.

II. KINETIC ANALYSIS FOR DISPERSION RELATION

Let us consider the propagation of two beams of large-amplitude upper-hybrid-laser radiation along the same direction ($\mathbf{k}_1' \| \mathbf{k}_2' \| \hat{\mathbf{x}}$) in a hot, collisionless, and homogeneous plasma immersed in an external static magnetic field in the transverse direction ($\mathbf{B}_s \| \hat{\mathbf{z}}$):

$$\mathbf{E}'_{1,2}=\mathbf{E}''_{1,2}\exp[-i(\omega'_{1,2}t-k'_{1,2}x)] , \tag{1}$$

where

$$k'_{1,2}=\frac{\omega'_{1,2}}{c}\left[1-\frac{\omega_p^2}{\omega'^2_{1,2}}\frac{\omega'^2_{1,2}-\omega_p^2}{\omega'^2_{1,2}-\omega_p^2-\omega_c^2}\right]^{1/2} , \tag{2}$$

$\omega'_{1,2}$ and $k'_{1,2}$ are the angular frequencies and wave numbers of the two-incident-laser radiation, $\omega_p=(4\pi e^2 n_0^0/m)^{1/2}$ is the electron plasma frequency, and $\omega_c=eB_s/mc$ is the electron gyrofrequency; $-e$, m, n_0^0, and c being the electronic charge, mass, unperturbed equilibrium electron density, and the speed of light in a vacuum, respectively. On account of the nonlinear interaction of the incident electromagnetic waves in the plasma, a longitudinal electrostatic electron plasma wave

$$\mathbf{E}_0(\mathbf{k}_0,\omega_0)=-\hat{\mathbf{x}}ik_0\phi_0\exp[-i(\omega_0t-k_0x)] , \tag{3}$$

is excited at the difference frequency through the optical mixing or forward Raman-scattering mechanism, where $\mathbf{k}_0=\mathbf{k}_1'-\mathbf{k}_2'$, $\omega_0=\omega_1'-\omega_2'$, and ϕ_0 is the amplitude of the electrostatic potential of the generated wave. A small amount of the energy of the incident waves may also be up-converted into the sum frequency generation, which we neglect in this work. The linear dispersion relation of the excited electron plasma wave at the difference frequency can be written as

$$\omega_0^2=\omega_{\rm UH}^2+3k_0^2v_{\rm th}^2 , \tag{4}$$

where $\omega_{\rm UH}=(\omega_p^2+\omega_c^2)^{1/2}$ and $v_{\rm th}=(T_e/m)^{1/2}$ are the upper-hybrid frequency and the thermal speed of electrons in the magnetized plasma; T_e being the electron temperature expressed in units of the Boltzmann constant. We consider this excited longitudinal electrostatic electron plasma wave at the difference frequency of two laser beams as the pump wave $(\mathbf{k}_0,\omega_0)$ which couples parametrically[6] with a low-frequency density perturbation present in the plasma due to a variety of reasons, such as an ion-acoustic mode $(\mathbf{k},\omega)$ in the plasma and generates two high-frequency (Stokes and anti-Stokes) scattered sideband modes $(\mathbf{k}_{1,2},\omega_{1,2})$:

$$\mathbf{k}_{1,2}=\mathbf{k}_0\mp\mathbf{k}\ , \qquad \omega_{1,2}=\omega_0\mp\omega\ . \tag{5}$$

The nonlinear growth of the ion-acoustic mode in the transverse direction will lead to the filamentation of the initially uniform electron plasma wave.

For the hot magnetized plasma the Larmor radius of electrons may be larger than any of the wavelengths of the decay waves involved, $k_\perp\rho_e,k_{1\perp}\rho_e,k_{2\perp}\rho_e\gtrsim 1$ where $\rho_e=v_{\rm th}/\omega_c$. Hence, the fluid model of plasmas breaks down and one has to solve the full Vlasov equation for the nonlinear response of electrons in the plasma, corresponding to the decay waves.

