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Study of fluctuations in transient optical bistability

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The effect of external noise on the dynamic behavior of an intrinsic optical-bistable system is studied experimentally. Under conditions of critical slowing down we observe a transient bimodality in the intensity distribution as well as a reduction of the switching delay due to fluctuations. Our experimental results are in good qualitative agreement with theoretical predictions.

Optical-bistable systems have been proposed as models for the study of nonequilibrium thermodynamics, but they also inspire widespread interest due to their potential for practical applications.[1] In both respects it is important to have a detailed knowledge of the effect of fluctuations on the switching behavior of the system because noise obviously plays a crucial role close to the deterministic bifurcation points. The influence of fluctuations on the steady state and transient behavior of optical-bistable systems has been analyzed in a large number of theoretical papers.[1] One of the most prominent features of noise effects in steady-state optical bistability (OB) is the appearance of two peaks in the output-intensity probability distribution. Experimentally, however, the steady-state situation is difficult to investigate because the lifetimes of the metastable states involved are extremely long.

Recently, Broggi and Lugiato discussed a different situation that is accessible to experimental observation: In their work the effect of external amplitude noise on the transient switching in OB was analyzed with parameters chosen close to one of the bifurcation points, but on the monostable side.[2] It is well known that in this regime an optical-bistable device displays critical slowing down; i.e., the equilibrium value is approached with considerable delay τ_D after a long lethargic stage.[3-6] By numerically solving the corresponding Fokker-Planck equation Broggi and Lugiato showed that in the presence of noise the probability distribution becomes double peaked in a sizable time interval during the approach to the single-peaked steady-state distribution. In the present paper we report on an experimental investigation of this so-called "transient noise-induced OB."

It is worth mentioning that this type of transient bimodality is not expected to be a specific property of OB, but should be a phenomenon of a quite general type: It is believed to occur whenever the temporal evolution of a physical, chemical, biological, etc., system involves a long lethargic stage followed by an abrupt change to a final stable state. In fact, the phenomenon was first predicted in the stochastic analysis of explosive behavior in a chemical system.[7] As pointed out by Broggi and Lugiato in this context, an optical-bistable device is just one candidate for a study of this effect.

The optical-bistable device we consider here (Fig. 1) has been described in some detail elsewhere.[6] It consists of sodium vapor (number density $\simeq 10^{12}$ cm^{-3}) in an argon atmosphere ($P_{\mathrm{Ar}} \simeq 30$ kPa) placed inside a confocal Fabry-Perot resonator. A cw dye laser serves to resonantly excite the sodium $D1$ line; the light is circularly polarized. An electro-optic modulator (EOM) fed by a step generator is used to create step inputs of light. Additional intensity fluctuations can be introduced by feeding a broadband noise source to the same EOM. It employs a pseudorandom generator and produces noise with a Gaussian distribution of amplitudes and a bandwidth of 20 kHz with 120 lines/Hz.

The experiments reported here were performed under absorptive conditions. Only very moderate optical power is required, since optical pumping between the ground-state Zeeman sublevels of sodium gives rise to a strong nonlinearity. The dynamic response of our device is governed by the response time of the medium (bad cavity limit), which in turn is related to the rather slow relaxation process of the ground-state orientation z due to diffusion.[6] Under the conditions of our experiment these diffusive losses occur on a time scale of about 10 μsec. This is a convenient scale for detailed dynamic studies.

Very much in the manner of Ref. 6, the input intensity is rapidly switched from near zero to a value slightly above the upper switching point. The output intensity displays the typical behavior of critical slowing down (see inset of Fig. 2). In our experiment τ_D is of the order of a few hundred μsec. The setup shown in Fig. 1 allows the sampling of the output intensity at a fixed time T after the switch on of the input intensity. By repeating the process many times, a histogram is accumulated in the microcomputer representing a probability distribution of output intensities.

