Theory of interparticle correlations in dense, high-temperature plasmas. V. Electric and thermal conductivities

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On the basis of the quantum-statistical formulation of electronic transport, we calculate the electric and thermal conductivities of plasmas in a wide range of densities and temperatures where it is essential to take into account the varied degrees of electron degeneracy and local-field corrections describing the strong Coulomb-coupling effects. The physical implications of the results are investigated through comparison with other theories and experiments. For utility in the practical applications, we derive the analytic formulas parametrizing the computed results accurately for the generalized Coulomb logarithms appearing in those conductivities.

I. INTRODUCTION

In a recent series of publications,[1] Mitake, Yan, and the present authors developed a general theory in which the interparticle correlations in dense, high-temperature multicomponent plasmas were formulated systematically over a wide range of plasma parameters; various correlation functions, the thermodynamic quantities, and the stopping power against an injected charged particle were thereby calculated. The parameter domain of interest covered those appropriate to the inertial-confinement-fusion experiments[2] and the interior of the main-sequence stars.

In this paper we wish to extend those theories and now consider the problems of the electronic transport in such high-density plasmas. We thus calculate the electric and thermal conductivities of the plasmas over the same parameter domain as treated in the previous papers:[1] Plasmas are assumed to be fully ionized; the relativistic effects in the electrons and the quantum-mechanical effects in the ions[3] are ignored.

The salient physical features which we account for in the theory are the varied degrees of Fermi degeneracy in the electrons and the strong Coulomb-coupling effects described by the various local-field corrections in the density-density response functions.[4] We shall show that the resulting theory is capable of describing the transport coefficients accurately over a wide range of the density and temperature parameters.

We begin with the quantum-statistical description of the electrons and approach the transport problems through consideration of their scattering in the random potential fields produced by the ions. In Sec. II, we compile the basic formulas for the electric and thermal conductivities obtained in this approach. In Sec. III, the generalized Coulomb logarithms characterizing those conductivities are investigated and are explicitly calculated for various combinations of the density and temperature parameters; analytic formulas parametrizing those numerical results are also presented. The results of the theory are compared with those of other theories in Sec. IV, and with experimental data in Sec. V. Derivations or a heuristic account of some of the basic formulas are described in the Appendixes. In this paper we follow the notation and convention adopted in the previous papers,[1] unless otherwise specified.

II. ELECTRIC AND THERMAL CONDUCTIVITIES

We consider a fully ionized two-component plasma (TCP) consisting of the electrons (mass $m_e = m$, charge $-e$, number density $n_e = n$) and the ions (mass $m_i = M$, charge Ze, number density $n_i = n/Z$). As we did in the previous papers[1] of this series, we define a set of dimensionless parameters as

$$\Gamma = \frac{(Ze)^2}{k_B T}\left[\frac{4\pi n_i}{3}\right]^{1/3}, \tag{1}$$

$$\theta = \frac{2mk_B T}{\hbar^2}(3\pi^2 n_e)^{-2/3}, \tag{2}$$

$$r_s = \left[\frac{3}{4\pi n_e}\right]^{1/3}\frac{me^2}{\hbar^2} = \frac{1}{2}\left[\frac{9\pi}{4}\right]^{2/3} Z^{-5/3}\Gamma\theta . \tag{3}$$

These, respectively, mean the Coulomb-coupling constant of the ions, the Fermi degeneracy parameter, and the density parameter of the electrons.

The electric resistivity ρ stemming from scattering of the electronic current by the random potential fields produced by the ions is expressed as

$$\rho = \frac{m^2 n_i}{12\pi^3\hbar^3 e^2 n_e^2}\int_0^\infty dk\, k^3 f_0(k/2)\left|\frac{v_{ei}(k)}{\epsilon(k)}\right|^2 S_{ii}(k) . \tag{4}$$

Here $S_{ii}(k)$ is the static structure factor of the ions in the TCP, defined and calculated in the previous papers,[1]

$$f_0(k) = \left[\exp\left[\frac{\hbar^2 k^2}{2mk_B T} - \alpha\right] + 1\right]^{-1} \tag{5}$$

is the Fermi distribution function with the normalization

$$n_e=\frac{1}{\pi^2}\int_0^\infty dk\, k^2 f_0(k) \tag{6}$$

which determines $\alpha=\mu/k_BT$,

$$v_{ei}(k)=-4\pi Ze^2/k^2 \tag{7}$$

is the Fourier transform of the bare electron-ion interaction, and

$$\epsilon(k)=1+[1-G_e(k)]\frac{4me^2}{\pi\hbar^2k^3}\int_0^\infty dq\, qf_0(q)\ln\left|\frac{2q+k}{2q-k}\right| \tag{8}$$

is the electronic screening function of the electron-ion interactions.[5] In Eq. (8), $G_e(k)$ represents the local-field correction between electrons, defined and discussed in the previous papers.[1] A derivation of Eq. (4) is given in Appendix A.

Expressions analogous to Eq. (4) were obtained earlier by Boercker, Rogers, and DeWitt.[6,7] The major difference appears to lie in the use of the Fermi distribution, Eq. (5), and the exact ionic structure factor for the TCP in Eq. (4), while Boercker *et al.* rely on a certain approximate relation between the classical dielectric functions and the ionic structure factor.

In the limit of $T\to 0$, the Fermi function $f_0(k)$ becomes a unit step function, so that Eq. (4) reduces to the Ziman formula,[8]

$$\rho=\frac{m^2n_i}{12\pi^3\hbar^3e^2n_e^2}\int_0^{2k_F}dk\, k^3\left|\frac{v_{ei}(k)}{\epsilon(k)}\right|^2 S_{ii}(k)\ , \tag{9}$$

where

$$k_F=(3\pi^2n_e)^{1/3}\ . \tag{10}$$

In the classical limit $\theta\gg 1$ for the electrons, Eq. (4) takes the expression

$$\rho=\left[\frac{m}{2\pi k_BT}\right]^{3/2}\frac{n_i}{3e^2mn_e}\times\int_0^\infty dk\, k^3\left|\frac{v_{ei}(k)}{\epsilon(k)}\right|^2 S_{ii}(k)\exp\left[-\frac{\hbar^2k^2}{8mk_BT}\right]\ . \tag{11}$$

The exponential factor in the integrand of Eq. (11) introduces a natural convergence of the integration in the large-k domain, with an effective cutoff approximately at the inverse of the thermal de Broglie wavelength, $\hbar/(2mk_BT)^{1/2}$. The concept of large-k cutoff by such a quantum-mechanical diffraction effect is valid for high-temperature plasmas with $k_BT\gtrsim(Ze^2)^2m/2\hbar^2$. Otherwise a usual classical cutoff at about the inverse of the Landau length, Ze^2/k_BT, is applicable.

