Theory of interparticle correlations in dense, high-temperature plasmas. I. General formalism

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This is the first in a series of papers in which we carry out a systematic study of multiparticle correlation effects in the dense, high-temperature plasmas, appropriate to the inertially-confined-fusion experiments and the interior of the main-sequence stars. In this paper (paper I), we develop a general density-response formalism with inclusion of varied degrees of the electron degeneracy and the local-field corrections (LFC's) describing the strong Coulomb-coupling effects. An explicit theoretical scheme of calculating the static LFC's is developed on the basis of the hypernetted-chain approximation; useful expressions for the dynamic LFC's are proposed.

I. INTRODUCTION

Theoretical study of the interparticle correlations in dense, high-temperature plasmas is an essential problem in clarifying the basic properties of those high-density materials[1] which we encounter in the inertially-confined-fusion experiments[2,3] and in the interior of the main-sequence stars. The fundamental quantities in describing the correlation properties are the radial distribution functions and the static- and dynamic-structure factors[1,4,5] in multicomponent plasmas. With the knowledge of those correlation functions one calculates various thermodynamic functions and thereby determines the equation of state; transport coefficients, the stopping power against an injected charged particle, and the distribution of electric microfields can likewise be evaluated.

A most simplified treatment of a dense plasma consists in assuming a classical one-component plasma (OCP) model[1,6] in which a single species of ions is considered and the electrons are regarded as a rigid, uniform background neutralizing the average space-charge field of the ions. The correlation properties of such an OCP have been extensively studied, thanks to the progress in the Monte Carlo simulation technique[7] and the recent advancement in the analytic theories.[8,9] Accurate information is now available on the correlation functions, the thermodynamic properties, and other basic quantities.[1,6]

For the description of the actual high-density plasma system under present consideration, however, the OCP model amounts to something of an oversimplification, since the electrons do form a polarizable medium.[1,10–13] The treatment of interionic correlations with inclusion of the electronic screening effect is a fairly complex problem, owing to varied degrees of electron degeneracy and involvement of the local-field corrections (LFC's) due to strong Coulomb coupling.[1,13]

The present paper (paper I), dealing with a general formalism, is the first in a series of papers in which we intend to carry out a systematic study of multiparticle correlation effects in the dense, high-temperature plasmas. In the subsequent papers we shall investigate the numerical results of the explicit calculation for various physical quantities, such as the correlation functions, thermodynamic variables, the stopping power, transport coefficients, and the microfield distributions.

In Sec. II we specify the parameter domain for the plasmas under present consideration and remark on relative magnitudes of various characteristic parameters. In Sec. III we establish a general framework of the density-response formalism for multicomponent plasmas and introduce the LFC's describing the strong Coulomb-coupling effects. In Sec. IV correlation functions are introduced and are expressed in terms of the density-response functions. In Sec. V we present a theoretical scheme of calculating the static LFC's explicitly in the parameter domain of interest; approximate expressions for the dynamic LFC's are proposed in Sec. VI. Section VII concludes with a summary.

II. PLASMA PARAMETERS

We consider a multicomponent plasma at temperature T consisting of electrons and various kinds of ions. The particle species is distinguished by the subscript μ, and we reserve $\mu=1$ for the electrons. The electric charge, the mass, and the number density of each species are denoted by $Z_\mu e$, m_μ, and n_μ; the condition for the macroscopic charge neutrality, $\sum_\mu Z_\mu n_\mu=0$ is assumed.

In this series of papers we confine ourselves to the consideration of those plasmas in which the ions may be regarded as classical particles obeying the classical statistics. This implies that the thermal de Broglie wavelength $\hbar(m_\mu k_B T)^{-1/2}$ is much smaller than the average interparticle spacing, which we estimate at the Wigner-Seitz radius of the electrons,

$$a_1=(4\pi n_1/3)^{-1/3} . \tag{1}$$

Hence the condition stated above reads

$$\Lambda_\mu \equiv a_1(m_\mu k_B T)^{1/2}/\hbar \gg 1 \quad (\mu \geq 2) , \tag{2}$$

where k_B is the Boltzmann constant.