To describe the nonlinear response of electrons to the four-wave parametric process in the magnetized collisionless plasma, we express the Vlasov equation[7] in its equivalent form in terms of the guiding-center coordinates $\mathbf{x}_g$, the magnetic moment μ, the polar angle θ of the perpendicular velocity (i.e., the angle $\mathbf{v}_\perp$ makes with the x axis), and the parallel momentum p_z,

$$\frac{\partial F}{\partial t}+\dot{\mathbf{x}}_g\cdot\frac{\partial F}{\partial \mathbf{x}_g}+\dot{\mu}\frac{\partial F}{\partial \mu}+\dot{\theta}\frac{\partial F}{\partial \theta}-eE_z^t\frac{\partial F}{\partial p_z}=0\ , \tag{6}$$

where

$$\mathbf{x}_g=\mathbf{x}-\mathbf{v}_\perp\times\boldsymbol{\omega}_c/\omega_c^2\ , \tag{7}$$

$$\mu=mv_\perp^2/2\omega_c\ , \tag{8}$$

$$F=f_0^0+f_0(\mathbf{k}_0,\omega_0)+f(\mathbf{k},\omega)+f_1(\mathbf{k}_1,\omega_1)+f_2(\mathbf{k}_2,\omega_2)\ , \tag{9}$$

the overdot denoting the derivative of the quantity involved with respect to time. As (μ,θ), (x_g,y_g), and (p_z,z) constitute the canonical set of variables, Eq. (6) follows directly from the continuity equation of electron density in the six-dimensional space of the new variables. In Eq. (9) f_0^0 is the unperturbed equilibrium distribution function, taken to be Maxwellian at temperature T_e; f_0, f_1, and f_2 are the high-frequency responses to the pump and the two scattered sidebands and f is the response of the low-frequency perturbation mode. Using the equation of motion for electrons we can write

$$\dot{\mu}=-\frac{e}{\omega_c}(\mathbf{E}_\perp^t\cdot\mathbf{v}_\perp)=-\frac{\partial H}{\partial\theta}\ , \tag{10}$$

$$\dot{\theta}=\frac{\partial H}{\partial\mu}=\omega_c+\frac{e}{mv_\perp}(E_x^t\sin\theta-E_y^t\cos\theta)\ , \tag{11}$$

$$\dot{\mathbf{x}}_g=\frac{e}{m\omega_c^2}\mathbf{E}_\perp^t\times\boldsymbol{\omega}_c\ , \tag{12}$$

$$H=\mu\omega_c+p_z^2/2m-e\Phi^t\ , \tag{13}$$

where the superscript t refers to the total field and $\Phi^t=\Phi_0(\mathbf{k}_0,\omega_0)+\Phi(\mathbf{k},\omega)+\Phi_1(\mathbf{k}_1,\omega_1)+\Phi_2(\mathbf{k}_2,\omega_2)$ is the total potential of the four electrostatic waves involved. Using the identity

$$\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]\equiv\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\times\sum_n\exp[in(\theta-\delta)]J_n(k_\perp\rho)\ , \tag{14}$$

where J_n is the Bessel function of order n and $\rho=v_\perp/\omega_c$ we can express

$$\mathbf{E}^t=-i\mathbf{k}_0\phi_0\exp[-i(\omega_0t-k_0x_g)]\sum_n\exp(in\theta)J_n^0-i\mathbf{k}\phi\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta)]J_n$$
$$-i\mathbf{k}_1\phi_1\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1)]J_n^1-i\mathbf{k}_2\phi_2\exp[-i(\omega_2t-\mathbf{k}_2\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_2)]J_n^2\ , \tag{15}$$

$$F=n_0^0(m/2\pi T_e)^{3/2}\exp(-mv^2/2T_e)+\exp[-i(\omega_0t-k_0x_g)]\sum_n\exp(in\theta)f_n^0+\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta)]f_n$$
$$+\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_1)]f_n^1+\exp[-i(\omega_2t-\mathbf{k}_2\cdot\mathbf{x}_g)]\sum_n\exp[in(\theta-\delta_2)]f_n^2\ . \tag{16}$$

In Eqs. (15) and (16) ϕ's are the amplitudes of the electrostatic potentials, $J_n=J_n(k_\perp\rho)$, $J_n^0=J_n(k_0\rho)$, $J_n^1=J_n(k_{1\perp}\rho)$, $J_n^2=J_n(k_{2\perp}\rho)$, and δ, δ_1, and δ_2 are the angles between the x axis and $\mathbf{k}_\perp$, $\mathbf{k}_{1\perp}$, and $\mathbf{k}_{2\perp}$, respectively. Using Eqs. (15) and (16) we can write

