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Damping of cylindrical phonons in extense fermion matter

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A previous model, designed so as to simulate the coupling between a one-dimensional, quantal harmonic excitation in a fermion heat bath, is extended to consider three-dimensional vibrations with axial symmetry. This investigation addresses future studies on the relaxation of collective modes in finite Fermi systems such as deformed nuclei. For this application, the equations of irreversible dynamics in the proposed model are extracted and their solutions are examined in a range of parameters appropriate for the nuclear fluid. The formulation, techniques, and discussion are far more general and can be applied to a variety of problems arising in statistical mechanics and many-body physics.

I. INTRODUCTION

The relaxation of quantal macroscopic degrees of freedom in many-body systems or fields continues to be a subject of enthusiastic research. Although this issue has traditionally received most inputs from quantum optics,[1] the observation of transport processes in the nuclear fluid[2] offer new insights and roads to enriching our comprehension of its different facets. In particular, atomic nuclei, that are many-body, however finite, quantal systems, provide an interesting evidence of damped collective motion through the giant resonances, among which the oldest and most popular, the dipole-charge mode,[3] has induced many theorists in the field to search for explanations of its width for a couple of decades. As an alternative to the official nuclear models that inspire the most accepted answers to that question, namely the hydrodynamic and the microscopic descriptions of the dipole relaxation, a scheme has recently been proposed that primarily suggests the possibility of regarding the macroscopic vibration under study as a quantal oscillator, that undergoes Brownian motion in the nuclear environment.[4] With this idea in mind, the model was developed according to several procedures of nonequilibrium statistical mechanics[5–7] and the relaxation of a one-dimensional oscillator in an equilibrated fermionic reservoir, that could be conceived as nuclear matter for the applications of interest, was established.[4] In the first stage, a standard particle-phonon interaction was adopted as the coupling mechanism. Further extensions of the primary model imply the evaluation and analysis of the collision frequencies of the fermions in presence of the oscillator,[8] a test application to spherical nuclei,[9] the examination of the competition between dissipative and diffusive events in the equilibration of a charge mode in a nucleus,[10] the investigation of the solutions of the master equation for a time-dependent harmonic oscillator[11] and the coupling of the vibration to a low-frequency oscillation. The latter is aimed at representing a surface collective mode in a finite nucleus.[12]

In the present work, we advance one more step in the field of quantal Brownian motion in fermionic systems and propose an extension of Ref. 4 intending most realistic applications. Our motivation lies in the experimental fact that giant dipole resonances in deformed nuclei split into two or three components, according to the nucleus being axially symmetric or not. We then begin the study of the equilibration dynamics of a three-dimensional, axially degenerated harmonic oscillator immersed in an extended fermion heat bath to which it couples via single-phonon creation or destruction.[4] Notwithstanding the actual calculations that are performed for nuclear parameters and ranges, the formulation is general and only makes use of assertions, techniques, and approximations widely approved in nonequilibrium statistical mechanics.[1,5–7,13,14]

This paper is organized as follows. In Sec. II the details of the physical model are described. In Sec. III we briefly explain how the equations of irreversible motion for the oscillator and the fermion system are extracted within the current approximations. It is seen that the oscillator decouples into two modes, the one lying along the symmetry axis, the other corresponding to a circularly degenerated vibration lying on the equatorial plane. The irreversible road toward equilibration is then described by two mutually independent master equations for the one- and two-dimensional oscillators, respectively. However, they are coupled together by the kinetics of the heat bath, whose total collision rate admits contributions from either vibration. The generator of evolution of the latter turns out to be a constant, non-Hermitian matrix in the weak coupling limit supplemented with the assumption of perfectly elastic collisions between each oscillator component and the fermions. As for the one-dimensional component along the z axis, this is exactly the situation discussed in Ref. 4 and is no further considered here. The evolution of the oscillator in the plane is associated to a spectral problem that can be analytically solved, if one takes advantage of the particular random-walk pattern related to the given master equation and conveniently chooses a set of boundary conditions. This is explained and analyzed in Sec. IV. In Sec. V, some numerical calculations aimed at illustrating the model in the range of nuclear parameters are presented and discussed, while Sec. VI contains the major conclusions and perspectives.

II. DESCRIPTION OF THE MODEL

The model proposed for the preliminary study faced in this work considers a boson system coupled to a fermionic heat bath, the former being associated to a three-dimensional (3D) oscillator that simulates a harmonic excitation in a finite deformed nucleus. The fermionic reservoir represents equilibrated nuclear matter at a given temperature T, or in other words, a Fermi gas described by an equilibrium single-particle (SP) density,

$$\rho_i \equiv \rho(\epsilon_i) = \frac{1}{1+e^{(\epsilon_i-\epsilon_F)/T}} , \tag{2.1}$$

where $\epsilon_F = 38$ MeV and T can amount to a few MeV (hereafter, the Boltzmann's constant k is chosen as the unit of entropy).