The strength of Coulomb coupling in the classical ion system may be measured by a dimensionless parameter[1,6]

$$\Gamma \equiv \langle Z^{5/3} \rangle e^2 / a_1 k_B T \ , \tag{3}$$

where

$$\langle Z^{5/3} \rangle = \sum_{\mu \geq 2} n_\mu Z_\mu^{5/3} \Big/ \sum_{\mu \geq 2} n_\mu \ . \tag{4}$$

We shall consider those plasmas with weak ($\Gamma \leq 0.1$) to intermediate ($0.1 < \Gamma \leq 10$) coupling. In these circumstances, as we shall elaborate later in Sec. V, the hypernetted-chain (HNC) approximation[14] provides an accurate description of the static correlations between ions.

For the electron system we assume first of all that the relativistic effects are negligible. This means that the parameter domain under consideration is restricted to

$$E_F < mc^2, \quad k_B T < mc^2 \ , \tag{5}$$

where

$$E_F = \frac{\hbar^2}{2m} (3\pi^2 n_1)^{2/3} \tag{6}$$

is the Fermi energy of the electrons at $T=0$ and m is the rest mass of an electron.

We further assume that the atomic nuclei in the plasma are all in the fully ionized states, that is, the atoms do not retain their orbital electrons. Such a state may be achieved either through the pressure ionization

$$E_F > Z_\mu^2 \frac{me^4}{2\hbar^2} \quad (\mu \geq 2) \tag{7}$$

or through the thermal ionization

$$k_B T > Z_\mu^2 \frac{me^4}{2\hbar^2} \quad (\mu \geq 2) \ . \tag{8}$$

In light of the condition for Saha equilibrium, Eq. (8) appears a bit too restrictive numerically than actually necessary. At any rate, when none of those conditions for total ionization are met, we must deal with a plasma system in which the atomic nuclei retain some of their orbital electrons.

The electron system in the plasma is characterized by two parameters: the dimensionless density parameter,

$$r_s = a_1 m e^2 / \hbar^2 \ , \tag{9}$$

and the degeneracy parameter,

$$\theta = k_B T / E_F \ . \tag{10}$$

The parameter r_s plays the role of the Coulomb-coupling constant for a degenerate electron system.[1]

In Fig. 1 we show the relative magnitudes of various parameters on the density-temperature plane for two-component electron-ion (-proton) plasmas, where $Z_1 = -1$, $Z_2 = 1$, and $n_1 = n_2 \equiv n$. The hatched areas are excluded from our consideration because of Eqs. (5), (7), and (8). In A, bounded by $\theta = 10$ from the right, the electrons are nondegenerate and obey the classical statistics; the Coulomb-coupling constant Γ of the electrons is the same as that of the ions and takes on a value less than 0.1 for the bulk of the domain. In B, bounded between $\theta = 10$ and $\theta = 0.1$, the electrons are partially degenerate; the Coulomb coupling of the ions as well as of the electrons can be intermediate. In C, bounded between $\theta = 0.1$ and $\Gamma = 10$, the Coulomb coupling of the classical ions is intermediate, while that of the degenerate electrons is weak to intermediate ($10^{-2} \leq r_s \leq 1$). On all of the domains A, B, and C, which we shall be concerned with, the condition (2) is well satisfied.

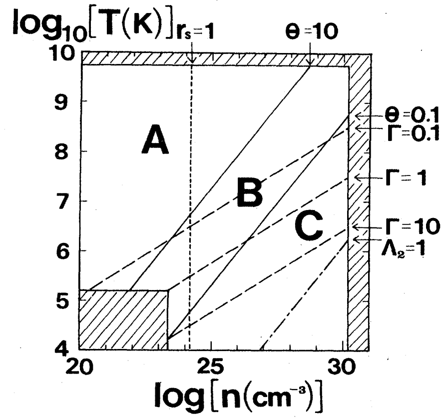

FIG. 1. Relative magnitudes of various characteristic parameters on the number density vs temperature plane for electron-proton plasmas. The parameters Λ_2, Γ, r_s, and θ are defined by Eqs. (2), (3), (9), and (10) in the text.