$$\dot{\mu}=ie\phi_0\exp[-i(\omega_0t-k_0x_g)]\sum_n n\exp(in\theta)J_n^0+ie\phi\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n n\exp[in(\theta-\delta)]J_n$$
$$+ie\phi_1\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n n\exp[in(\theta-\delta_1)]J_n^1+ie\phi_2\exp[-i(\omega_2t-\mathbf{k}_2\cdot\mathbf{x}_g)]\sum_n n\exp[in(\theta-\delta_2)]J_n^2\ , \tag{17}$$

$$\dot{\theta}=\omega_c-\frac{e\phi_0 k_0}{mv_\perp}\exp[-i(\omega_0 t-k_0 x_g)]\sum_n \exp(in\theta)J_n^{0'}-\frac{e\phi k_\perp}{mv_\perp}\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta)]J_n'$$
$$-\frac{e\phi_1 k_{1\perp}}{mv_\perp}\exp[-i(\omega_1 t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta_1)]J_n^{1'}-\frac{e\phi_2 k_{2\perp}}{mv_\perp}\exp[-i(\omega_2 t-\mathbf{k}_2\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta_2)]J_n^{2'} , \tag{18}$$

$$\dot{x}_g=-\frac{ie}{m\omega_c}\left[k_\perp\phi\sin\delta\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta)]J_n+k_{1\perp}\phi_1\sin\delta_1\exp[-i(\omega_1 t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta_1)]J_n^1\right.$$
$$\left.+k_{2\perp}\phi_2\sin\delta_2\exp[-i(\omega_2 t-\mathbf{k}_2\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta_2)]J_n^2\right] , \tag{19}$$

$$\dot{y}_g=\frac{ie}{m\omega_c}\left[k_0\phi_0\exp[-i(\omega_0 t-k_0 x_g)]\sum_n \exp(in\theta)J_n^0+k_\perp\phi\cos\delta\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta)]J_n\right.$$
$$+k_{1\perp}\phi_1\cos\delta_1\exp[-i(\omega_1 t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta_1)]J_n^1$$
$$\left.+k_{2\perp}\phi_2\cos\delta_2\exp[-i(\omega_2 t-\mathbf{k}_2\cdot\mathbf{x}_g)]\sum_n \exp[in(\theta-\delta_2)]J_n^2\right] , \tag{20}$$

$$\dot{z}_g=p_z/m , \tag{21}$$

where the prime denotes derivative with respect to its argument.

Now, since the maximum growing modes propagate in the plane perpendicular to the external magnetic field,[8] we take $k_z=0$. Using Eqs. (15)–(20) in the Vlasov equation (6) we obtain the following linear response of electrons:

$$f_{0n}^l=-\frac{e\phi_0}{T_e}\frac{n\omega_c}{\omega_0-n\omega_c}J_n^0 f_0^0 , \tag{22}$$

$$f_n^l=-\frac{e\phi}{T_e}\frac{n\omega_c}{\omega-n\omega_c}J_n f_0^0 , \tag{23}$$

$$f_{1n}^l=-\frac{e\phi_1}{T_e}\frac{n\omega_c}{\omega_1-n\omega_c}J_n^1 f_0^0 , \tag{24}$$

$$f_{2n}^l=-\frac{e\phi_2}{T_e}\frac{n\omega_c}{\omega_2-n\omega_c}J_n^2 f_0^0 . \tag{25}$$

Using Eqs. (22)–(25) in Eq. (6) we obtain the nonlinear part of the distribution function for the low-frequency mode as

$$f_n^{\rm nl}=\frac{\exp(in\delta)}{\omega-n\omega_c}\sum_l\left[-\frac{e\phi_1^* k_{1\perp}}{2mv_\perp}(n+l)\exp(il\delta_1)J_l^{1'}f_{n+l}^0+\frac{e\phi_0 k_0}{2mv_\perp}(l-n)\exp[i(l-n)\delta_1]J_l^{0'}f_{l-n}^{1*}\right.$$
$$-\frac{e\phi_1^*}{2}l\exp(il\delta_1)J_l^1\frac{\partial f_{n+l}^0}{\partial\mu}+\frac{e\phi_0}{2}l\exp[i(l-n)\delta_1]J_l^0\frac{\partial f_{l-n}^{1*}}{\partial\mu}+\frac{ie\phi_1^* k_0 k_{1y}}{2m\omega_c}\exp(il\delta_1)J_l^1 f_{n+l}^0$$
$$-\frac{ie\phi_0 k_0 k_{1y}}{2m\omega_c}\exp[i(l-n)\delta_1]J_l^0 f_{l-n}^{1*}+\frac{e\phi_2 k_{2\perp}}{2mv_\perp}(l-n)\exp(-il\delta_2)J_l^{2'}f_{l-n}^{0*}$$
$$-\frac{e\phi_0^* k_0}{2mv_\perp}(n+l)\exp[-i(n+l)\delta_2]J_l^{0'}f_{n+l}^2+\frac{e\phi_2}{2}l\exp(-il\delta_2)J_l^2\frac{\partial f_{l-n}^{0*}}{\partial\mu}$$
$$-\frac{e\phi_0^*}{2}l\exp[-i(n+l)\delta_2]J_l^0\frac{\partial f_{n+l}^2}{\partial\mu}+\frac{ie\phi_2 k_0 k_{2y}}{2m\omega_c}\exp(-il\delta_2)J_l^2 f_{l-n}^{0*}$$
$$\left.-\frac{ie\phi_0^* k_0 k_{2y}}{2m\omega_c}\exp[-i(n+l)\delta_2]J_l^0 f_{n+l}^2\right] , \tag{26}$$