It is well known[9–11] that the magnitude of the electric resistivity calculated according to the classical scheme of Eq. (11) takes on a value 1.97 times as large as the Spitzer value[12,13] for $Z=1$, because possible deformation of the electronic distribution function is not appropriately taken into account in the calculation leading to Eq. (4). The factor

$$A=1.97 \tag{12}$$

thus corresponds to the ratio between the electric resistivity calculated in a single Sonine polynomial approximation and that evaluated by a summation of infinite terms in the Sonine polynomial expansion for $Z=1$. In the comparison with experiments, to be carried out later in Sec. V, we shall take account of the factor A given by Eq. (12).

We now turn to consideration of the thermal conductivity for TCP. As we show its derivation in Appendix A, we have an expression

$$\frac{1}{\kappa}=\frac{\theta^{9/2}}{16\sqrt{2}\pi\Sigma^2}\frac{m^{1/2}n_i}{k_B(k_BT)^{5/2}n_e}\times\int_0^\infty dk\, k^3\left|\frac{v_{ei}(k)}{\epsilon(k)}\right|^2 S_{ii}(k)\times\int_{k/2k_F}^\infty dx\, x(x^2-\lambda)^2\frac{\partial f_0(k_Fx)}{\partial\alpha}\ , \tag{13}$$

corresponding to Eq. (4) for the electric resistivity. In Eq. (13), we define

$$\Sigma=\frac{7}{4}\frac{\theta^{9/2}}{I_{1/2}(\alpha)}\{I_{5/2}(\alpha)I_{1/2}(\alpha)-\tfrac{25}{21}[I_{3/2}(\alpha)]^2\}\ , \tag{14}$$

$$\lambda=\frac{5}{3}\theta\frac{I_{3/2}(\alpha)}{I_{1/2}(\alpha)}\ , \tag{15}$$

and

$$I_\nu(\alpha)=\int_0^\infty dz\frac{z^\nu}{\exp(z-\alpha)+1} \tag{16}$$

are the Fermi integrals.

In the limit of $T\to 0$, Eq. (13) reduces to

$$\frac{1}{\kappa}=\frac{m^2n_i}{4\pi^5\hbar^3n_e^2k_B^2T}\int_0^{2k_F}dk\, k^3\left|\frac{v_{ei}(k)}{\epsilon(k)}\right|^2 S_{ii}(k)\ . \tag{17}$$

A procedure of this derivation is described in Appendix B. The ratio between Eqs. (9) and (17) yields the usual Wiedemann-Franz relation[14] at $\theta\ll 1$, i.e.,

$$\kappa\rho=\frac{\pi^2}{3}\left[\frac{k_B}{e}\right]^2T\ . \tag{18}$$

In the classical limit $\theta\gg 1$, we find from Eq. (13)

$$\frac{1}{\kappa}=\frac{13}{75(2\pi)^{3/2}}\frac{m^{1/2}n_i}{k_B(k_BT)^{5/2}n_e}\times\int_0^\infty dk\, k^3\left|\frac{v_{ei}(k)}{\epsilon(k)}\right|^2 S_{ii}(k)\exp\left[-\frac{\hbar^2k^2}{8mk_BT}\right]\ . \tag{19}$$

Again we remark that the value calculated through Eq. (19) is 1.66 times as large as the Spitzer value[12,13] for $Z=1$, due to the fact that Eq. (13) has been obtained in the single Sonine polynomial approximation.

III. GENERALIZED COULOMB LOGARITHMS

In this section and hereafter we confine ourselves to consideration of TCP with $Z=1$ for simplicity. We rewrite Eqs. (4) and (13) as

$$\rho=4\left[\frac{2\pi}{3}\right]^{1/2}\frac{\Gamma^{3/2}}{\omega_p}L_E\ , \tag{20}$$

$$\frac{1}{\kappa}=\frac{52(6\pi)^{1/2}}{75}\frac{e^2}{k_B^2T}\frac{\Gamma^{3/2}}{\omega_p}L_T\ , \tag{21}$$

where $\omega_p=(4\pi ne^2/m)^{1/2}$. We shall call L_E and L_T the generalized Coulomb logarithms, which are defined and calculated according to

$$L_E(\Gamma,\theta)=\frac{3\sqrt{\pi}\theta^{3/2}}{4}\int_0^\infty\frac{dk}{k}f_0(k/2)\frac{S_{ii}(k)}{|\epsilon(k)|^2}\ , \tag{22}$$

$$L_T(\Gamma,\theta)=\frac{75\sqrt{\pi}}{104}\frac{\theta^{9/2}}{\Sigma^2}\int_0^\infty\frac{dk}{k}\frac{S_{ii}(k)}{|\epsilon(k)|^2}\times\int_{k/2k_F}^\infty dx\,x(x^2-\lambda)^2\times\frac{\partial f_0(k_Fx)}{\partial\alpha}\ . \tag{23}$$

In the classical limit $\theta\gg1$, both L_E and L_T approach the same expression:

$$L_0=\int_0^\infty\frac{dk}{k}\frac{S_{ii}(k)}{|\epsilon(k)|^2}\exp\left[-\frac{\hbar^2k^2}{8mk_BT}\right]\ . \tag{24}$$