The time evolution of the combined system is generated by the Hamiltonian

$$H = H_F + H_B + H_{BF} , \tag{2.2}$$

with the fermionic contribution

$$H = \sum_i \epsilon_i b_i^\dagger b_i + \tfrac{1}{4} \sum_{i,j,k,l} V_{ijkl} b_i^\dagger b_j^\dagger b_l b_k , \tag{2.3}$$

and the boson term

$$\begin{aligned} H_B &= H_x + H_y + H_z \\ &= (\Gamma_x^\dagger \Gamma_x + \Gamma_y^\dagger \Gamma_y + 1)\hbar\Omega + (\Gamma_z^\dagger \Gamma_z + \tfrac{1}{2})\hbar\Omega_z . \end{aligned} \tag{2.4}$$

The operator b_i ($\Gamma_\nu^\dagger$) creates a SP (a one-phonon) state on the corresponding vacuum, b_i (Γ_ν) being its Hermitian adjoint. Equation (2.4) displays another of the working hypothesis; namely, the 3D oscillator exhibits axial symmetry. Thus, the model here presented is well suited for the description of resonant decay in axially deformed nuclei. In this situation, it is convenient to introduce the circular operators that respectively create right- and left-handed polarizations in the (x,y) plane,

$$\Gamma_R = 1/\sqrt{2}(\Gamma_x - i\Gamma_y) , \tag{2.5a}$$

$$\Gamma_L = 1/\sqrt{2}(\Gamma_x + i\Gamma_y) \tag{2.5b}$$

in terms of which the boson Hamiltonian reads,

$$H = (\Gamma_R^\dagger \Gamma_R + \Gamma_L^\dagger \Gamma_L + 1)\hbar\Omega + (\Gamma_z^\dagger \Gamma_z^\dagger + \tfrac{1}{2})\hbar\Omega_z . \tag{2.6}$$

It is clear that Γ_R, Γ_L, and their adjoints satisfy the boson commutation rules and are independent, $[\Gamma_R, \Gamma_L] = [\Gamma_x, \Gamma_y] = 0$. In this representation, the axial component of the angular momentum of the oscillator reads,

$$L_z = \hbar(\Gamma_R^\dagger \Gamma_R - \Gamma_L^\dagger \Gamma_L) . \tag{2.7}$$

Since L_z is a constant of the motion, one can build up a basis $|n,m,n_z\rangle$ (Ref. 15) where $n = n_R + n_L$, $m = n_R - n_L$, and

$$H\,|n,m,n_z\rangle = [(n+1)\hbar\Omega + (n_z + \tfrac{1}{2}\hbar\Omega_z]\,|n,m,n_z\rangle , \tag{2.8a}$$

$$L_z\,|n,m,n_z\rangle = \hbar m , \tag{2.8b}$$

$$\Gamma_R^\dagger\,|n,m,n_z\rangle = \sqrt{(n+m)/2}\,|n+1,m+1,n_z\rangle , \tag{2.8c}$$

$$\Gamma_L^\dagger\,|n,m,n_z\rangle = \sqrt{(n-m)/2}\,|n+1,m-1,n_z\rangle . \tag{2.8d}$$

Notice that $|m| \le n$ and m varies in steps of two units.[15]

This basis will be the one adopted to expand the evolving density operator of the oscillator, $\hat\rho_B(t)$ as

$$\hat\rho_B(t) = \sum_{\substack{n,m,n_z,\\ n',m',n_z'}} \rho_{nmn_z,n'm'n_z'}(t)\,|n,m,n_z\rangle\langle n',m',n_z'| . \tag{2.9}$$

The matrix elements $\rho_{IJ}(t)$ will be irreversible driven to equilibration by an interaction that we assume to have a standard particle-phonon structure,[4]

$$H_{FB} = \sum_{\alpha,\mu} (\lambda_{\alpha\mu}^{(R)} \Gamma_R^\dagger b_\mu^\dagger b_\alpha + \lambda_{\alpha\mu}^{(L)} \Gamma_L^\dagger b_\mu^\dagger b_\alpha + \lambda_{\alpha\mu}^{(z)} \Gamma_z^\dagger b_\mu^\dagger b_\alpha) + \text{H.c.} \tag{2.10}$$

As usual[4,8–12] SP labels $\mu(\alpha)$ are assigned to represent the lowest- (highest-) lying levels in a collision vertex; at zero temperature $\mu(\alpha)$ explicitly denotes SP states located below (above) the Fermi level.

III. THE EQUATIONS OF MOTION

The technique here utilized to extract the equations of motion for the boson and fermion subsystems has been already employed in the one-dimensional (1D) case.[4,8–12] It is inspired in the reduction procedure giving rise to the quantal Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy and it has been shown[4] that it is equivalent to a projection method[1,5,6,13,14] with time-dependent projectors.[17] From the operational point of view, what one does is to reduce the undesired degrees of freedom out of the density matrix of the system $\hat\rho$ to obtain the subsystem densities,

$$\hat\rho_B(t) = \mathrm{Tr}_F \hat\rho(t) , \tag{3.1a}$$

$$\hat\rho_F(t) = \mathrm{Tr}_B \hat\rho(t) . \tag{3.1b}$$

Reduction of the Schrödinger—von Neumann equation of motion $i\hbar\dot{\hat\rho} = L\hat\rho$ with the Liouvillian

$$L = [H,\ ] = L_B + L_F + L_{BF} , \tag{3.2}$$

gives rise to the coupled equations for $Q = B$ or F,

$$i\hbar\dot{\hat\rho}_Q = \mathscr{L}_Q \hat\rho_Q + K_Q(\hat\rho) , \tag{3.3}$$