where the asterisk denotes the complex conjugate of the quantity involved. We can obtain the linear and nonlinear density perturbations associated with the low-frequency mode $(\mathbf{k},\omega)$ from the relation

$$n(\mathbf{k},\omega)=2\pi \exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]\sum_n \int_0^{\infty}\int_{-\infty}^{+\infty} f_n J_n v_\perp \, dv_\perp \, dv_z \; . \tag{27}$$

Now, it is noticed that $n^{\rm nl}=0$ for a dipole pump ($k_0=0$). In the next approximation considering the first-order effect of small k_0 in Eq. (27), we obtain the following expression for the nonlinear density fluctuation at $(\mathbf{k},\omega)$:

$$\begin{aligned} n^{\rm nl}=&\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x})]\left[\frac{n_0^0 e^2\exp(i\delta)}{2mT_e(\omega-\omega_c)}\right] \\ &\times\left[\frac{m\omega_c^2\phi_0\phi_1^*\exp(i\delta_1)}{2T_e(\omega_1-\omega_c)}[I_0(b_0)\exp(-b_0)-1]-\frac{ik_0k_{1y}\phi_0\phi_1^*}{\omega_0-\omega_c}I_0(b_1)\exp(-b_1)\right. \\ &+\frac{ik_0k_{1y}\phi_0\phi_1^*\exp(i\delta_1)}{\omega_1-\omega_c}I_0(b_0)\exp(-b_0)+\frac{m\omega_c^2\phi_0^*\phi_2\exp(-i\delta_2)}{2T_e(\omega_2-\omega_c)}[I_0(b_0)\exp(-b_0)-1] \\ &\left.-\frac{ik_0k_{2y}\phi_0^*\phi_2}{\omega_0-\omega_c}I_0(b_2)\exp(-b_2)+\frac{ik_0k_{2y}\phi_0^*\phi_2\exp(-i\delta_2)}{\omega_2-\omega_c}I_0(b_0)\exp(-b_0)\right] \\ \simeq& A\phi_0\phi_1^*+B\phi_0^*\phi_2 \; , \end{aligned} \tag{28}$$

where

$$A=\frac{in_0^0e^2k_0k_{1y}\exp[i(\delta+\delta_1)]}{2mT_e(\omega-\omega_c)(\omega_1-\omega_c)}I_0(b_0)\exp(-b_0) \; , \tag{29}$$

$$B=\frac{in_0^0e^2k_0k_{2y}\exp[i(\delta-\delta_2)]}{2mT_e(\omega-\omega_c)(\omega_2-\omega_c)}I_0(b_0)\exp(-b_0) \; , \tag{30}$$

I_0 is the zeroth-order modified Bessel function of first kind, $b_0=k_0^2v_{\rm th}^2/\omega_c^2 \ll 1$ for the usual plasma parameters but $b_{1,2}=k_{1,2}^2v_{\rm th}^2/\omega_c^2 \gg 1$ for short-wavelength perturbation. In deriving Eq. (28) we have retained only the dominating terms having resonant denominators, viz., $(\omega-\omega_c)$.