Assuming the weak-coupling cases $\Gamma\ll1$, we may substitute the Debye-Hückel formulas[15]

$$S_{ii}(k)=\frac{k^2+k_D^2}{k^2+2k_D^2}\ , \tag{25}$$

$$\epsilon(k)=1+\frac{k_D^2}{k^2} \tag{26}$$

in Eq. (24), to find

$$L_0=\exp(\zeta)E_1(\zeta)-\tfrac{1}{2}\exp(\zeta/2)E_1(\zeta/2)\ . \tag{27}$$

Here

$$k_D^2=4\pi ne^2/k_BT\ , \tag{28}$$

$$\zeta=\frac{\hbar^2k_D^2}{4mk_BT}=\frac{3}{2}\left[\frac{4}{9\pi}\right]^{2/3}\frac{\Gamma}{\theta}\ , \tag{29}$$

and

$$E_1(x)=\int_x^\infty\frac{dt}{t}\exp(-t) \tag{30}$$

is the exponential integral. Since $\zeta\ll1$, we may expand Eq. (27) as

$$L_0=-\tfrac{1}{2}\ln\zeta-\tfrac{1}{2}(\gamma+\ln2)-\tfrac{3}{4}\zeta(\ln\zeta-1+\gamma+\tfrac{1}{3}\ln2)+O(\zeta^2,\zeta^2\ln\zeta)\ , \tag{31}$$

where $\gamma=0.57721\ldots$ is Euler's constant. The leading terms in this expansion were obtained by Kivelson and DuBois[16] with the aid of the quantum-mechanical version of the Lenard-Balescu-Guernsey equation; those terms were used later by Williams and DeWitt[10] to construct a connecting formula for the Coulomb logarithm between the high-temperature ($k_BT>me^4/2\hbar^2$) and the low-temperature ($k_BT<me^4/2\hbar^2$) domains.

We have explicitly carried out numerical calculations of Eqs. (22) and (23) for 21 parametric combinations ranging over $0.003\le\Gamma\le2$ at $\theta=0.1$, 1, and 10. The functional values of $G_e(k)$ and $S_{ii}(k)$ necessary for those calculations have been obtained in the hypernetted chain (HNC) scheme[17,18] developed in the previous papers[1] of this series. The results of the calculations are listed in Table I. We note that $L_T/L_E\to1$ in the classical limit ($\theta\gg1$) and that $L_T/L_E\to75/13\pi^2=0.5845$ in the limit of complete Fermi degeneracy ($\theta\to0$).

For convenience in practical applications, we derive analytic formulas which parametrize those computed results: For the generalized Coulomb logarithm in the electric resistivity, we find an expression applicable over the domain $\Gamma\le2$ and $\theta\ge0.1$,

$$L_E(\Gamma,\theta)=\frac{a(\theta)\ln\Gamma+b(\theta)+c(\theta)\Gamma}{1+d(\theta)\Gamma^3}\ , \tag{32}$$

where

$$a(\theta)=\frac{-\theta^{3/2}}{2\theta^{3/2}-0.579\,23\theta+0.232\,72\theta^{1/2}+1.4853}\ ,$$

$$b(\theta)=\frac{\theta^3(\frac{1}{2}\ln\theta-0.186\,03)+1.2704\theta^{3/2}}{\theta^3+1.8993\theta^2+4.3243\theta+1}\ ,$$

$$c(\theta)=\theta^{3/2}\frac{0.694\,60\theta^{1/2}+0.242\,28}{\theta^2+1.7768}\ , \tag{33}$$

$$d(\theta)=\theta^{3/2}\frac{0.135\,50\theta^{1/2}+0.083\,521}{\theta^2+0.367\,97}\ .$$

For the generalized Coulomb logarithm in the thermal conductivity, we obtain again for $\Gamma\le2$ and $\theta\ge0.1$

TABLE I. The generalized Coulomb logarithms L_E and L_T for the electric and thermal conductivities defined by Eqs. (22) and (23).

θ	Γ	L_E	L_T	L_T/L_E
10	0.003	3.893	3.591	0.9224
10	0.01	3.255	2.954	0.9075
10	0.03	2.700	2.402	0.8896
10	0.1	2.143	1.850	0.8633
10	0.2	1.852	1.563	0.8440
10	0.5	1.530	1.249	0.8163
10	1	1.354	1.076	0.7947
1	0.01	1.599	1.391	0.8699
1	0.03	1.249	1.048	0.8391
1	0.05	1.094	0.8985	0.8213
1	0.1	0.8940	0.7083	0.7923
1	0.2	0.7123	0.5402	0.7584
1	0.3	0.6180	0.4551	0.7364
1	0.5	0.5145	0.3639	0.7073
1	0.7	0.4566	0.3140	0.6877
1	1	0.4045	0.2700	0.6675
0.1	0.03	0.097 63	0.062 98	0.6451
0.1	0.1	0.073 29	0.046 80	0.6386
0.1	0.3	0.052 84	0.033 26	0.6294
0.1	1	0.034 55	0.021 26	0.6153
0.1	2	0.026 81	0.016 23	0.6054

$$L_T(\Gamma,\theta)=\frac{p(\theta)\ln\Gamma+q(\theta)+r(\theta)\Gamma}{1+s(\theta)\Gamma^3}\ , \tag{34}$$

where

$$p(\theta)=\frac{-\theta^{3/2}}{2\theta^{3/2}+0.0292\,20\theta-1.46661\theta^{1/2}+2.6858}\ ,$$

$$q(\theta)=\frac{\theta^3(\frac{1}{2}\ln\theta-0.186\,03)-0.987\,87\theta^2+0.874\,22\theta^{3/2}}{\theta^3+4.93127\theta^2+\theta+1}\ ,$$

$$r(\theta)=\theta^{3/2}\frac{0.636\,07\theta^{1/2}+0.033\,439}{\theta^2-0.361\,86\theta+1}\ , \tag{35}$$

$$s(\theta)=\theta^{3/2}\frac{0.031\,856\theta^{1/2}+0.424\,60}{\theta^2-0.299\,33\theta+0.5}\ .$$

Those fitting formulas reproduce all the computed values in Table I with digressions of less than 2% for L_E and 1% for L_T.

The analytic forms of those formulas retain the following features.

(i) In the classical ($\theta\gg1$) and weak-coupling ($\Gamma\ll1$) limit, Eqs. (32) and (34) reproduce the first two terms on the right-hand side of Eq. (31).