with the free-flow generator or effective Liouvillian

$$\mathscr{L}_Q = L_Q + \mathrm{Tr}_{-Q}(L_{BF}\hat\rho_{-Q}) , \tag{3.4}$$

(here $-Q$ denotes the complement of the Q coordinates) and the coupling term $K_Q(\hat\rho)$. Furthermore, a hypothesis of the Markovian type, in other words, an assumption regarding the microscopic and macroscopic scales in the problem has to be introduced in order to extract the kernel of irreversible evolution in Eq. (3.3). One supposes that the duration τ_c of the collision event between the oscillator and a fermion is much shorter than any period or observable time scale, disregards the information concerning those microscopic, short-lived processes, and quickly writes down a formal expression for $\tilde{K}_Q$, the kinetic operator,

$$\tilde{K}_Q[(\hat{\rho}_Q\hat{\rho}_{-Q})(t)]\simeq K_Q[\rho(t)],\quad t\gg\tau_c\ . \tag{3.5}$$

The details of the calculation are given elsewhere[4,18] and for the model under consideration, straightforward application of this technique gives rise, in the weak coupling ($H_{BF}\ll H_B,H_F$)-plus-sharp resonance (energy-conserving interaction vertex) limit, to the coupled system of equations of irreversible motion

$$\dot{\rho}_{nm}=W_+^{(R)}(\rho_{n+1,m+1}+\rho_{n+1,m-1}-2\rho_{nm})+W_-^{(R)}(\rho_{n-1,m-1}+\rho_{n-1,m+1}-2\rho_{nm})\ , \tag{3.6a}$$

$$\dot{\rho}_{n_z}=W_+^{(z)}(\rho_{n_z+1}-\rho_{n_z})+W_-^{(z)}(\rho_{n_z-1}-\rho_{n_z})\ , \tag{3.6b}$$

$$\begin{aligned}\dot{\rho}_A=\rho_A^{(\mathrm{kin})}+\sum_{\alpha,\mu,i}|\lambda^{(i)}|^2\pi\delta(\Omega^{(i)}-\omega_{\alpha\mu})\{&[(1-\rho_0^{(i)})\rho_\mu(1-\rho_\alpha)-(1-\rho_N^{(i)})\rho_\alpha(1-\rho_\mu)]\delta_{A\mu}\\&+[(1-\rho_N^{(i)})\rho_\alpha(1-\rho_\mu)-(1-\rho_0^{(i)})\rho_\mu(1-\rho_\alpha)]\delta_{A\alpha}\}\ ,\end{aligned} \tag{3.6c}$$

with $i=R$, z, and

$$\rho_N^{(R)}=\sum_{m=-N}^{N}\rho_{Nm}^{(R)}\ . \tag{3.6d}$$

Equations (3.6) are valid in the very close to equilibration regime where interference effects (off-diagonal elements of either density matrix) have already vanished.[16] It is clear from Eqs. (2.8) and (2.9) that the occupation probabilities of the oscillator states are factorizable,

$$\rho_{nmn_z}(t)=\rho_{nm}(t)\rho_{n_z}(t)\ . \tag{3.7}$$

The transition probabilities (per unit time) $W_\pm^{(i)}$ read

$$W_+^{(i)}=\sum_{\alpha,\mu}|\lambda_{\alpha\mu}^{(i)}|^2\pi\delta(\Omega^{(i)}-\omega_{\alpha\mu})\rho_\mu(1-\rho_\alpha)\ , \tag{3.8a}$$

$$W_-^{(i)}=\sum_{\alpha,\mu}|\lambda_{\alpha,\mu}^{(i)}|^2\pi\delta(\Omega^{(i)}-\omega_{\alpha\mu})\rho_\alpha(1-\rho_\mu)\ , \tag{3.8b}$$

where i denotes either R or z. Notice that according to the hypothesis of axial symmetry, we must choose $\lambda_{\alpha\mu}^{(R)}=\lambda_{\alpha\mu}^{(L)}$, this being the reason why only the label R appears in (3.6a). We observe that while Eq. (3.6b) is simply the master equation for a 1D oscillator that performs quantal Brownian motion in a fermionic heat reservoir, Eq. (3.6a) generalizes the former to the 2D situation with axial symmetry. Thus, the latter contains events related to creation or destruction of nodes ($n\pm1$) as well as of polarization quanta ($m\pm1$). In addition, one should take notice that in the extended kinetic equation for the fermions Eq. (3.6c) the contributions from collision events of the particles with the different components of the 3D vibration [namely, those in the (x,y) plane and along the z axis] simply add up, reflecting the independence of these components. Furthermore, we have assumed that $\mathbf{q},\mathbf{q}_z$ are the respective components of the oscillator momenta, related to the corresponding frequencies by an acousticlike dispersion relation,[4]

$$\Omega=c_s|\mathbf{q}|\ , \tag{3.9a}$$

$$\Omega_z=c_{s_z}|\mathbf{q}_z|\ , \tag{3.9b}$$

where c_s,c_{s_z} are the sound velocities in the given directions in the nuclear environment.