III. DENSITY-RESPONSE FORMALISM

To analyze the interparticle correlations in a multicomponent plasma we use the density-response formalism or the dielectric formulation[4,5] in the framework of the linear-response theory. We thus apply to the plasma a weak (fictitious) external potential field $V_\mu(\mathbf{r},t)$ which couples *only* to the density field $\rho_\mu(\mathbf{r})$ of the μ-species particles. The extra Hamiltonian arising from the presence of such external fields is then written as

$$H_{\text{ext}}(t) = \sum_\mu \int d\mathbf{r}\, \rho_\mu(\mathbf{r}) V_\mu(\mathbf{r},t) \ . \tag{11}$$

We assume that the plasma state in the absence of the external fields is translationally invariant both in space and in time. We thus work in terms of Fourier components, so that

$$\tilde{V}_\mu(\mathbf{k},\omega) = \int d\mathbf{r} \int_{-\infty}^{\infty} dt\, V_\mu(\mathbf{r},t) \exp[-i(\mathbf{k}\cdot\mathbf{r} - \omega t)] \ , \tag{12}$$

for example. The periodic boundary conditions for a unit volume are adopted; the volume of the $\mathbf{r}$ integration in Eq. (12) is unity.

The external disturbances $\tilde{V}_\mu(\mathbf{k},\omega)$ induce the deviations $\delta\rho_\mu(\mathbf{k},\omega)$ of the density fields from the unperturbed values. The linear-response relations

$$\delta\rho_\mu(\mathbf{k},\omega) = \sum_\nu \chi_{\mu\nu}(k,\omega) \tilde{V}_\nu(\mathbf{k},\omega) \tag{13}$$

then define the density-density response functions $\chi_{\mu\nu}(k,\omega)$ between the μ and ν species of the particles. The longitudinal, frequency and wave-number-dependent, dielectric response function $\epsilon(k,\omega)$, which is defined as the ratio between the total and external potential fields in the plasma,[1,4,5] is given by

$$\frac{1}{\epsilon(k,\omega)}=1+v(k)\sum_{\mu,\nu} Z_\mu Z_\nu \chi_{\mu\nu}(k,\omega) , \tag{14}$$

where $v(k)=4\pi e^2/k^2$.

We find it useful to express the density-density response functions in terms of the screened response functions and the LFC's. To do so we consider the effective potential $\phi_{\mu\nu}(\mathbf{k},\omega)$ on a μ-species particle produced by the density fluctuation $\delta\rho_\nu(\mathbf{k},\omega)$ in the ν-species particles, which may be written as

$$\phi_{\mu\nu}(\mathbf{k},\omega)=Z_\mu Z_\nu v(k)[1-G_{\mu\nu}(k,\omega)]\delta\rho_\nu(\mathbf{k},\omega) . \tag{15}$$

This potential generally differs from the bare Coulomb potential $Z_\mu Z_\nu v(k)\delta\rho_\nu(k,\omega)$ because of the microscopic correlation effects involved; the difference is here measured by the dynamic LFC, $G_{\mu\nu}(k,\omega)$. The screened density-response function $\chi_\mu^{(0)}(k,\omega)$ is then introduced according to

$$\delta\rho_\mu(\mathbf{k},\omega)=\chi_\mu^{(0)}(k,\omega)\left[\widetilde{V}_\mu(\mathbf{k},\omega)+v(k)\sum_\nu Z_\mu Z_\nu[1-G_{\mu\nu}(k,\omega)]\delta\rho_\nu(\mathbf{k},\omega)\right] . \tag{16}$$

The function $\chi_\mu^{(0)}(k,\omega)$ thus describes a density response of the μ- species particles against a renormalized potential field $\widetilde{V}_\mu(\mathbf{k},\omega)+\sum_\nu\phi_{\mu\nu}(\mathbf{k},\omega)$ in the plasma. For the analysis of the electron-hole liquid in semiconductors, Vashishta, Bhattacharyya, and Singwi[15] have used a formulation analogous to Eq. (16) where the ω dependence of $G_{\mu\nu}(k,\omega)$ has been ignored.

Expression for $\chi_{\mu\nu}(k,\omega)$ may be obtained through a comparison between Eqs. (13) and (16). For simplicity in notation we write the results for a two-component electron-ion plasma, suppressing the frequency and wavenumber arguments ω and k:

$$\chi_{11}=\chi_1^{(0)}[1-Z_2^2 v\chi_2^{(0)}(1-G_{22})]/D , \tag{17a}$$

$$\chi_{22}=\chi_2^{(0)}[1-Z_1^2 v\chi_1^{(0)}(1-G_{11})]/D , \tag{17b}$$

$$\chi_{12}=Z_1Z_2 v\chi_1^{(0)}\chi_2^{(0)}(1-G_{12})/D , \tag{17c}$$

$$\chi_{21}=Z_1Z_2 v\chi_1^{(0)}\chi_2^{(0)}(1-G_{21})/D , \tag{17d}$$

where

$$D=[1-Z_1^2 v\chi_1^{(0)}(1-G_{11})][1-Z_2^2 v\chi_2^{(0)}(1-G_{22})] -Z_1^2Z_2^2 v^2\chi_1^{(0)}\chi_2^{(0)}(1-G_{12})(1-G_{21}) . \tag{18}$$