(ii) In the limit of complete Fermi degeneracy ($\theta\to0$), L_E and L_T behave proportionally to $\theta^{3/2}$, as Eq. (22) illustrates.

(iii) In the strong-coupling regime ($\Gamma\gg1$), both Eq. (32) and Eq. (34) take forms proportional to Γ^{-2}. This is a consequence of the ion-sphere scaling in the interparticle correlations for a strong-coupling plasma;[4] a heuristic account of this effect is given in Appendix C.

IV. COMPARISON WITH OTHER THEORIES

The electric conductivity of a strongly coupled hydrogen plasma has been investigated recently by a number of authors.[5–7,19–22]

Baus, Hansen, and Sjögren[20] (BHS) treated the electron-ion plasma as a classical system of pseudoparticles interacting by those effective pair potentials which simulate quantum diffraction effects at short distances. Starting with a memory-function formalism coupled with the Green-Kubo formula, they obtained an approximate expression for the generalized Coulomb logarithm as

$$L_{\rm BHS}=\int_0^\infty\frac{dk}{k}\frac{[S_{ee}(k)S_{ii}(k)-S_{ei}^2(k)]}{(1+\hbar^2k^2/2\pi mk_BT)^2}\ . \tag{36}$$

The static structure factors, $S_{ee}(k)$, $S_{ii}(k)$, and $S_{ei}(k)$, were determined by solving the HNC equations for the system of pseudoparticles. As they noted, the weak-coupling (Debye-Hückel) limit of Eq. (36) differs slightly from Eq. (31).

On the basis of the quantum kinetic theory for the current-current correlation functions, Boercker, Rogers, and DeWitt[6,7] (BRD) obtained an expression similar to Eq. (4). In the classical strong-coupling regime they thus proposed a generalized Coulomb logarithm in the form

$$L_{\rm BRD}=-\frac{1}{4\pi e^2}\int_0^\infty dk\,k\frac{u_{ei}(k)}{|\epsilon_0(k)|^2}S_{ii}(k)\exp\left[-\frac{\hbar^2k^2}{8mk_BT}\right]\ , \tag{37}$$

TABLE II. Values of σ^* calculated in various schemes at $r_s=0.4$. "Present" uses Eq. (32) in place of L in Eq. (39); BHS, Eq. (36); BRD, Eq. (37); and MD refers to the molecular-dynamics values obtained by Hansen and McDonald (Ref. 21).

Γ	θ	Present	BHS	BRD	MD
0.05	4.34	14.97	14.18	16.2	
0.1	2.17	7.95	7.13	8.61	
0.2	1.09	4.97	3.99	5.33	
0.5	0.434	3.86	2.30	4.13	3.6
1.0	0.217	3.85	1.87	5.29	
2.0	0.109	3.99		12.3	

where $\epsilon_0(k)$ is the classical limit of Eq. (8),

$$u_{ei}(k)=-k_BTc_{ei}(k)\ , \tag{38}$$

and $c_{ei}(k)$ refers to the Fourier transform of the direct correlation function between the electrons and the ions. The functions, $S_{ii}(k)$, $u_{ei}(k)$ and $\epsilon_0(k)$, in Eq. (37) were then determined in the HNC scheme. In the weak-coupling limit, $u_{ei}(k)$ approaches $v_{ei}(k)$, so that Eq. (37) reproduces Eq. (31).

In Tables II and III, we compare the values of a normalized electric conductivity

$$\sigma^*=1.93\left[\frac{3\pi}{2}\right]^{1/2}\frac{1}{4\pi\Gamma^{3/2}L} \tag{39}$$

calculated in various theories at $r_s=0.4$ and 1.0. For the present work, we take $L=L_E$, Eq. (22) or (32); for BHS, $L=L_{\rm BHS}$, Eq. (36); and for BRD, $L=L_{\rm BRD}$, Eq. (37). We also list the values obtained in the molecular dynamics (MD) simulation carried out by Hansen and McDonald.[21] The particular choice of the form, Eq. (39), follows the example set by BHS (Ref. 20) and BRD.[7]

We remark at this stage that the calculation scheme of BHS, BRD, or even MD cannot in principle predict a correct value of the electric conductivity for $\theta\lesssim1$ because the classical statistics is used for the electrons. When the effect of Fermi degeneracy is weak ($\theta>1$), however, we find that the values listed in Tables II and III show fairly good agreement with each other.

The electric and thermal conductivities of dense plasmas with strong Coulomb coupling (for the ions) and with strong Fermi degeneracy (for the electrons) have been studied by Minoo, Deutsch, and Hansen (MDH),[19] by Itoh *et al.*,[22] and by Ichimaru and Iyetomi,[5] on the basis of Ziman formulas as given by Eqs. (9) and (17). In Figs. 1–3, we compare the present results, Eqs. (20) and (21) with Eqs. (32) and (34) extrapolated into the small-θ domain,

TABLE III. Same as Table II, but with $r_s=1.0$.

Γ	θ	Present	BHS	BRD	MD
0.05	10.9	12.00	11.72	12.7	
0.1	5.43	5.90	5.57	6.16	
0.2	2.72	3.19	2.87	3.36	
0.5	1.09	1.74	1.43	2.07	2.15
1.0	0.543	1.47	0.99	2.13	
2.0	0.272	1.56	0.82	3.72	1.1

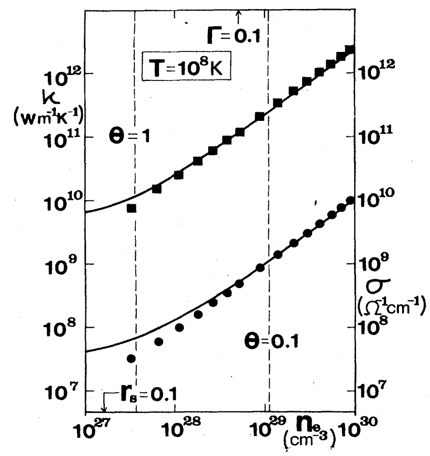

FIG. 1. Comparison of the electronic transport coefficients of hydrogen plasmas at $T=10^8$ K. Above, the thermal conductivity κ: curve refers to the present result based on Eqs. (21), (34), and (35); solid squares, MDH (Ref. 19). Below, the electric conductivity $\sigma=1/\rho$: curve is our result based on Eqs. (20), (32), and (33); solid circles, MDH (Ref. 19).

with those of MDH.[19] As one would expect, a good agreement is observed for $\theta \lesssim 0.3$, say.