IV. THE SPECTRAL PROBLEM

As a prior step to any application we wish to examine the eigenvalue problem of the dynamical system (3.6) and the evolution in the linear regime. Such a regime is automatically ensured in the so-called sharp-resonance approximation[4,8–12] here adopted, that assumes the particle-phonon interaction to be perfectly elastic. This assumption shows up in the δ distributions that force energy conservation in the transition probabilities [Eqs. (3.8)] and in the generalized kinetics of the heat reservoir [Eq. (3.6)]. In particular, the latter shows that when particle-phonon collisions are effective, the collision frequency driving the fermion system to equilibrium is infinite. This fact, a limiting behavior of slightly energy-broadened events, has been analyzed in the context of the 1D oscillator[4,18] where it has been shown that those fermions in momentum space lying in selected planes perpendicular to the z axis achieve instantaneous thermalization; overall equilibration in $\mathbf{k}$ space can then proceed via particle-particle collisions. Furthermore, this thermalization does not imply any temperature change in the heat bath, since the amount of transferred energy is finite. In this case, the circularly degenerated oscillator in the (x,y) plane is associated in the current view, to a cylindrical acoustic wave stemming from the z axis in configuration space. In other words, the geometry of the present model refers to an infinite cylinder, rather than to an infinite cube, of nuclear matter. This means that the phonon wave function is proportional to a Bessel function $J_0(qr)$; consequently, the matrix element $\lambda_{\alpha\mu}$ of the particle-phonon interaction should be of the form,

$$\lambda_{\alpha\mu}^{(R)}=\lambda_{\alpha\mu}^{0(R)}\delta(k_{\alpha_z}-k_{\mu_z})\delta(k-q)\ , \tag{4.1}$$

where k is the modulus of the projection of the relative momentum $\mathbf{k}_\alpha-\mathbf{k}_\mu$ on the (x,y) plane. In due turn, this implies that two particles with momenta $\mathbf{k}_\alpha$ and $\mathbf{k}_\mu$ can participate in a collision event with the plane oscillator only if (a) the momenta lie on a plane $k_z=cte$ and (b) for given projection $\mathbf{k}_{\alpha_{xy}}$ of $\mathbf{k}_\alpha$ on the (x,y) plane, $\mathbf{k}_{\mu_{xy}}$ must lie on a circumference of radius q centered at $\mathbf{k}_{\alpha_{xy}}$. Now, as one combines the energy conservation requirement at the collision vertex, one finds the unique solution,

$$|\mathbf{k}_{\alpha_{xy}}|^2=|\mathbf{k}_{\mu_{xy}}|^2+\frac{2m\Omega}{\hbar}\ , \tag{4.2a}$$

$$|\mathbf{k}_{\alpha_{xy}}|^2+|\mathbf{k}_{\mu_{xy}}|^2-2|\mathbf{k}_{\alpha_{xy}}|\ |\mathbf{k}_{\mu_{xy}}|\cos\varphi=q^2\ , \tag{4.2b}$$

$$|\mathbf{k}_{\alpha_{xy}}|>q\left[\frac{mc_s^2}{\hbar\Omega}+\frac{1}{2}\right]=k_0\ , \tag{4.2c}$$

where φ is the polar angle between $\mathbf{k}_{\alpha_{xy}}$ and $\mathbf{k}_{\mu_{xy}}$. Thus, the transition rates $W_{\pm}^{(R)}$ reduce to one integral along the z axis and one integral with respect to $\mathbf{k}_{\alpha_{xy}}$

$$W_{+}^{(R)}=16\pi^2|\lambda^0|^2\int_{k_0}^{\infty}\int_{-\infty}^{\infty}\rho_\mu(1-\rho_\alpha)\left|\frac{q}{|\mathbf{k}_{\alpha_{xy}}|\ |\mathbf{k}_{\mu_{xy}}|\sin\varphi}\right|dk_z\,d\,|\mathbf{k}_{\alpha_{xy}}|\ , \tag{4.3}$$

where $|\mathbf{k}_{\mu_{xy}}|$ and φ are given by Eqs. (4.2a) and (4.2b), respectively, and

$$W_{-}^{(R)}=\exp(-\hbar\Omega/T)W_{+}^{(R)}\ . \tag{4.4}$$

In the present work, the major interest is to solve Eq. (3.6a) for the 2D oscillator. For numerical applications, it is necessary to truncate the infinite system of coupled equations (3.6a), this procedure induces boundary conditions whose appropriate selection may lead to analytical resolution of the spectral problem as in the 1-dim case. In the following we show that this actually happens with the truncation indicated in Fig. 1(a), for sufficiently large N. The allowed transitions are represented by the arrows, the upward (downward) going ones being weighted by $W_{-}^{(R)}(W_{+}^{(R)})$. In addition, a minus sign has to be introduced if the arrow goes out the node whose derivative one is writing. One can easily write any boundary condition with the aid of the drawing; for example, along the edge $0<n<N$, $m>0$ one has

$$\begin{aligned}\dot{\rho}_{nn}=&\ W_{+}^{(R)}(\rho_{n+1,n+1}+\rho_{n+1,n-1}-\rho_{nn})\\&+W_{-}^{(R)}(\rho_{n-1,n-1}-2\rho_{nn})\ .\end{aligned} \tag{4.5}$$