The dielectric function, Eq. (14), takes the form

$$\frac{1}{\epsilon}=1+\frac{v}{D}[Z_1^2\chi_1^{(0)}+Z_2^2\chi_2^{(0)} +Z_1^2Z_2^2 v\chi_1^{(0)}\chi_2^{(0)}(G_{11}+G_{22}-G_{12}-G_{21})] . \tag{19}$$

In the random-phase approximation[1,4,5] (RPA), one sets $G_{\mu\nu}(k,\omega)=0$, and uses the free-particle polarizability[16]

$$\chi_\mu^{(0)}(k,\omega)=-\frac{m_\mu(3\pi^2 n_\mu)^{1/3}}{2\pi^2\hbar^2 K} \times\int_0^\infty dx\frac{x}{1+\exp[(x^2-M)/\Theta]} \times\ln\left[\frac{(2xK+K^2)^2-\Omega^2}{(2xK-K^2)^2-\Omega^2}\right] \tag{20}$$

with

$$K=k/(3\pi^2 n_\mu)^{1/3} , \tag{21a}$$

$$\Omega=2m_\mu(\omega+i0)/\hbar(3\pi^2 n_\mu)^{2/3} , \tag{21b}$$

$$\Theta=2m_\mu k_B T/\hbar^2(3\pi^2 n_\mu)^{2/3} , \tag{21c}$$

for the screened response function. The positive infinitesimal $+0$ in Eq. (21b) ensures the causal boundary conditions in the evaluation of Eq. (20). The dimensionless chemical potential M in Eq. (20) is to be determined through a numerical solution to the equation

$$\frac{1}{3}=\int_0^\infty dx\frac{x^2}{1+\exp[(x^2-M)/\Theta]} \tag{22}$$

resulting from the normalization condition. In the classical limit $\Theta\gg 1$, Eq. (20) tends to the Vlasov formula;[1,5] in the degeneracy limit $\Theta\to 0$, it reduces to the Lindhard expression.[1,4,17]

In the present formulation we retain the representation of the screened response functions as given by Eq. (20). All the microscopic correlation effects beyond the RPA are thus lumped into $G_{\mu\nu}(k,\omega)$. Once those LFC's are determined, the strong-coupling theory of a dense plasma is completed.

IV. CORRELATION FUNCTIONS

The dynamic-structure factors $S_{\mu\nu}(k,\omega)$ in a multicomponent plasma are calculated as the Fourier transforms in space and time of density-density correlation functions:[4,5,18]

$$S_{\mu\nu}(k,\omega)=\frac{1}{4\pi}\int d\mathbf{r}\int_{-\infty}^{\infty}dt\langle[\rho_\mu(\mathbf{r}'+\mathbf{r},t'+t)\rho_\nu(\mathbf{r}',t')+\rho_\nu(\mathbf{r}',t')\rho_\mu(\mathbf{r}'+\mathbf{r},t+t')]\rangle\exp[-i(\mathbf{k}\cdot\mathbf{r}-\omega t)] . \tag{23}$$

Here $\langle\cdots\rangle$ refers to the expectation value in the equilibrium state. The fluctuation-dissipation theorem[19,20] (FDT) relates those structure factors and the imaginary parts of the density-density response functions as[5,18]

$$S_{\mu\nu}(k,\omega)=-\frac{\hbar}{2\pi}\coth\left[\frac{\hbar\omega}{2k_BT}\right]\mathrm{Im}\chi_{\mu\nu}(k,\omega) . \tag{24a}$$

In the classical limit $\hbar\omega\ll k_BT$, those relations reduce to

$$S_{\mu\nu}(k,\omega) = -\frac{k_B T}{\pi\omega}\mathrm{Im}\chi_{\mu\nu}(k,\omega)\ . \tag{24b}$$

The static-structure factors $S_{\mu\nu}(k)$ are defined and calculated as

$$S_{\mu\nu}(k) = \frac{1}{\sqrt{n_\mu n_\nu}}\int_{-\infty}^{\infty} d\omega\, S_{\mu\nu}(k,\omega)\ . \tag{25a}$$