We have thus shown that the parametrization expressed by Eqs. (32)—(35) is capable of describing accurately the electric and thermal conductivities of TCP with $Z=1$ over a wide range of the plasma parameters.

V. COMPARISON WITH EXPERIMENT

In a remarkable experiment, Ivanov, Mintsev, Fortov, and Dremin[23] measured the Coulomb conductivity of nonideal plasmas which were produced by a dynamic method based on compression and irreversible heating of gases in the front of high-power ionizing shock waves. Gases used were argon, xenon, neon, and air; those were regarded as forming singly ionized ($Z=1$) plasmas. Each of the experimental values $\sigma_{\rm expt}$ for the Coulomb conductivity listed in Table IV derives from an average of five to ten independent measurements and is attached to a 10—50% error bar.

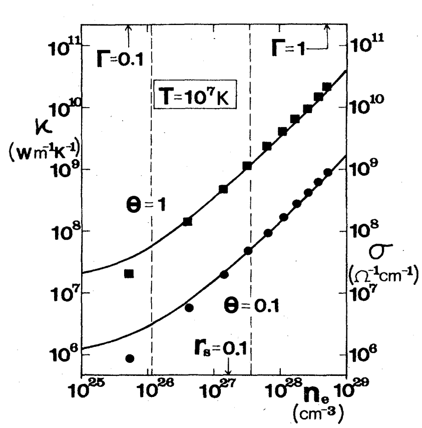

FIG. 2. The electronic transport coefficients at $T=10^7$ K; otherwise the same as in Fig. 1.

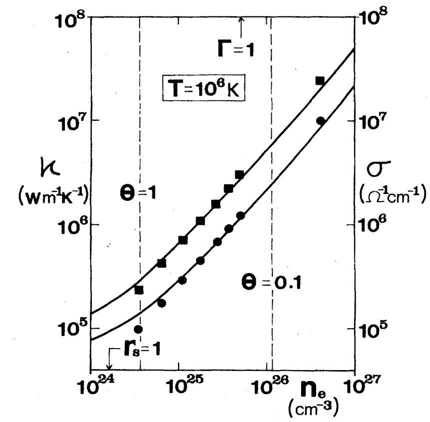

FIG. 3. The electronic transport coefficients at $T=10^6$ K; otherwise the same as in Fig. 1.

We compare those experimental values with the theoretical predictions of Sec. III. Since the classical statistics applies to the electrons for all the cases of the experiment, we take account of the factor, Eq. (12), and write

$$\sigma=1.97\left[\frac{3\pi}{2}\right]^{1/2}\frac{\omega_p}{4\pi\Gamma^{3/2}L}\ . \tag{40}$$

When L_E given by eq. (32) is substituted in place of L, we denote the resulting value of Eq. (40) as $\sigma_{\rm theor}$. When the first two terms on the right-hand side of Eq. (31) are used for L in Eq. (40), the resulting value of σ is called σ_0. In Table IV, the calculated values of $\sigma_{\rm theor}$ and σ_0 are listed.

In the weak-coupling domain $\Gamma<1$, we find that $\sigma_{\rm expt}$ is fairly well represented by σ_0, except for the second and the third cases of the Ar experiments in Table IV. In the four strong-coupling cases ($\Gamma>1$) of Xe, however, σ_0 shows a large departure from $\sigma_{\rm expt}$, which increases systematically with Γ.

In the comparison between $\sigma_{\rm expt}$ and $\sigma_{\rm theor}$, such a systematic discrepancy is completely erased, and we now find that the values of

$$\delta=|\sigma_{\rm expt}-\sigma_{\rm theor}|/\sigma_{\rm expt} \tag{41}$$

are confined within 0.31 for all the 15 cases of the experiment. In view of the large error bars associated with the experimental data, we find such an overall agreement to be rather remarkable. We wish to emphasize in this connection that the generalized Coulomb logarithms are functions of two parameters Γ and θ, rather than of a single parameter Γ, even for those plasmas where the electrons may obey the classical statistics.

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TABLE IV. Experimental values σ_{expt} of the electric conductivity for Ar, Xe, Ne, and air plasmas measured by Ivanov *et al.* (Ref. 23) and the theoretical predictions σ_{theor} and σ_0 based on Eq. (40).

Gas	T (10^3 K)	n_e (cm^{-3})	Γ	θ	σ_{expt} ($\Omega^{-1}\text{cm}^{-1}$)	σ_{theor} ($\Omega^{-1}\text{cm}^{-1}$)	σ_0 ($\Omega^{-1}\text{cm}^{-1}$)
Ar	22.2	2.8×10^{19}	0.368	56.9	190	200	218
	20.3	5.5×10^{19}	0.505	33.2	155	203	231
	19.3	8.1×10^{19}	0.604	24.4	170	209	246
	19.0	1.4×10^{20}	0.736	16.7	255	234	290
	17.8	1.7×10^{20}	0.838	13.7	245	232	301
Xe	30.1	2.5×10^{20}	0.564	17.9	450	442	518
	27.5	5.9×10^{20}	0.822	9.24	680	506	680
	27.0	7.9×10^{20}	0.922	7.47	740	546	789
	26.1	1.4×10^{21}	1.15	4.93	690	657	1204
	25.1	1.6×10^{21}	1.26	4.34	780	660	1389
	24.6	2.0×10^{21}	1.38	3.66	1040	728	1957
	22.7	2.0×10^{21}	1.50	3.38	930	694	2352
Ne	19.8	1.1×10^{19}	0.303	94.6	130	148	158
	19.6	1.9×10^{19}	0.367	65.0	165	160	175
Air	11.0	1.3×10^{18}	0.267	218	60	53.1	56.0

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APPENDIX A: QUANTUM-MECHANICAL TRANSPORT EQUATIONS FOR ELECTRONS

We consider the kinetic equation for the electrons in random potential fields $\tilde{\phi}_t(\mathbf{r},t)$. Let $a_{\mathbf{p}}^{\dagger}$ and $a_{\mathbf{p}}$ be the creation and annihilation operators for an electron in a plane-wave state with momentum $\mathbf{p}$. The spin indices are suppressed for simplicity and we shall take account of spin dependence only through its degree of freedom 2. We shall be working on physical quantities Fourier-transformed in space and time, so that

$$\tilde{\phi}_t(\mathbf{r},t)=\sum_{\mathbf{k}}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\phi_t(\mathbf{k},\omega)\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]\ , \tag{A1}$$

where we adopt the periodic boundary conditions appropriate to a cube of unit volume in the spatial Fourier transformation.