A typical result is shown in Fig. 2. Here histograms are

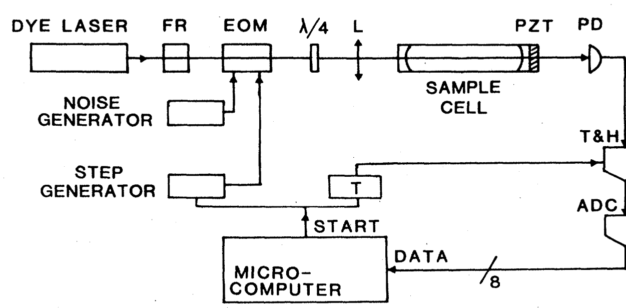

FIG. 1. Schematic of the experimental setup: resonator length: 15 cm; intracavity beam waist: 120 μm; mirror reflectivities: 0.75 and 0.9; FR: Faraday rotator; EOM: electro-optic modulator; L: mode matching lens; PZT: piezoceramic translator; PD: photodiode; ADC: analog-to-digital converter; T&H: track-and-hold amplifier; T: sampling delay.

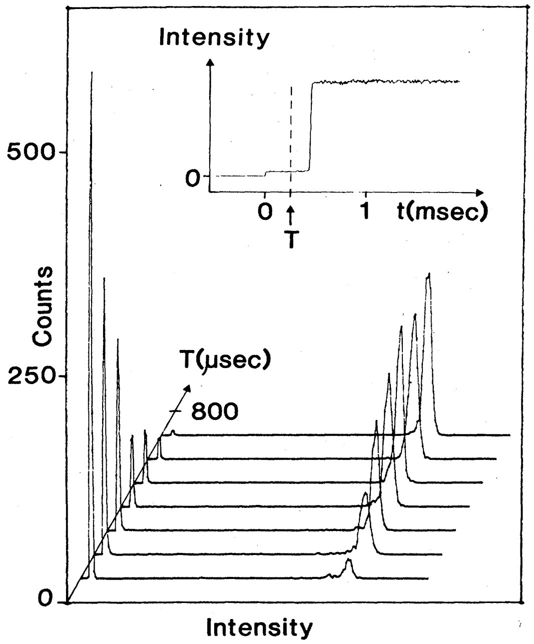

FIG. 2. Measured histograms of output intensities for different times T after switch on of the light. Experimental parameters are as follows: (average) input light power p_{in}: 12 mW; rms noise level: $0.2p_{in}$; number of intensity values on the abscissa: 256.

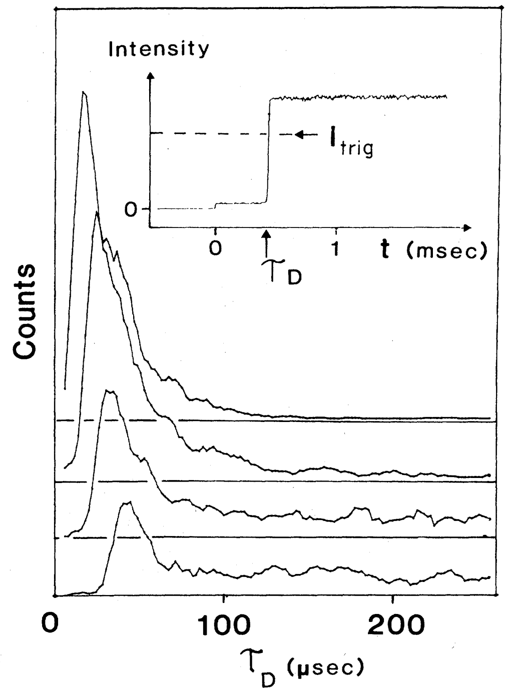

FIG. 3. Measured histograms of switching delay τ_D for different noise levels. Experimental parameters are as follows: p_{in}: 32 mW; rms noise levels from top to bottom: $0.21p_{in}$, $0.094p_{in}$, $0.065p_{in}$, and with external noise; number of time values on the abscissa: 256.

given for different times T. Deterministically, each of these should consist of one and only one δ-function-like peak, at low intensity for $T < \tau_D^{\rm det}$ and at high intensity for $T > \tau_D^{\rm det}$ with $\tau_D^{\rm det}$ denoting the deterministic value of τ_D. Noise obviously does not only broaden the peaks, but it also causes the probability distribution to be double peaked for a considerable range of intermediate T values. This is just the type of behavior predicted in Ref. 2.