In the classical limit we find

$$S_{\mu\nu}(k) = -\frac{k_B T}{\sqrt{n_\mu n_\nu}}\chi_{\mu\nu}(k,0)\ . \tag{25b}$$

The radial distribution functions $g_{\mu\nu}(r)$, describing the joint probability functions of finding the $\mu-$ and ν-species particles at a distance r, and the associated pair-correlation functions $h_{\mu\nu}(r)$ are then given by

$$\begin{aligned} g_{\mu\nu}(r) &= 1 + h_{\mu\nu}(r) \\ &= 1 + \frac{1}{\sqrt{n_\mu n_\nu}}\int \frac{d\mathbf{k}}{(2\pi)^3}[S_{\mu\nu}(k) - \delta_{\mu\nu}]\exp(i\mathbf{k}\cdot\mathbf{r})\ , \end{aligned} \tag{26}$$

where $\delta_{\mu\nu}$ represents Kronecker's delta. Those correlation functions will be explicitly evaluated and discussed for dense plasmas in the subsequent paper (paper II).[21]

Basic physical quantities of dense plasmas can be calculated with the knowledge of the correlation and response functions. For example, the density of Coulomb-interaction energy in a two-component plasma is given by

$$\begin{aligned} E_{\mathrm{int}}(e^2) = \tfrac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^3}\Bigg[\int_{-\infty}^{\infty} d\omega \frac{\hbar}{2\pi}\coth\left[\frac{\hbar\omega}{2k_B T}\right] &\times \mathrm{Im}\left[1 - \frac{1}{\epsilon(k,\omega)}\right] \\ &- (Z_1^2 n_1 + Z_2^2 n_2)v(k)\Bigg]\ , \end{aligned} \tag{27}$$

as a function of the strength e^2 of the Coulomb interaction. The Helmholtz free energy F per unit volume is then calculated according to the integration over the coupling constant as

$$F = F(e_1^2) + \int_{e_1^2}^{e^2} dx \frac{E_{\mathrm{int}}(x)}{x}\ . \tag{28}$$

We shall investigate the thermodynamic properties of dense plasmas on the basis of those formulas in paper III.[22]

V. APPROXIMATION SCHEME FOR STATIC LFC

The dielectric formulation described in the foregoing sections is completed when the LFC's introduced in Eq. (15) are determined. As Eq. (25b) implies, the static values

$$G_{\mu\nu}(k) \equiv G_{\mu\nu}(k,0) \tag{29}$$

of the LFC are essential to the evaluation of the thermodynamic quantities for a classical plasma. Because of this significance and partly becuase of their conceptual simplicity, the dynamic LFC's are sometimes[1] approximated by their static values, Eq. (29). In this section we develop a theoretical scheme by which the static LFC's are calculated for the dense plasmas of our interest (cf. Sec. II).

The present scheme relies on the observation that the HNC scheme[14,23] offers an extremely accurate description of correlations in an intermediate to weakly coupled ($\Gamma \le 10$) classical OCP; comparison illustrating such an accuracy has been discussed in the literature.[1,6,8,9,13] For $\Gamma \le 1$, the HNC results are virtually exact; even for $1 < \Gamma \le 10$, the HNC scheme reproduces interaction energies of OCP with relative errors less than 1%.[9] In the present plasma system the ions, interacting via screened Coulomb forces, cannot be regarded as an OCP; the screening effect then acts to widen the range of Γ for the validity of the HNC calculation.[12]

In the HNC scheme the interparticle correlations with binary interaction $\phi(r)$ are analyzed through the HNC equation

$$g(r) = \exp\{-[\phi(r)/k_B T] + h(r) - c(r)\} \tag{30}$$

coupled with the Ornstein-Zernike relation

$$h(r) = c(r) + n\int d\mathbf{r}'\, c(|\mathbf{r}-\mathbf{r}'|)h(r')\ , \tag{31}$$

where $h(r) = g(r) - 1$ and $c(r)$ is the direct correlation function. We have described the HNC scheme for a one-component system; its extension to a multicomponent system is straightforward.

A principal problem involved in the calculation of the static LFC is the degeneracy of the electrons. In an earlier theory[13] we evaluated the LFC for the electrons accurately in the domains A and C of Fig. 1 and then interpolated the results into the partially degenerate domain B.