This means we are in presence of a square lattice with perfectly reflecting walls and nearest-neighbor interactions. A $\pi/2$ rotation by means of the change of variables,

$$n_R=(n+m)/2\ , \tag{4.6a}$$

$$n_L=(n-m)/2 \tag{4.6b}$$

gives the lattice geometry shown in Fig. 1(b). It becomes clear that in the (n_R,n_L) plane the 2D oscillator factorizes in two 1D ones and one is in a condition to profit of the already known result.[4,11] The eigenvalues and eigenvectors of the dynamical matrix (3.6a) can be straightforwardly written as

$$\begin{aligned}\lambda_{sr}&=-2[W_{+}+W_{-}-\sqrt{W_{+}W_{-}}(\cos\varphi_s+\cos\varphi_r)]\\&=\lambda_s+\lambda_r\ ,\end{aligned} \tag{4.7a}$$

$$\begin{aligned}V_{sr}^{kl}=&\ \frac{2W_{+}}{N\sqrt{|\lambda_s\lambda_r|}}\alpha^{(k+l)}[\alpha\sin(k\varphi_s)-\sin(k-1)\varphi_s]\\&\times[\alpha\sin(l\varphi_r)-\sin(l-1)\varphi_r]\ ,\end{aligned} \tag{4.7b}$$

where

$$\varphi_s=s\pi/N,\ \ s=1,2,\ldots,N-1 \tag{4.8a}$$

$$\alpha=\sqrt{W_{-}/W_{+}}\ . \tag{4.8b}$$

Hereafter, we drop the labels (R) on the transition rates

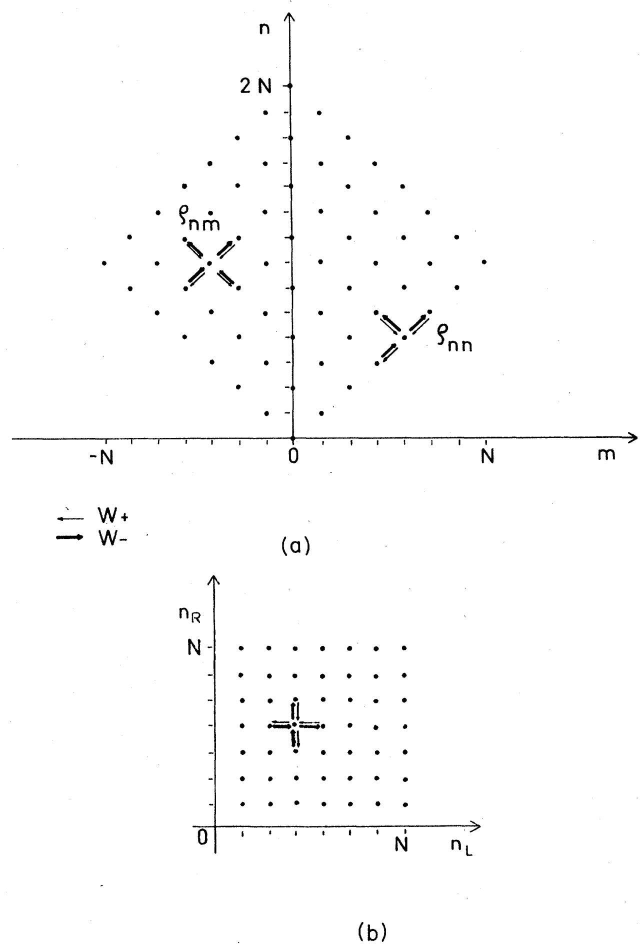

FIG. 1. (a) Representation of the boundary conditions selected for the search of analytical solutions of the master equation (3.6a). Each node represents a state (n,m) of the plane oscillator spectrum populated with an occupation ρ_{nm}. The arrows indicate the allowed transitions and we have chosen to truncate the spectrum along the edges of a square with vertices (0,0), (N,N), $(2N,0)$, and $(N,-N)$. (b) The same lattice rotated according to the transformation in Eqs. (4.6).

since no confusion will arise. The eigenvector V^{00} associated to the eigenvalue $\lambda_{NN}=0$ (or, as one usually denotes it, $\lambda_{00}=0$) corresponds to the equilibrium distribution ρ_0 and turns out to be

$$\rho_{0k,l}=\left[\frac{1-\alpha}{1-\alpha^N}\right]^2\alpha^{2(k+l-1)} . \tag{4.9}$$

The time evolution of the 2D density matrix is then given by the superposition

$$\rho(t)=\rho_0+\sum_{r,s} c_{sr}V^{sr}\exp(\lambda_{sr}t) , \tag{4.10}$$

where the amplitudes c_{sr} are the roots of the linear system

$$\rho_{kl}(0)=\rho_{0kl}+\sum_{s,r} c_{sr}V^{sr}_{kl} . \tag{4.11}$$

Related quantities of interest are (a) the boson energy,

$$\begin{aligned} E_B(t)&=\hbar\Omega\sum_{n,m} n\rho_{nm}(t)\\ &=\hbar\Omega\sum_{n_R,n_L}(n_R+n_L)\rho_{n_R,n_L}(t) , \end{aligned} \tag{4.12}$$