The Hamiltonian for the electron system is written as

$$H=2\sum_{\mathbf{p}}\frac{p^2}{2m}a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}} -e\sum_{\mathbf{k}}\sum_{\mathbf{p}}\int\frac{d\omega}{2\pi}\phi_t(\mathbf{k},\omega)\xi(\mathbf{p},-\mathbf{k})\exp(-i\omega t)\ , \tag{A2}$$

where

$$\xi(\mathbf{p},\mathbf{k})=2a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}+\hbar\mathbf{k}} \tag{A3}$$

is the electron-hole pair operator. The Heisenberg equation of motion for this operator reads

$$i\frac{\partial}{\partial t}\xi(\mathbf{p},\mathbf{q}) =(\Delta_{\mathbf{p}}^{\mathbf{q}}\varepsilon_p)\xi(\mathbf{p},\mathbf{q}) +\frac{e}{2}\sum_{\mathbf{k}}\int\frac{d\omega}{2\pi}\Delta_{\mathbf{p}}^{\mathbf{k}}\{\xi(\mathbf{p},\mathbf{q}-\mathbf{k}),\phi_t(\mathbf{k},\omega)\}\exp(-i\omega t)\ . \tag{A4}$$

Here

$$\varepsilon_p=p^2/2m\ , \tag{A5}$$

$$\{A,B\}=AB+BA\ , \tag{A6}$$

and we have introduced a difference operator $\Delta_{\mathbf{p}}^{\mathbf{k}}$ in the momentum space, so that in general

$$\Delta_{\mathbf{p}}^{\mathbf{k}}f(\mathbf{p})=\frac{1}{\hbar}[f(\mathbf{p}+\hbar\mathbf{k})-f(\mathbf{p})]\ . \tag{A7}$$

We now transform Eq. (A4) into the transport equations of our interest. To do so we take a statistical average, denoted by $\langle\ \rangle$, of this equation. For the potential field, we assume

$$\phi_t(\mathbf{k},\omega)=\langle\phi_t(\mathbf{k},\omega)\rangle+\phi(\mathbf{k},\omega) =2\pi i\frac{\mathbf{k}}{k^2}\cdot\mathbf{E}\delta_{\mathbf{k},0}\delta(\omega)+\phi(\mathbf{k},\omega) \tag{A8}$$

so that a uniform dc electric field $\mathbf{E}$ is applied to the system; $\phi(\mathbf{k},\omega)$ then represents the fluctuating internal potential fields produced by the electrons and ions. We also note that the Wigner distribution $F(\mathbf{r},\mathbf{p})$ is given by

$$F(\mathbf{r},\mathbf{p})=\sum_{\mathbf{q}}\langle\xi(\mathbf{p},\mathbf{q})\rangle\exp(i\mathbf{q}\cdot\mathbf{r})\ . \tag{A9}$$

We are concerned with the analyses of the transport processes in the presence of the dc electric field $\mathbf{E}$ or the weak temperature gradient $\partial T/\partial\mathbf{r}$. The Wigner distribution in these circumstances can be calculated by consideration of contributions only from the vicinity of $\mathbf{q}=\mathbf{0}$ in Eq. (A4); we thus find

$$\frac{\partial}{\partial t}F(\mathbf{r},\mathbf{p})=-\frac{\partial \varepsilon_p}{\partial \mathbf{p}}\cdot\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial \mathbf{r}}+e\mathbf{E}\cdot\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}} -i\frac{e}{2}\sum_{\mathbf{k}}\int\frac{d\omega}{2\pi}\Delta_{\mathbf{p}}^{\mathbf{k}}\langle\{\xi(\mathbf{p},-\mathbf{k}),\phi(\mathbf{k},\omega)\}\rangle \times\exp(-i\omega t)\ . \tag{A10}$$

Introducing the Fourier components of the electron density fluctuations in the phase space through

$$\delta N(\mathbf{k},\omega;\mathbf{p})=\int_{-\infty}^{\infty}dt\,\xi(\mathbf{p},\mathbf{k})\exp(i\omega t)\ , \tag{A11}$$

we obtain the transport equation,

$$\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial t}+\frac{\mathbf{p}}{m}\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial \mathbf{r}}-e\mathbf{E}\cdot\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}} =ie\int\frac{d\mathbf{k}}{(2\pi)^3}\int d\omega\,\Delta_{\mathbf{p}}^{\mathbf{k}}\langle\delta N\,\phi^*(\mathbf{k},\omega;\mathbf{p})\rangle\ , \tag{A12}$$

where $\langle\delta N\,\phi^*(\mathbf{k},\omega;\mathbf{p})\rangle$ is a spectral function defined via[15]

$$\langle\delta N(\mathbf{k},\omega;\mathbf{p})\phi(-\mathbf{k},\omega')\rangle=(2\pi)^2\langle\delta N\,\phi^*(\mathbf{k},\omega;\mathbf{p})\rangle\delta(\omega+\omega')\ . \tag{A13}$$