In the present series of papers, in light of the numerical comparison exemplified in Fig. 1, we adopt the point of view that the degeneracy of the electrons can be taken into account through the free-electron polarizability $\chi_1^{(0)}(k,\omega)$, while the LFC's are determined semiclassically in the way consistent with various polarizabilities. For a two-component plasma ($Z_1 = -1$, $Z_2 = Z$), we may thus calculate the LFC's and the dielectric response function through the following steps of calculations.

(i) Solve the HNC equations, Eqs. (30) and (31), for the static-structure factor$S_e(k)$ of the electron OCP and calculate $G_e(k)$ by[1]

$$G_e(k) = 1 + \frac{k^2}{k_e^2}\left[1 - \frac{1}{S_e(k)}\right]\ , \tag{32}$$

where $k_e^2 = 4\pi n_1 e^2/k_B T$. Set the electronic dielectric function as

$$\epsilon_e(k,\omega) = 1 - \frac{v(k)\chi_1^{(0)}(k,\omega)}{1 + v(k)G_e(k)\chi_1^{(0)}(k,\omega)}\ . \tag{33}$$

(ii) Solve the HNC equations for the ion-ion structure factor $S_{22}(k)$ with the interionic potential given by[13]

$$\phi_i(r) = \frac{(Ze)^2}{2\pi^2}\int d\mathbf{k}[k^2\epsilon_e(k,0)]^{-1}\exp(i\mathbf{k}\cdot\mathbf{r})\ . \tag{34}$$

(iii) Note Eq. (17b) and the classical FDT, Eq. (25b), i.e.,

$$S_{22}(k)=-\frac{k_BT}{n_2}\chi_{22}(k,0)\ . \tag{35}$$

(iv) Calculate $h_{12}(r)$ through the diagrammatic summation of Fig. 2, where the single and double circles represent the electron and ion coordinates, respectively, the filled circle corresponds to the particle coordinate to be integrated, and $l_{12}(r)$ describes an electron-ion bond involving only the electronic coordinates at intermediate stages. In the linear-response approximation, the Fourier component of the electron-ion bond is given by

$$\tilde{l}_{12}(k)=\frac{Z}{n_1}\left[1-\frac{1}{\epsilon_e(k,0)}\right]\ . \tag{36}$$

Use of this expression in Fig. 2 yields

$$h_{12}(r)=\frac{Z}{n_1}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[1-\frac{1}{\epsilon_e(k,0)}\right]S_{22}(k)\exp(i\mathbf{k}\cdot\mathbf{r})\ , \tag{37}$$

whence we find

$$S_{12}(k)=Z^{1/2}S_{22}(k)\left[1-\frac{1}{\epsilon_e(k,0)}\right]\ . \tag{38}$$

(v) Note Eqs. (17c) and (25b), to write

$$S_{12}(k)=-\frac{k_BT}{\sqrt{n_1n_2}}\chi_{12}(k,0)\ . \tag{39}$$

(vi) Calculate the pure electron-electron bond by

$$l_{11}(r)=\frac{1}{n_1}\int\frac{d\mathbf{k}}{(2\pi)^3}[\sigma_{11}(k)-1]\exp(i\mathbf{k}\cdot\mathbf{r}) \tag{40}$$

with

$$\sigma_{11}(k)=-\frac{k_BT}{n_1}\frac{\chi_1^{(0)}(k,0)}{1-v(k)[1-G_e(k)]\chi_1^{(0)}(k,0)}\ . \tag{41}$$

(vii) Calculate $h_{11}(r)$ through the diagrammatic summation described in Fig. 3, which yields

$$S_{11}(k)=\sigma_{11}(k)+Z\left[1-\frac{1}{\epsilon_e(k,0)}\right]^2S_{22}(k)\ . \tag{42}$$

(viii) Note Eqs. (17a) and (25b), to write

$$S_{11}(k)=-\frac{k_BT}{n_1}\chi_{11}(k,0)\ . \tag{43}$$

(ix) The LFC's are determined from the solution to Eqs. (35), (39), and (43); the result is

$$G_{11}(k)=G_e(k)\ , \tag{44a}$$

$$G_{22}(k)=\frac{1}{\epsilon_e(k,0)}+\frac{k_BTk^2}{4\pi n_2(Ze)^2}\left[1-\frac{1}{S_{22}(k)}\right]\ , \tag{44b}$$

$$G_{12}(k)=G_{21}(k)=0\ . \tag{44c}$$

(x) Finally, substituting Eqs. (44) in place of the dynamic LFC's in Eq. (19), we find

$$\epsilon(k,\omega)=1-\frac{v(k)\chi_1^{(0)}(k,\omega)}{1+v(k)G_{11}(k)\chi_1^{(0)}(k,\omega)}-\frac{Z^2v(k)\chi_2^{(0)}(k,\omega)}{1+Z^2v(k)G_{22}(k)\chi_2^{(0)}(k,\omega)} \tag{45}$$