(b) the boson entropy,

$$S_B=-\sum_{n,m}\rho_{nm}(t)\ln\rho_{nm}(t) , \tag{4.13}$$

and (c) the time-dependent effective frequency $\nu_E(t)$ defined as the logarithmic derivative of the energy,

$$\nu_E(t)=\dot{E}_B(t)/E_B(t) . \tag{4.14}$$

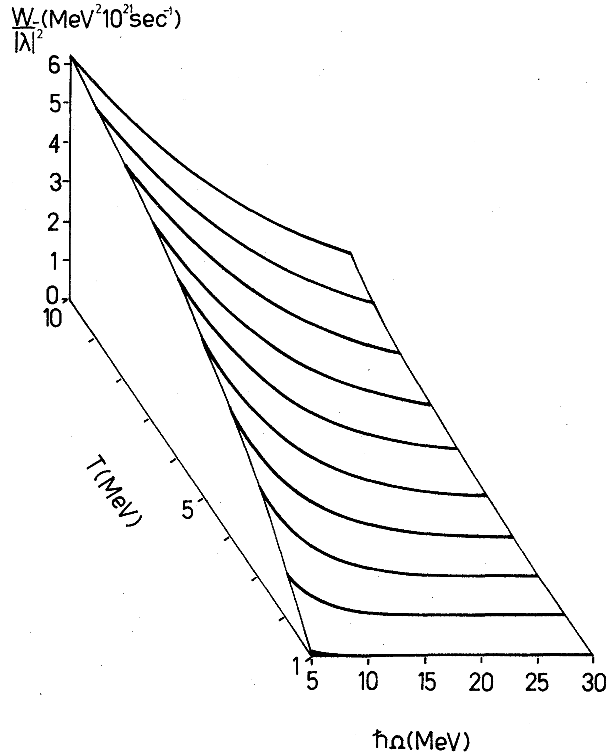

FIG. 3. Same as in Fig. 2 but for the upward transition rate W.

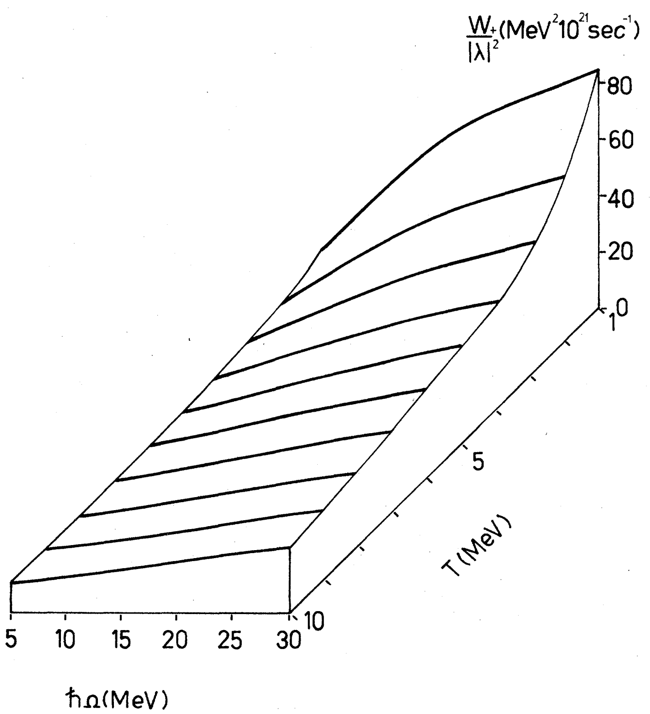

FIG. 2. The downwards transition rate W_+ as a function of phonon energy and heat-bath temperature.

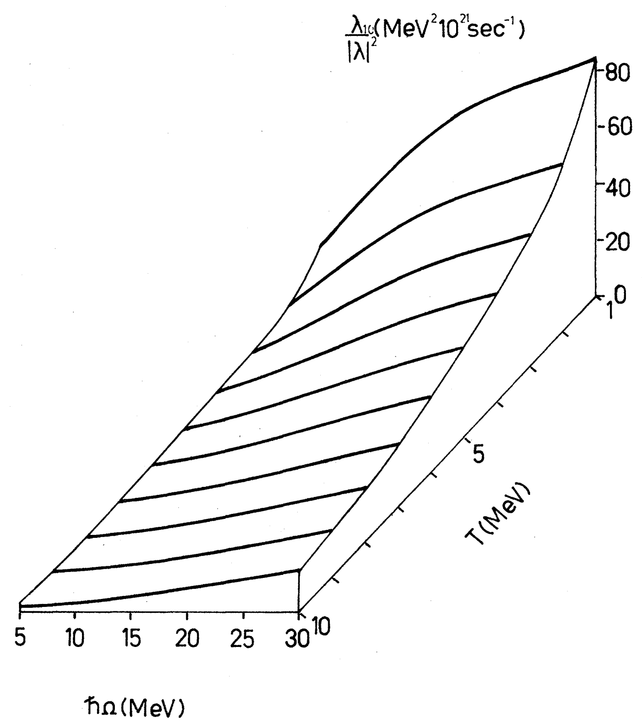

FIG. 4. Same as in Figs. 2 and 3 for the lowest nonvanishing eigenvalue λ_{10}.