The right-hand side of Eq. (A12) can be expressed in terms of the spectral function $\langle|\phi^2|(\mathbf{k},\omega)\rangle$ of the potential fluctuations, as we note that a linear-response solution to Eq. (A4) at $\mathbf{q}=\mathbf{k}$ ($\neq\mathbf{0}$) is given by

$$\delta N(\mathbf{k},\omega;\mathbf{p})=\frac{\Delta_{\mathbf{p}}^{\mathbf{k}}F(\mathbf{r},\mathbf{p})}{\omega-\omega_{\mathbf{pk}}+i\eta}e\phi(\mathbf{k},\omega)\ , \tag{A14}$$

where $F(\mathbf{r},\mathbf{p})=\langle\xi(\mathbf{p},\mathbf{0})\rangle$,

$$\omega_{\mathbf{pk}}=\Delta_{\mathbf{p}}^{\mathbf{k}}\varepsilon_p=(\mathbf{k}\cdot\mathbf{p}/m)+(\hbar k^2/2m)\ , \tag{A15}$$

and η in the denominator of Eq. (A14) represents a positive infinitesimal ensuring a causal electron-density response against a potential fluctuation. Equation (A12) finally reduces to

$$\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial t}+\frac{\mathbf{p}}{m}\cdot\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial \mathbf{r}}-e\mathbf{E}\cdot\frac{\partial F(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}} =ie^2\int\frac{d\mathbf{k}}{(2\pi)^3}\int d\omega\,\Delta_{\mathbf{p}}^{\mathbf{k}}\left[\frac{\Delta_{\mathbf{p}}^{\mathbf{k}}F(\mathbf{r},\mathbf{p})}{\omega-\omega_{\mathbf{pk}}+i\eta}\right] \times\langle|\phi^2|(\mathbf{k},\omega)\rangle\ . \tag{A16}$$

The electric resistivity ρ and the thermal conductivity κ due to the electronic transport can be calculated by a solution to Eq. (A16) under specific boundary conditions. For this purpose we set the Wigner distribution $F(\mathbf{r},\mathbf{p})$ as a summation of an unperturbed distribution $F_0(p^2)$ and a perturbation $F_1(\mathbf{p})$ arising from the presence of the electric field or the temperature gradient:

$$F(\mathbf{r},\mathbf{p})=F_0(p^2)+F_1(\mathbf{p})\ , \tag{A17}$$

where

$$F_0(p^2)=\frac{2}{(2\pi\hbar)^3}\left[\exp\left[\frac{\varepsilon_p}{k_BT}-\alpha\right]+1\right]^{-1} \tag{A18}$$

is the Fermi distribution at a finite temperature. Although we have suppressed the spatial coordinates $\mathbf{r}$ on the right-hand side of Eq. (A17), we take T and α ($=\mu/k_BT$) in Eq. (A18) to be functions of $\mathbf{r}$ in the calculation of the thermal conductivity, when Eq. (A17) is substituted on the left-hand side of Eq. (A16).

For the calculation of the electric resistivity we set $\partial/\partial t=0$, $\partial/\partial\mathbf{r}=0$, and $F(\mathbf{r},\mathbf{p})=F_0(p^2)$ on the left-hand side of Eq. (A16). Correspondingly we substitute a displaced Fermi distribution $F(\mathbf{r},\mathbf{p})=F_0(|\mathbf{p}-m\mathbf{u}|^2)$ on its right-hand side, so that the electric current density is given by

$$\mathbf{J}=-n_e e\mathbf{u}\ . \tag{A19}$$

We intend to solve Eq. (A16) for $\rho=E/J$ in the limit of both E and J approach zero.

We thus multiply both sides of Eq. (A16) by $\mathbf{p}$ and carry out integration with respect to $\mathbf{p}$. Performing partial integrations by noting

$$\Delta_{\mathbf{p}}^{\mathbf{k}}[f(\mathbf{p})g(\mathbf{p})]=f(\mathbf{p})\Delta_{\mathbf{p}}^{\mathbf{k}}g(\mathbf{p})+g(\mathbf{p})\Delta_{\mathbf{p}}^{\mathbf{k}}f(\mathbf{p}) +\hbar[\Delta_{\mathbf{p}}^{\mathbf{k}}f(\mathbf{p})][\Delta_{\mathbf{p}}^{\mathbf{k}}g(\mathbf{p})]\ , \tag{A20}$$

$$\int d\mathbf{p}\,\Delta_{\mathbf{p}}^{\mathbf{k}}f(\mathbf{p})=0\ , \tag{A21}$$

we find

$$\rho=\lim_{u\to0}\frac{1}{(en_eu)^2} \times\int\frac{d\mathbf{k}}{(2\pi)^3}\int d\omega\,\mathbf{k}\cdot\mathbf{u}\langle|\phi^2|(\mathbf{k},\omega)\rangle\mathrm{Im}\chi_0(\mathbf{k},\tilde{\omega})\ , \tag{A22}$$

where

$$\tilde{\omega}=\omega-\mathbf{k}\cdot\mathbf{u}\ , \tag{A23}$$

$$\chi_0(\mathbf{k},\omega)=-\int d\mathbf{p}\frac{1}{\omega-\omega_{\mathbf{pk}}+i\eta}\Delta_{\mathbf{p}}^{\mathbf{k}}F_0(p^2)\ . \tag{A24}$$

We now assume that $m_e \ll m_i$ and that the spectrum $\langle|\phi^2|(\mathbf{k},\omega)\rangle$ of the random potential fields is produced by the ions with the structure factor $S_{ii}(k)$. Taking account of the screening action of the electrons via Eqs. (7) and (8), we express

$$\langle|\phi^2|(\mathbf{k},\omega)\rangle=n_i\left|\frac{v_{ei}(k)}{\varepsilon(k)}\right|^2S_{ii}(k)\delta(\omega)\ . \tag{A25}$$

We also note in the low-frequency limit

$$\mathrm{Im}\chi_0(\mathbf{k},\omega)\to-2\pi^2m^2\frac{\omega}{k}F_0\left[\left[\frac{\hbar k}{2}\right]^2\right]\ . \tag{A26}$$

Substitution of Eqs. (A25) and (A26) into Eq. (A22) yields Eq. (4).

For the calculation of the thermal conductivity we assume T and α are functions of $\mathbf{r}$ and substitute the Fermi distribution, Eq. (A18), into the left-hand side of Eq.

(A16); the first term vanishes because we assume $\partial/\partial t = 0$. Since the pressure

$$P = \int d\mathbf{p} \frac{p^2}{3m} F_0(p^2) \tag{A27}$$

should be kept uniform (independent of $\mathbf{r}$), we have a relation

$$\frac{\partial \alpha}{\partial \mathbf{r}} = -\frac{5}{3} \frac{I_{3/2}(\alpha)}{I_{1/2}(\alpha)} \frac{\partial \ln T}{\partial \mathbf{r}} , \tag{A28}$$

where $I_\nu(\alpha)$ are the Fermi integrals, Eq. (16).