In a similar way, using the guiding-center coordinates the nonlinear response at the high-frequency sidebands are given by

$$\begin{aligned} f_{1n}^{\rm nl}=&\frac{\exp(in\delta_1)}{\omega_1-n\omega_c}\sum_l\left[-\frac{ek_\perp\phi^*}{2mv_\perp}(n+l)\exp(il\delta)J_l'f_{n+l}^0-\frac{e\phi^*}{2}l\exp(il\delta)J_l\frac{\partial f_{n+l}^0}{\partial\mu}+\frac{e\phi_0k_0}{2mv_\perp}(l-n)\exp[i(l-n)\delta]J_l^{0'}f_{l-n}^*\right. \\ &\left.+\frac{e\phi_0}{2}l\exp[i(l-n)\delta]J_l\frac{\partial f_{l-n}^*}{\partial\mu}-\frac{ie\phi_0k_0k_y}{2m\omega_c}\exp[i(l-n)\delta]J_l^0f_{l-n}^*+\frac{ie\phi^*k_0k_y}{2m\omega_c}\exp(il\delta)J_lf_{n+l}^0\right] , \end{aligned} \tag{31}$$

$$\begin{aligned} f_{2n}^{\rm nl}=&\frac{\exp(in\delta_2)}{\omega_2-n\omega_c}\sum_l\left[-\frac{ek_\perp\phi}{2mv_\perp}(n-l)\exp(-il\delta)J_l'f_{n-l}^0+\frac{e\phi}{2}l\exp(-il\delta)J_l\frac{\partial f_{n-l}^0}{\partial\mu}\right. \\ &-\frac{e\phi_0k_0}{2mv_\perp}(n-l)\exp[-i(n-l)\delta]J_l^{0'}f_{n-l}+\frac{e\phi_0}{2}l\exp[-i(n-l)\delta]J_l^0\frac{\partial f_{n-l}}{\partial\mu} \\ &\left.+\frac{ie\phi_0k_0k_y}{2m\omega_c}\exp[-i(n-l)\delta]J_l^0f_{n-l}-\frac{ie\phi k_0k_y}{2m\omega_c}\exp(-il\delta)J_lf_{n-l}^0\right] . \end{aligned} \tag{32}$$

Thus the nonlinear density perturbations at the high-frequency sidebands are given by

$$n_1^{\rm nl}\simeq C\phi_0\phi^* \; , \tag{33}$$

$$n_2^{\rm nl}\simeq D\phi_0\phi \; , \tag{34}$$

where

$$C=\frac{in_0^0e^2k_0k_y\exp[i(\delta+\delta_1)]}{2mT_e(\omega-\omega_c)(\omega_1-\omega_c)}I_0(b_0)\exp(-b_0) \; , \tag{35}$$

$$D=-\frac{in_0^0e^2k_0k_y\exp[-i(\delta-\delta_2)]}{2mT_e(\omega-\omega_c)(\omega_2-\omega_c)}I_0(b_0)\exp(-b_0) \; . \tag{36}$$

Using Eqs. (28), (33), and (34) in Poisson's equation we obtain the following nonlinear coupled equations:

$$\epsilon\phi=-(4\pi e/k^2)(A\phi_0\phi_1^*+B\phi_0^*\phi_2) \; , \tag{37}$$

$$\epsilon_1\phi_1=-(4\pi e/k_1^2)C\phi^*\phi_0 \; , \tag{38}$$

$$\epsilon_2\phi_2 = -(4\pi e/k_2^2)D\phi\phi_0\ , \tag{39}$$

where the linear dielectric functions are given by

$$\epsilon = 1 - \frac{\omega_p^2}{k^2 v_{\rm th}^2}\sum_n \left[\frac{n\omega_c}{\omega - n\omega_c}\right] I_n(b)\exp(-b)\ , \tag{40}$$

$$\epsilon_1 = 1 - \frac{\omega_p^2}{k_1^2 v_{\rm th}^2}\sum_n \left[\frac{n\omega_c}{\omega_1 - n\omega_c}\right] I_n(b_1)\exp(-b_1)\ , \tag{41}$$

$$\epsilon_2 = 1 - \frac{\omega_p^2}{k_2^2 v_{\rm th}^2}\sum_n \left[\frac{n\omega_c}{\omega_2 - n\omega_c}\right] I_n(b_2)\exp(-b_2)\ . \tag{42}$$