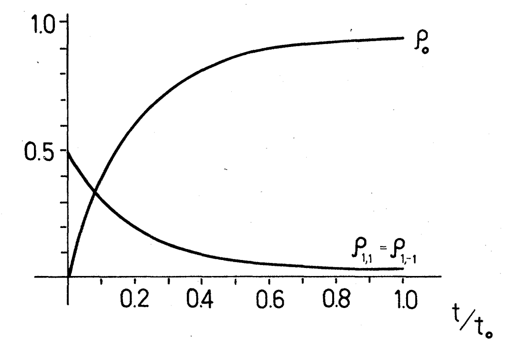

FIG. 5. Time evolution of the significant nonvanishing components of the density matrix ρ_{nm}. The time unit is the period of the unperturbed oscillator, $t_0=2\pi/\Omega$.

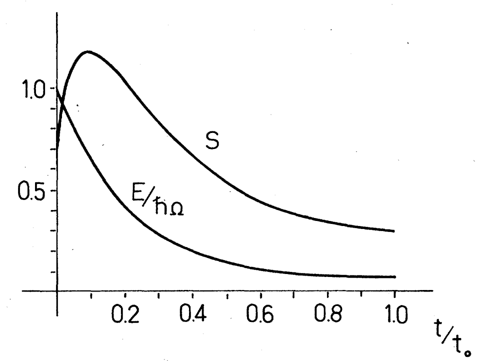

FIG. 6. Time evolution of the oscillator energy (in units of $\hbar\Omega$) and of the entropy (in units of the Boltzmann constant k).

V. CALCULATIONS AND DISCUSSION

We have performed a set of calculations ordered in two groups. (i) The transition rates W_+, W_-, and the lowest lying nonzero eigenvalue $\lambda_{10}=\lambda_{01}$ have been computed as functions of the cylindrical boson energy $\hbar\Omega$ and the equilibrium temperature T.

(ii) The time evolution of the density and the quantities listed at the end of the preceding section have been evaluated for several combinations of the parameters $\hbar\Omega$ and T, so as to sample the whole range of interest of physical applications. The sound velocity c_s in Eq. (3.9a) has been chosen as $c/4$ (where c is the speed of light in vacuum, equal to 300 fm/10^{-21} sec, and in all cases the initial condition has been set as

$$\rho_{nm}(0)=\tfrac{1}{2}\delta_{n1}(\delta_{m,-1}+\delta_{m1}) . \tag{5.1}$$

Figures 2—4 exhibit the transition rates W_+, W_-, and the eigenvalue λ_{10} as functions of $\hbar\Omega$ and T. As an illustration of the time evolution, we show several graphs corresponding to the middle range parameters $\hbar\Omega=18$ MeV and $T=5$ MeV for a plane oscillator initially promoted to its first excited level. The decay of the occupation probability $\rho_{11}=\rho_{1,-1}$ as well as the excitation of the neighboring states is depicted in Fig. 5. In Fig. 6, the energy and the entropy profiles as time elapses are displayed, while the effective frequency ν_E is shown in Fig. 7.

From sets of time-evolution data such as those exhibited in Figs. 5—7, we can extract a few parameters characteristic of the approach to equilibration. Typically, we define a relaxation time or mean life τ_X for the magnitude X, so that

$$X(\tau_X)=\begin{cases} X(\infty)+[X(0)-X(\infty)]/e & \text{if } X \text{ decays ,} \\ (1-1/e)X(\infty) & \text{if } X \text{ populates .}\end{cases} \tag{5.2}$$

Table I exhibits the values of $\tau_{\rho_{11}}$, τ_{ρ_0}, and τ_E for several phonon energies and heat-bath temperatures. For the sake of comparison, we give as well the corresponding numbers for the largest relaxation time eigenvalue λ_{10}^{-1}. All times are given in the nuclear time unit 10^{-21} sec while transition rates and eigenvalues are given in MeV$^2\,10^{21}\,\text{sec}^{-1}/|\lambda|^2$. In addition, in this table we give the saturation (i.e., asymptotic) values of the populations, energy, and entropy.

From inspection of Figs. 2—4 one verifies that, according to Eq. (4.7a), the lowest-lying eigenvalue λ_{10} almost coincides with the downwards transition rate W_+ for low temperatures, where the upwards-going probability measured by W_- is almost vanishing. Correspondingly, increasing the temperature favors the departure of λ_{10} with respect to W_+, due to the growing importance of reexcitation processes. The curvature of either W_+ or λ_{10} reflects the dependence on the exponent appearing in the equilibrium densities ρ_α and ρ_μ, in the integral (4.3). By contrast, the transition rate W_- is smoothed down by the

TABLE I. The effective lifetimes τ_x (in units of 10^{-21} sec) defined in Eq. (5.2) for the first two nonvanishing components of the density matrix and the energy, the inverse of the smallest nonvanishing eigenvalue of the evolution matrix (in units of 10^{-21} sec) and the asymptotic or saturation values of energy (in MeV) and entropy (in units of the Boltzmann's constant k) on a sample of phonon energies $\hbar\Omega$ and heat-bath temperatures T, both in MeV.