In Ref. 2 the distribution of τ_D also has been calculated. Therefore, the experimental setup was slightly modified in order to measure the duration it takes the output intensity to reach a preassigned threshold level $I_{\rm trig}$. This time is a measure of τ_D (see inset of Fig. 3). Again from many shots histograms are obtained. Typical results for different noise levels are shown in Fig. 3. The curve at the bottom is obtained with no external noise added; it thus represents the distribution of τ_D due to the intrinsic noise in our system.

When we calculate the mean value $\langle \tau_D \rangle$ from the measurements, we find that it is reduced by increased noise as predicted in Ref. 2. This is an important result, since it implies that $\tau_D^{\rm det}$ generally cannot be obtained by an experiment involving noise. By improving the statistics the accuracy of $\langle \tau_D \rangle$ can be improved, but there is no obvious way of concluding on $\tau_D^{\rm det}$. This has to be kept in mind when predictions based on deterministic models[4] are to be checked.

Our experimental results reproduce the most prominent features of Fig. 5 of Ref. 2: (i) There is a broad asymmetric distribution of τ_D; (ii) the most probable as well as the mean value of τ_D is, by and large, decreasing with increasing noise levels; (iii) surprisingly enough, the distribution of τ_D can become narrower with increasing noise levels: The long tail in the distribution vanishes with increasing noise levels. The theoretical approach of Broggi and Lugiato, however, cannot directly be applied to our experimental findings because the situations analyzed in Ref. 2 and in this work are quite different. In Ref. 2 the good cavity limit is assumed with the nonlinear medium consisting of two-level atoms. In contrast, our experiment is performed in the bad cavity limit and the nonlinear mechanism is due to Zeeman pumping of the sodium ground state.

As pointed out above, the temporal variation of our system is essentially governed by the slow time evolution of the ground-state orientation z due to diffusive motion. If we approximately describe the decay of z by the lifetime τ_1 (with $\tau_1 \simeq 10$ μsec in our experiment), we find the equation of motion:[8,9]

$$\dot{z} = -(1/\tau_1 + P)z + P \quad . \tag{1}$$

Equation (1) represents a rate equation[10] with P denoting an optical pumping rate. In the purely absorptive case, P is related to the pumping rate $P_{\rm in}$ without resonator by an Airy function $A(z)$:[9]

$$P = A(z)P_{\rm in} = \frac{1-R_f}{(1-\sqrt{R_f R_b}e^{-A(1-z)})^2} P_{\rm in} \quad . \tag{2}$$

A is the optical density of the sample in very weak light fields and R_f (R_b) is the reflection coefficient of the front (back) mirror. It should be noted that the treatment of the dispersive or a mixed case is easily accomplished.

The temporal variations in the noise-modulated input intensity of our experiment give rise to fluctuations of $P_{\rm in}$:

$$P_{\rm in} = \bar{P}_{\rm in}(1+\zeta) \quad , \tag{3}$$

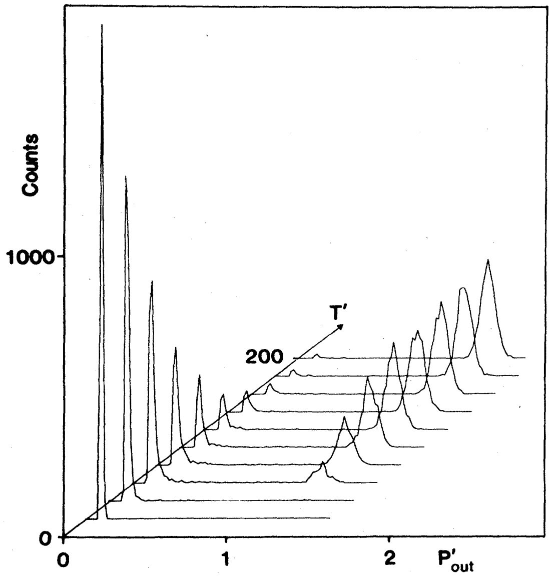

FIG. 4. Calculated histograms of $P'_{out} = P_{out}\tau_1$ for different $T' = T/\tau_1$ after switch on of the light. P'_{out} and T' are proportional to the output intensity and sampling time, respectively. Parameters are as follows: $P'_{in} = 3.5965$; $\tau_1/\tau_C = 14.3$; rms noise level $\sigma = 0.02$; number of P'_{out} values on the abscissa: 100; $A = 1$; $R_f = R_b = 0.94$.

with $\bar{P}_{in}$ denoting an average pumping rate; ζ is the fluctuating term describing noise contributions. Since Eq. (1) contains a term $zP(z)$, the fluctuations are of multiplicative nature, while additive noise was assumed in Ref. 2.