Eliminating ϕ, ϕ_1, and ϕ_2 from Eqs. (37)–(39) we obtain the following nonlinear dispersion relation for the low-frequency electrostatic mode:

$$\epsilon = \frac{\mu_1}{\epsilon_1} + \frac{\mu_2}{\epsilon_2}\ , \tag{43}$$

where

$$\mu_1 = \frac{|v_0/v_{\rm th}|^2 \omega_p^4 \omega_0^2 \sin\delta\sin\delta_1}{4kk_1 v_{\rm th}^2(\omega-\omega_c)^2(\omega_1-\omega_c)^2}[I_0(b_0)\exp(-b_0)]^2\ , \tag{44}$$

$$\mu_2 = \frac{|v_0/v_{\rm th}|^2 \omega_p^4 \omega_0^2 \sin\delta\sin\delta_2}{4kk_2 v_{\rm th}^2(\omega-\omega_c)^2(\omega_2-\omega_c)^2}[I_0(b_0)\exp(-b_0)]^2\ , \tag{45}$$

$$|v_0|^2 = e^2\phi_0\phi_0^* k_0^2/m^2\omega_0^2\ . \tag{46}$$

III. GROWTH RATES OF FILAMENTATION INSTABILITY

When the low-frequency perturbation and the two high-frequency sidebands are the normal modes, they must satisfy $\epsilon = 0 = \epsilon_1 = \epsilon_2$ in the linear limit. However, on account of the nonlinear coupling in the presence of the pump wave, the angular frequencies of the decay waves get modified and we can have the following expansions:[6–8]

$$\begin{aligned} &\omega = \omega + i\gamma\ ,\\ &\epsilon \simeq i(\gamma+\Gamma)\partial\epsilon/\partial\omega\ ,\\ &\epsilon_{1,2} \simeq i(\gamma+\Gamma_{1,2})\partial\epsilon_{1,2}/\partial\omega_{1,2}\ , \end{aligned} \tag{47}$$

where Γ, Γ_1, and Γ_2 are the linear damping rates of the decay waves[6]

$$\Gamma = \left[\frac{\pi m}{8m_i}\right]^{1/2} kC_s + \left[\frac{T_e}{T_i}\right]^{3/2}\exp\left[-\frac{1}{2}\left[3+\frac{T_e}{T_i}\right]\right]\ , \tag{48}$$

$$\Gamma_{1,2} = \frac{\pi^{1/2}\omega_p}{2k_{1,2}^3\lambda_D^3}\exp\left[-\frac{1}{2k_{1,2}^2\lambda_D^2} - \frac{3}{4}\right]\ , \tag{49}$$

$\lambda_D = v_{\rm th}/\omega_p$, C_s is the ion-acoustic velocity, and T_i is the temperature of the ions in the plasma. Substituting the expansions, Eqs. (47), in the dispersion relation, Eq. (43), we obtain the growth rate of the decay waves from the following relation:

$$\gamma(\gamma+\Gamma) \simeq \gamma_0^2 = -\frac{1}{\partial\epsilon/\partial\omega}\left[\frac{\mu_1}{\partial\epsilon_1/\partial\omega_1} + \frac{\mu_2}{\partial\epsilon_2/\partial\omega_2}\right]\ , \tag{50}$$

where γ_0 is the growth rate of the instability in the absence of the linear damping of the decay waves. In writing Eq. (50) we have made an approximation, $\Gamma_1 = \Gamma_2 \simeq 0$. Using Eqs. (40)–(42) one may obtain

$$\begin{aligned} \frac{\partial\epsilon}{\partial\omega} &\simeq \frac{\omega_p^2}{k^2 v_{\rm th}^2}\frac{4\omega_c^2\omega}{(\omega^2-\omega_c^2)^2} I_1(b)\exp(-b)\ ,\\ \frac{\partial\epsilon_1}{\partial\omega_1} &\simeq \frac{\omega_p^2}{k_1^2 v_{\rm th}^2}\frac{4\omega_c^2\omega_1}{(\omega_1^2-\omega_c^2)^2} I_1(b_1)\exp(-b_1)\ ,\\ \frac{\partial\epsilon_2}{\partial\omega_2} &\simeq \frac{\omega_p^2}{k_2^2 v_{\rm th}^2}\frac{4\omega_c^2\omega_2}{(\omega_2^2-\omega_c^2)^2} I_1(b_2)\exp(-b_2)\ , \end{aligned} \tag{51}$$

where we have kept dominating terms up to $n = \pm 1$ only.

