Filamentation instability of the electron plasma wave at the difference frequency of two laser beams in a plasma

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A longitudinal electrostatic electron plasma wave excited at the difference frequency of two laser beams is shown to be unstable against filamentation instability in a hot collisionless plasma. The relativistic Vlasov equation has been solved to obtain the nonlinear response of electrons. It is noticed that for a considerable power density of the excited electron plasma wave the growth rate of the filamentation instability is quite high.

The nonlinear excitation of electrostatic electron plasma waves at the beat frequency of two high-frequency electromagnetic waves in a plasma has been extensively studied[1–8] because of its potential role in future high-energy plasma-laser accelerators and in other studies related to space and laboratory plasmas. In this Brief Report we show that an excited electron plasma wave at the difference frequency of two-laser radiation is effectively unstable against filamentation instability due to the presence of ion acoustic perturbations in a plasma. Since the temperature of electrons[6] in the beat wave accelerator may be as high as ~ 100 keV, we use the relativistic Vlasov equation to obtain the nonlinear response of electrons in the plasma.

We consider the propagation of two beams of high-power laser radiation along the same direction in a uniform, hot, collisionless and unmagnetized plasma:

$$\mathbf{E}'_{1,2}=\mathbf{E}''_{1,2}\exp[-i(\omega'_{1,2}t-k'_{1,2}z)] \ , \qquad (1)$$

where

$$k'_{1,2}\simeq(\omega'_{1,2}/c)(1-\omega_p^2/\omega'^2_{1,2})^{1/2} \ .$$

$\omega'_{1,2}$ and $k'_{1,2}$ are the angular frequencies and the wave vectors of the incident laser radiation, and $\omega_p=(4\pi e^2 n_0^0/m_0)^{1/2}$ is the electron plasma frequency, $-e$, m_0, n_0^0, and c being the electronic charge, rest mass, equilibrium unperturbed electrón density, and the velocity of light in a vacuum, respectively. On account of the nonlinear relativistic interaction of the incident waves, a longitudinal electron plasma wave ($\mathbf{k}_0,\omega_0$; $\mathbf{k}_0=\mathbf{k}'_1-\mathbf{k}'_2$, $\omega_0=\omega'_1-\omega'_2$) will be generated at the difference frequency through the forward Raman scattering, parametric instabilities, or the resonant excitation mechanism:

$$\omega_0^2=\omega_p^2+3k_0^2 v_e^2 \ , \qquad (2)$$

where $v_e\simeq(T_e/m_0)^{1/2}$ is the electron thermal speed, T_e being the electron temperature in units of Boltzmann constant. Now, we assume that this excited electrostatic electron plasma wave acts as a driving source (pump wave), couples parametrically with the low-frequency density perturbation present in a plasma due to a variety of reasons, such as ion acoustic mode $(\mathbf{k},\omega)$, and produces two electrostatic scattered sidebands

$$\mathbf{k}_{1,2},\omega_{1,2};\ \ \mathbf{k}_{1,2}=\mathbf{k}\mp\mathbf{k}_0,\ \ \omega_{1,2}=\omega\mp\omega_0 \ .$$

The nonlinear growth of the ion acoustic mode in the transverse direction will lead to the filamentation of the initial uniform beam of the electron plasma wave.

The response of electrons to the four-wave parametric process is governed by the nonlinear relativistic Vlasov equation[9]

$$\frac{\partial f}{\partial t}+\frac{\mathbf{v}}{\gamma}\cdot\nabla f-\frac{e\mathbf{E}}{m_0}\cdot\nabla_v f=0 \ , \qquad (3)$$

where $\mathbf{v}=\gamma\dot{\mathbf{r}}$ and

$$\gamma=(1-\dot{r}^2/c^2)^{-1/2}=(1+v^2/c^2)^{1/2} \ .$$

The motion of ions is neglected. The linear response of electrons to the four electrostatic waves involved may be written as

$$\begin{aligned} f_0^L&=-e\Phi_0 f_0^0(\mathbf{k}_0\cdot\mathbf{v})(1+\mathbf{k}_0\cdot\mathbf{v}/\gamma\omega_0)/T_e\omega_0 \ ,\\ f_1^L&=-e\Phi_1 f_0^0(\mathbf{k}_1\cdot\mathbf{v})(1+\mathbf{k}_1\cdot\mathbf{v}/\gamma\omega_1)/T_e\omega_1 \ ,\\ f_2^L&=-e\Phi_2 f_0^0(\mathbf{k}_2\cdot\mathbf{v})(1+\mathbf{k}_2\cdot\mathbf{v}/\gamma\omega_2)/T_e\omega_2 \ ,\\ f^L&=-e\Phi f_0^0(\mathbf{k}\cdot\mathbf{v})(1+\mathbf{k}\cdot\mathbf{v}/\gamma\omega)/T_e\omega \ , \end{aligned} \qquad (4)$$

where $\omega_{1,2}>\mathbf{k}_{1,2}\cdot\mathbf{v}$ has been assumed,[10] Φ's are the potentials of the electrostatic waves involved, and f_0^0 is the nonrelativistic Maxwellian distribution function[11] at the temperature T_e,

$$f_0^0=n_0^0(m_0/2\pi T_e)^{3/2}\exp(-m_0v^2/2T_e) \ . \qquad (5)$$

The nonlinear response of electrons at the low-frequency perturbation mode can be obtained from