In a computer experiment we repeatedly solved Eqs. (1)–(3) by calculating the response to a step of P_{in} with stochastic noise $\zeta\bar{P}_{in}$ superimposed. We made the simplifying assumption that ζ switched regularly after a time interval of duration τ_c with a Gaussian distribution of values (standard deviation σ) derived from a random number generator. In order to simulate white noise we chose $\tau_C = 0.1\tau_1$. In a way analogous to the experimental procedure we obtained histograms of the calculated values of τ_D, of the state variable z, and of $P_{out} = P\exp[-A(1-z)](1-R_b)$; P_{out} is proportional to the output intensity observed in the experiment.

Typical results of our calculation are displayed in Figs. 4 and 5: There is a double-peaked distribution of z and P in a well-defined time regime after switch on of P_{in} (Fig. 4), and there is an asymmetric distribution of τ_D shifting to shorter times with increasing noise level (Fig. 5). From our computer experiments we can also obtain distributions of τ_D for very low noise levels that were not accessible in the experiment. Here we find nearly symmetric distributions of τ_D, with a width increasing with increasing noise levels ($\sigma = 0.002$ and 0.005 in Fig. 5); τ_D^{det} obviously lies very close to the center of these narrow distributions. Let us point out that the long tails in the experimental distributions of τ_D (Fig. 3) do not show up in Fig. 5. Preliminary computer results indicate, however, that such tails may be

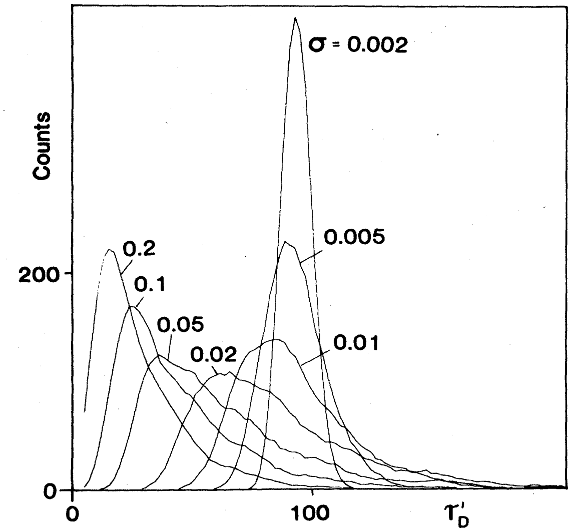

FIG. 5. Calculated histograms of normalized switching delay $T' = \tau_D/\tau_1$ for different noise levels σ. Otherwise parameters as in Fig. 4.

reproduced by choosing $\tau_C > \tau_1$ rather than $\tau_C << \tau_1$. Thus, we are led to conjecture that the experimentally observed tails are related to colored noise contributions like uncontrolled low-frequency fluctuations of acoustic origin.

Despite the simplicity of our theoretical model, Eqs. (1)–(3) seem to describe the main experimental features quite well though the agreement between calculations and experiment is still far from being quantitative. The main reason should be the fact that the atomic-diffusion process destroying the ground-state orientation cannot be represented adequately by a single time constant τ_1 as discussed previously.[6] Currently, we thus try to minimize the role of diffusion in order to facilitate a detailed comparison between theory and experiment. Moreover, experiments are in progress to study the effect of noise with different spectral distributions in transient OB.

In this paper we have given experimental evidence of the phenomenon of transient noise-induced optical bistability. We have further demonstrated that the distribution of delay times in the phenomenon of critical slowing down is asymmetric and very sensitive to noise. Moreover, our experiments clearly show that the most probable, as well as the mean delay, are severely shortened in the presence of strong external fluctuations. Finally, let us mention that we found similar results in the dynamic behavior of a bistable electronic circuit.[11] This indicates that the transient noise-induced phenomena reported here might in fact occur in a broad class of nonlinear systems.

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