		$\hbar\Omega=13$			$\hbar\Omega=18$			$\hbar\Omega=25$	
	$T=1$	$T=5$	$T=10$	$T=1$	$T=5$	$T=10$	$T=1$	$T=5$	$T=10$
$\tau(\rho_{11})$	0.020	0.042	0.034	0.016	0.036	0.038	0.013	0.032	0.038
$\tau(\rho_0)$	0.020	0.048	0.038	0.016	0.040	0.046	0.013	0.032	0.043
$\tau(E)$	0.020	0.052	0.044	0.016	0.042	0.056	0.013	0.033	0.047
$\|\lambda_{10}\|^{-1}$	0.020	0.089	0.210	0.016	0.056	0.136	0.013	0.037	0.085
$E_\infty \| \hbar\Omega$	0.0	0.16	0.74	0.0	0.056	0.4	0.0	0.01	0.18
S_∞/k	0.0	0.58	1.6	0.0	0.26	1.07	0.0	0.08	0.62

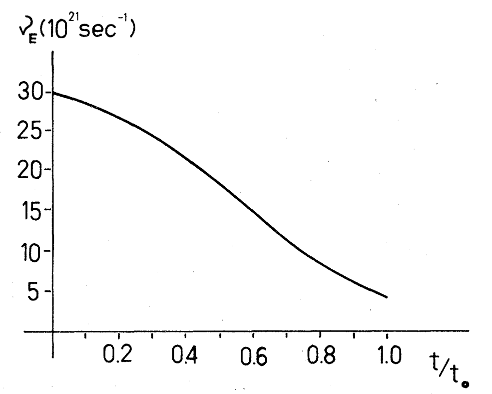

FIG. 7. Time evolution of the logarithmic derivative of the oscillator energy.

Boltzmann factor in Eq. (4.5), as a function of the phonon frequency.

The results of the time evolution are typical of this kind of calculation and display most expected features of equilibration. The population densities (Fig. 5) approach a canonical distribution for the given temperature with the effective time rates given in Table I. The energy and entropy (Fig. 6) reach saturation values that coincide up to three to four decimal figures with those predicted by the statistical equilibrium relations for the internal energy and entropy of a 2D oscillator,

$$U=\hbar\Omega\frac{e^{-\hbar\Omega/T}}{1-e^{-\hbar\Omega/T}}\,, \tag{5.3}$$

$$S=\frac{U}{T}-2\ln(1-e^{-\hbar\Omega/T})\,. \tag{5.4}$$

Due to the degeneracy of the initial pure state of the oscillator, the entropy does not vanish at $t=0$. We appreciate in Fig. 6 that it grows up to a maximum, from which it decays towards saturation below the original value. This behavior is related to the fact that the early evolution is mostly dissipative, as evidenced by the rather slight variation that the effective rate ν_E experiences in the short-time scale (Fig. 7), giving rise to entropy increase. As time proceeds and the system is driven towards equilibration, the entropy evolves to reach the statistical figure provided by Eq. (5.4) for the given temperature; if, as in the case under discussion, this value is smaller that the initial one, an entropy flow from the oscillator into the heat bath is established. On the other hand, we can observe in Fig. 7 the noticeable decrease in the effective decay rate ν_E in the long run, indicating an important presence of diffusionlike events accompanying the spread of the occupation density into the canonical distribution. This situation characterized by a fringe of the oscillator spectrum that becomes populated with some non-negligible probability, demands a large amount of intrinsic configurations to come into play, since there are very many different ways in which the particles in the heat bath can collide with the macroscopic system to maintain invariant the equilibrium density of the latter. This enlarging of fermion phase space gives rise to entropy production, as demanded by the second law of thermodynamics, in order to compensate for the negative flow associated to the entropy loss of the oscillator.

We now turn to the analysis of the mean lives defined in Eq. (5.2) whose values for a set of temperatures and frequencies are shown in Table I. It is not easy to establish any law regarding their variation as functions of $\hbar\Omega$ and T, since these relaxation times mainly reflect a weighted average of the different eigenvalues. It is clear that for the lowest temperatures, the three mean lives are identical and coincident with the largest relaxation time λ_{10}^{-1}. One can realize as well that for any frequency, heating the system introduces some amount of spread among these time parameters that becomes, however, less significant for higher phonon energies.

VI. SUMMARY AND CONCLUSIONS

In this paper we have undertaken a model study of the equilibration of a 3D quantal oscillator placed in a fermionic reservoir. Due to the relevance of this problem to applications in nuclear physics regarding resonance decay in deformed nuclei, we have conceived the heat bath as axially symmetric nuclear matter excited by the lowest cylindrical acoustic mode. The phonons of the wave propagating in the (x,y) plane are considered as the excitation quanta of a 2D oscillator lying in that plane and possessing angular degeneracy. Both the quantal and the statistical equilibrium problems of such an oscillator can be formulated and solved exactly.

We have extended a prior work devoted to a 1D harmonic modes and shown that under a given set of approximations and assumptions involving the Markovian and the weak coupling hypothesis, the dynamics of the combined system can be cast into coupled nonlinear equations of motion. The spectral problem of the master equation describing the evolution of the phonon population density can be exactly solved if one complements the former assumptions with the sharp-resonance approximation meaning energy conserving collision events. With the aid of the analytical eigenvalues and eigenvectors, that one easily finds if the boundary conditions are adequately chosen, it becomes a simple task to describe the motion of physical quantities of interest such as the distribution density of the 2D oscillator, its energy, entropy, and effective decay rate, which can be analyzed as functions of the selected parameters, namely the phonon energy and the temperature of the fermion reservoir.

The study here presented is of such a general character to be profited by other fields of physics requiring some version of the damping process of a quantal harmonic excitation in a many-body system. Further work along this line is in progress aiming at closer approximations to the inspiring problem of nuclear physics.

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