Hence, the growth rate of the four-wave parametric instability in absence of the linear damping of the decay waves is given by

$$\gamma_0^2 = -\frac{|v_0/v_{\rm th}|^2 kv_{\rm th}^2\omega_0^2(\omega+\omega_c)^2\sin\delta[I_0(b_0)\exp(-b_0)]^2}{64\omega_c^4\omega[I_1(b)\exp(-b)]}\left[\frac{k_1(\omega_1+\omega_c)^2\sin\delta_1}{\omega_1 I_1(b_1)\exp(-b_1)} + \frac{k_2(\omega_2+\omega_c)^2\sin\delta_2}{\omega_2 I_1(b_2)\exp(-b_2)}\right]\ . \tag{52}$$

When the low-frequency ion-acoustic mode $(\mathbf{k},\omega)$ is propagating in a direction transverse to the direction of propagation of the excited electron plasma wave, the nonlinear growth of the perturbation (ion-acoustic) may lead to the filamentation of the electron plasma wave. Therefore, in order to find the growth rate of the filamentation instability of the excited electron plasma wave at the difference frequency of two laser beams, we take $\delta = 90°$. For the short-wavelength low-frequency perturbation, $k \gg k_0$, we have $\delta_1 \simeq 90° \simeq -\delta_2$ from the parametric conditions $\mathbf{k}_{1,2} = \mathbf{k}_0 \mp \mathbf{k}$. Thus the expression for the growth rate of the filamentation instability reduces to the simple form for $b_0 \ll 1 \ll b, b_1, b_2$

$$\gamma_0 \simeq \pi^{1/2}|v_0/v_{\rm th}|kk_1 v_{\rm th}^2\omega_0(\omega_c+\omega)/4\omega_c^3\ . \tag{53}$$

It is noticed from Eq. (53) that the growth rate of the filamentation instability increases with $|v_0/v_{\rm th}|$, $v_{\rm th}$, ω_0, ω, and k, but decreases rapidly with increasing the external static magnetic field in the plasma. The growth rate is independent of the plasma density. From Eqs. (48) and (49) it is observed that the linear damping rates of the decay waves are small compared to the undamped growth rate for the usual plasma parameters. Hence the threshold of the parametric process is quite low. For typical plasma parameters in the laser-plasma beat-wave accelerators $\omega_0 \simeq 10^{13}$ rad s^{-1}, $\omega_c \simeq 10^{12}$ rad s^{-1} (corresponding to $B_s \simeq 100$ kG), $v_{th} \simeq 10^9$ cm s^{-1} (corresponding to $T_e \simeq 10$ keV), and $k \simeq 10^3$ cm^{-1}, the undamped growth rate of the filamentation instability turns out to be $\simeq 10^{12} | v_0 / v_{th} |$ rad s^{-1}.

IV. DISCUSSION

A high-amplitude excited electrostatic electron plasma wave at the difference frequency of two high-power extraordinary laser beams is found to be strongly unstable against the filamentation instability due to the nonlinear coupling of the beat wave with low-frequency short-wavelength ion-acoustic perturbation in a hot collisionless plasma in the presence of an external static magnetic field. However, the external magnetic field drastically reduces the growth rate of the filamentation instability. It is noticed that the growth rate of the instability is independent of the plasma density. Thus the external magnetic field which eliminates the upper limit of the energy gain of the plasma particles[5] reduces the filamentation instability of the excited beat wave in the plasma, thus enabling the excited electrostatic electron plasma wave to trap and accelerate the plasma particles to high energy uniformly.

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[1]T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).

[2]T. Tajima and J. M. Dawson, IEEE Trans. Nucl. Sci. **NS-28**, 3416 (1981).

[3]T. Tajima, C. Joshi, D. J. Sullivan, and B. B. Godfrey, in Proceedings of the European Committee for Future Accelerators—Rutherford Appleton Laboratory Meeting held at New College, Oxford, 1982 (unpublished).

[4]C. Joshi, T. Tajima, J. M. Dawson, H. A. Baldis, and N. A. Ebrahim, Phys. Rev. Lett. **47**, 1285 (1981).

[5]T. Katsouleas and J. M. Dawson, Phys. Rev. Lett. **51**, 392 (1983).

[6]C. S. Liu and P. K. Kaw, in *Advances in Plasma Physics,* edited by A. Simon and W. B. Thompson (Wiley, New York, 1976), Vol. 6.

[7]O. P. Sharma, V. K. Tripathi, and C. S. Liu, Phys. Fluids **25**, 1598 (1982).

[8]C. Grebogi and C. S. Liu, Phys. Fluids **23**, 1330 (1980).