In the right-hand side of Eq. (A16) we substitute Eq. (A17) with

$$F_1(\mathbf{p}) = \frac{3m^2}{2\pi(\hbar k_F)^7 \Sigma} \mathbf{q}_T \cdot \mathbf{p} \left[\frac{p^2}{(\hbar k_F)^2} - \lambda \right] \frac{(2\pi\hbar)^3}{2} \frac{\partial F_0(p^2)}{\partial \alpha} , \tag{A29}$$

where Eqs. (10), (14), and (15) have been used. We note that Eq. (A29) satisfies

$$\int d\mathbf{p} F_1(\mathbf{p}) = 0 , \tag{A30}$$

$$\int d\mathbf{p}\, \mathbf{p} F_1(\mathbf{p}) = \mathbf{0} , \tag{A31}$$

$$\int d\mathbf{p}\, \varepsilon_p \frac{\mathbf{p}}{m} F_1(\mathbf{p}) = \mathbf{q}_T , \tag{A32}$$

so that $\mathbf{q}_T$ represents the thermal energy flux transported by the electrons.

We thus set

$$\mathbf{q}_T = -\kappa \frac{\partial T}{\partial \mathbf{r}} \tag{A33}$$

and solve Eq. (A16) for κ. To do so we multiply both sides of Eq. (A16) by $\mathbf{p}\{[p^2/(\hbar k_F)^2] - \lambda\}$ and carry out the $\mathbf{p}$ integration. The third term on the left-hand side then produces an integral proportional to the left-hand side of Eq. (A31) and hence it vanishes. Assuming Eq. (A25) again, we finally obtain Eq. (13).

APPENDIX B: THERMAL CONDUCTIVITY IN THE LIMIT OF COMPLETE FERMI DEGENERACY

We begin with the expression for the thermal conductivity, Eq. (13). We define

$$I = \int_{k/2k_F}^{\infty} dx\, x (x^2 - \lambda)^2 \frac{\exp(x^2/\theta - \alpha)}{[\exp(x^2/\theta - \alpha) + 1]^2} . \tag{B1}$$

Setting $y = k/k_F$ and $z = x^2/\theta$, we find after a partial integration

$$I = \frac{\theta}{2} \left[\frac{y^2}{4} - \lambda \right]^2 \frac{1}{\exp(y^2/4\theta - \alpha) + 1} + \theta^2 \int_{y^2/4\theta}^{\infty} dz (\theta z - \lambda) \frac{1}{\exp(z - \alpha) + 1} . \tag{B2}$$

We apply the Sommerfeld expansion

$$\frac{1}{\exp(z - \alpha) + 1} = \Theta(\alpha - z) - \frac{\pi^2}{6} \frac{d}{dz} \delta(z - \alpha) + O(\alpha^{-4}) \tag{B3}$$

to the terms on the right-hand side of Eq. (B2), where $\Theta(\alpha - z)$ refers to the unit step function.

Keeping the first two leading contributions with respect to θ, we obtain

$$I = \frac{\pi^2}{6} \theta^3 \Theta(\alpha - y^2/4\theta) + O(\theta^5) . \tag{B4}$$

Since $\Sigma \to \pi^2\theta^3/6$, we have Eq. (17) in the limit of $\theta \to 0$.

APPENDIX C: COULOMB COLLISION IN STRONGLY COUPLED PLASMA

We consider collisions between two charged particles (Ze) in a one-component plasma; their reduced mass and relative velocity are μ and v in the center-of-mass system. The relation between the scattering angle χ and the impact parameter b is

$$\cot \left[\frac{\chi}{2} \right] = \frac{b \mu v^2}{(Ze)^2} , \tag{C1}$$

and the cross section for the momentum transfer is[15]

$$Q_m = \int_{\chi_{\min}}^{\pi} (1 - \cos\chi) \left[\frac{(Ze)^2}{2\mu v^2 \sin^2(\chi/2)} \right]^2 2\pi \sin\chi \, d\chi = 4\pi \left[\frac{(Ze)^2}{\mu v^2} \right]^2 \ln \left[\frac{1}{\sin(\chi_{\min}/2)} \right] . \tag{C2}$$

Here we have written the lower limit of the χ integration as $\chi_{\min}$, to avoid a logarithmic divergence.

When the plasma is weakly coupled ($\Gamma \ll 1$), one takes $\chi_{\min}$ to be that corresponding to $b \simeq k_D^{-1}$, so that $\ln[1/\sin(\chi_{\min}/2)]$ scales as $\sim \ln(1/\Gamma)$, the usual classical Coulomb logarithm for a weakly coupled plasma.[15]

When the plasma is strongly coupled ($\Gamma > 1$), the interparticle correlations begin to scale as the Wigner-Seitz (or the ion-sphere) radius[4]

$$a = (3/4\pi n)^{1/3} , \tag{C3}$$

and one can assume that the range of the electrostatic potential of the charged particle Ze is confined within a distance $\alpha_1 a$, where α_1 is a correction factor of the order of unity. We thus take $\chi_{\min}$ in Eq. (C2) to be that corresponding to $b = \alpha_1 a$, to find

$$Q_m = 2\pi \left[\frac{(Ze)^2}{\mu v^2} \right]^2 \ln \left[1 + \left[\frac{\alpha_1 a \mu v^2}{Z^2 e^2} \right]^2 \right] . \tag{C4}$$

Since $\Gamma > 1$, we may estimate $\alpha_1 a \mu v^2 / Z^2 e^2 < 1$ for a majority of the plasma particles. Retaining only the first term after expanding the logarithmic term in Eq. (C4), we find that the Coulomb logarithm now scales as $\sim \Gamma^{-2}$, a finding consistent with Eqs. (32) and (34). The cross section, Eq. (C4), becomes $\sim 2\pi(\alpha_1 a)^2$, a value close to the geometrical cross section between the ion spheres, $4\pi a^2$.

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