The expressions for the LFC's as obtained in Eqs. (44) are rather easy to interpret. Equation (44a) appears straightforward. We may rederive Eq. (44b) as follows: For the ion system with the screened interaction, Eq. (34), the LFC $G'_{22}(k)$ may be calculated as

$$G'_{22}(k)=1+\frac{k_BT\epsilon_e(k,0)k^2}{4\pi n_2(Ze)^2}\left[1-\frac{1}{S_{22}(k)}\right] \tag{46}$$

following the scheme of Eq. (32). Since $G_{22}(k)$ has been defined relative to the bare ion-ion interaction, however, we have $G_{22}(k)=G'_{22}(k)/\epsilon_e(k,0)$, which yields Eq. (44b).

Equation (44c) is a consequence of the linear-response approximation which we have adopted in Eq. (36) for a representation of the electron-ion bond. Strong-coupling effects between electron and ion beyond the RPA have been ignored in Eq. (36) and thus we have found Eq. (44c). Consequently the dielectric response function, as given by Eq. (45), takes a form in which the contributions of the electronic and ionic polarizabilities are added separately and no interference terms appear. Such a neglect of strong-coupling effects between electron and ion has been justified in a number of cases.[11,12] If the electron-ion bond is calculated in a scheme beyond the linear-response approximation (e.g., in a density-functional formalism[24]), one would naturally find a result different from Eq. (44c).

In summary we have presented in this section a complete theoretical scheme of calculating the static correlations and thermodynamic properties for dense plasmas in the parameter regime as described in Sec. II.

VI. DYNAMIC LFC

In the treatment of dynamic phenomena in dense plasmas, such as the stopping power against impinging charged particles, knowledge on the frequency dependence of the LFC's becomes indispensable. Formulation of the dynamic LFC's for a strongly coupled plasma is a diffi-

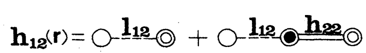

FIG. 2. Sum of diagrams for the electron-ion correlation function $h_{12}(r)$.

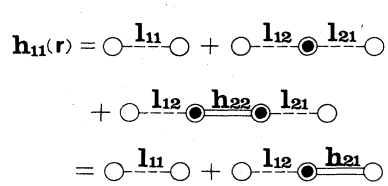

FIG. 3. Sum of diagrams for the electron-electron correlation function $h_{11}(r)$.

cult problem with formidable analytic complexities, and no exact solutions are as yet available. In this section, without delving into such a difficult problem, we attempt to derive approximate but useful expressions for the dynamic LFC's by noticing a number of exact asymptotic behaviors in their frequency dependence.

We begin by noting the high-frequency asymptotic expansion of the density-density response functions

$$\chi_{\mu\nu}(k,\omega)\rightarrow \sum_{n=1}^{\infty} \frac{M_{\mu\nu}^{(n)}(k)}{(\hbar\omega)^{n+1}} , \tag{47}$$

where the coefficients are given by the nth frequency moment of the imaginary part of $\chi_{\mu\nu}(k,\omega)$ as

$$M_{\mu\nu}^{(n)}(k)=-\hbar\int_{-\infty}^{\infty}\frac{d\omega}{\pi}(\hbar\omega)^n \mathrm{Im}\chi_{\mu\nu}(k,\omega) . \tag{48}$$

According to the frequency-moment sum rules,[1,25] Eq. (48) is expressed also in terms of the time-independent multiparticle correlation functions; $M_{\mu\nu}^{(n)}(k)$ identically vanishes for an even number of n in reflectionally invariant systems. For the multicomponent plasma, the first two nonvanishing terms are

$$M_{\mu\nu}^{(1)}(k)=2n_\mu \epsilon_\mu(k)\delta_{\mu\nu} , \tag{49}$$

$$\begin{aligned} M_{\mu\nu}^{(3)}(k)=&2n_\mu[\epsilon_\mu(k)]^2[4K_\mu+\epsilon_\mu(k)]\delta_{\mu\nu}\\ &+4n_\mu n_\nu \epsilon_\mu(k)\epsilon_\nu(k)Z_\mu Z_\nu v(k)[1-I_{\mu\nu}(k)] . \end{aligned} \tag{50}$$

Here K_μ is the average kinetic energy per a μ-species particle,

$$\epsilon_\mu(k)=\hbar^2k^2/2m_\mu , \tag{51}$$

$$\begin{aligned} I_{\mu\nu}(k)=&-\frac{1}{n_\mu n_\nu}\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{(\mathbf{k}\cdot\mathbf{q})^2}{k^2q^2}\\ &\times[\langle\tilde{\rho}_\mu(\mathbf{k}-\mathbf{q})\tilde{\rho}_\nu(\mathbf{q}-\mathbf{k})\rangle\\ &-\langle\rho_c(\mathbf{q})\tilde{\rho}_\mu(-\mathbf{q})\rangle\delta_{\mu\nu}/Z_\mu] , \end{aligned} \tag{52}$$

$$\rho_c(\mathbf{q})=\sum_\mu Z_\mu\tilde{\rho}_\mu(\mathbf{q}) , \tag{53}$$

and $\tilde{\rho}_\mu(\mathbf{k})$ denotes the Fourier component of the density field $\rho_\mu(\mathbf{r})$ for the μ-species particles.

In light of Eqs. (29), (47), and (50), we find the low- and high-frequency limiting behaviors of $G_{\mu\nu}(k,\omega)$ as

$$\lim_{\omega\rightarrow 0}G_{\mu\nu}(k,\omega)=G_{\mu\nu}(k) , \tag{54a}$$

$$\lim_{\omega\rightarrow\infty}G_{\mu\nu}(k,\omega)=I_{\mu\nu}(k) . \tag{54b}$$

Proposed formulas for $G_{\mu\nu}(k,\omega)$, satisfying those two constraints, are

$$G_{\mu\nu}(k,\omega)=\frac{\omega I_{\mu\nu}(k)+i\omega_{\mu\nu}G_{\mu\nu}(k)}{\omega+i\omega_{\mu\nu}} , \tag{55}$$

where $\omega_{\mu\nu}$ are the characteristic frequencies distinguishing between the low- and high-frequency domains in which, respectively, $G_{\mu\nu}(k,\omega)=G_{\mu\nu}(k)$ and $G_{\mu\nu}(k,\omega)=I_{\mu\nu}(k)$ are approximately valid.

Those characteristic frequencies may be determined once we find another constraint independent of Eqs. (54). A possible choice would be

$$\omega_{\mu\nu}^2=\tfrac{1}{2}(\omega_{p\mu}^2+\omega_{p\nu}^2)\zeta_{\mu\nu} \tag{56}$$

with

$$\omega_{p\mu}^2=4\pi n_\mu(Z_\mu e)^2/m_\mu , \tag{57}$$

where $\zeta_{\mu\nu}$ are factors of order unity.

The proposed ω dependence of $G_{\mu\nu}(k,\omega)$ in Eq. (55) satisfies the causal boundary conditions. In addition it derives another support from an explicit microscopic calculation of $G(k,\omega)$ carried out by Utsumi and Ichimaru[26] for the degenerate electron liquid. In the latter theory they performed a low-frequency expansion,

$$G(k,\omega)=G(k)+i\xi_1(k)\omega+\cdots , \tag{58}$$

and then determined the viscosity coefficient

$$\xi_1(k)=\left[\frac{\pi^2}{8}-1\right]\frac{(2\pi)^{1/2}}{3}\frac{1}{\Omega(k)}[G(k)-I(k)] , \tag{59}$$

where $\Omega(k)$ is a characteristic frequency in their theory. We find that the first two terms in the low-frequency expansion of Eq. (55), as applied to the degenerate electron liquid, agree with those in Eq. (58) since $\Omega(k)$ takes on values close to ω_p in the bulk of the k domain.

VII. CONCLUDING REMARKS

In the foregoing sections we have thus established a general theoretical scheme by which the static and dynamic correlational properties may be analyzed for the dense, high-temperature plasmas in the parameter domain as specified in Sec. II. In the succeeding papers we shall carry out numerical solutions to the equations obtained here and thereby investigate various fundamental properties of dense plasmas.

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