$$\frac{\partial f^{\rm NL}}{\partial t}+\frac{\mathbf{v}}{\gamma}\cdot\nabla f^{\rm NL}-\frac{e}{2m_0}(\mathbf{E}_0\cdot\nabla_v f_1^L+\mathbf{E}_1\cdot\nabla_v f_0^L+\mathbf{E}_0^*\cdot\nabla_v f_2^L+\mathbf{E}_2\cdot\nabla_v f_0^{L*})=0 \ , \qquad (6)$$

where the symbol * denotes complex conjugate of the quantity involved. Using the expansion $\gamma\simeq(1+v^2/2c^2)$, we solve Eq. (6) and integrate over the velocity space to obtain the nonlinear density perturbation associated with the low-frequency ion acoustic mode propagating in the direction (x axis) transverse to the propagation direction (z axis) of the pump wave

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$(\mathbf{k}_0, \omega_0)$,

$$n^{\rm NL} = (n_0^0 e^2 k k_0 / 2 m_0^2 \omega^2)(\beta_1 \Phi_0 \Phi_1 + \beta_2 \Phi_0^* \Phi_2) \;, \tag{7}$$

where

$$\beta_1 = k_{1x}\left[\left(\frac{k_0}{\omega_0^2} + \frac{2k_{1z}}{\omega_1^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right) - \frac{2k_{1z}}{\omega_1^2}\left(1 - \frac{5v_e^2}{c^2}\right)\right], \quad \beta_2 = k_{2x}\left[\left(\frac{k_0}{\omega_0^2} - \frac{2k_{2z}}{\omega_2^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right) + \frac{2k_{2z}}{\omega_2^2}\left(1 - \frac{5v_e^2}{c^2}\right)\right] .$$

In a similar manner we can solve Eq. (3) to obtain the nonlinear density fluctuations associated with the scattered sidebands as

$$n_1^{\rm NL} = \frac{n_0^0 e^2 \Phi \Phi_0^* k k_0}{2 m_0^2 \omega_1^2}\left(\frac{k_0 k_{1x}}{\omega_0^2} - \frac{k k_{1z}}{\omega^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right) , \tag{8}$$

$$n_2^{\rm NL} = \frac{n_0^0 e^2 \Phi \Phi_0 k k_0}{2 m_0^2 \omega_2^2}\left(\frac{k_0 k_{2x}}{\omega_0^2} + \frac{k k_{2z}}{\omega^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right) . \tag{9}$$

Substituting Eqs. (7)–(9) into the Poisson's equation and eliminating Φ, Φ_1, and Φ_2 we obtain the nonlinear dispersion relation for the low-frequency ion acoustic mode as

$$\epsilon = \mu_1/\epsilon_1 + \mu_2/\epsilon_2 \;, \tag{10}$$

where

$$\epsilon = 1 - \omega_p^2(1 - 5v_e^2/2c^2)/\omega^2, \quad \epsilon_1 = 1 - \omega_p^2(1 - 5v_e^2/2c^2)/\omega_1^2, \quad \epsilon_2 = 1 - \omega_p^2(1 - 5v_e^2/2c^2)/\omega_2^2 \;,$$

$$\mu_1 = \frac{|v_0/v_e|^2 \omega_p^4 \omega_0^2 k_{1x} v_e^2}{4\omega^2 \omega_1^2 k_1^2}\left(\frac{k_0 k_{1x}}{\omega_0^2} - \frac{k k_{1z}}{\omega^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right)\left[\left(\frac{k_0}{\omega_0^2} + \frac{2k_{1z}}{\omega_1^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right) - \frac{2k_{1z}}{\omega_1^2}\left(1 - \frac{5v_e^2}{c^2}\right)\right] ,$$

$$\mu_2 = \frac{|v_0/v_e|^2 \omega_p^4 \omega_0^2 k_{2x} v_e^2}{4\omega^2 \omega_2^2 k_2^2}\left(\frac{k_0 k_{2x}}{\omega_0^2} + \frac{k k_{2z}}{\omega^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right)\left[\left(\frac{k_0}{\omega_0^2} - \frac{2k_{2z}}{\omega_2^2}\right)\left(1 - \frac{6v_e^2}{c^2} - \frac{14v_e^4}{c^4}\right) + \frac{2k_{2z}}{\omega_2^2}\left(1 - \frac{5v_e^2}{c^2}\right)\right] ,$$

$$|v_0|^2 = e^2 k_0^2 \Phi_0 \Phi_0^* / m_0^2 \omega_0^2 \;.$$

In the presence of the pump wave $(\mathbf{k}_0, \omega_0)$ the angular frequency of the perturbation gets modified from the real value and one may expand

$$\omega = \omega + i\gamma, \quad \epsilon = i(\gamma + \Gamma)(\partial\epsilon/\partial\omega) \;,$$
$$\epsilon_{1,2} = i(\gamma + \Gamma_{1,2})(\partial\epsilon_{1,2}/\partial\omega_{1,2}) \;, \tag{11}$$

where Γ and $\Gamma_{1,2}$ are the linear damping rates of the decay waves. We neglect the linear damping of the high-frequency scattered electrostatic waves, so that the threshold of the instability is negligible. Substituting Eqs. (11) into Eq. (10) with the approximation $\Gamma_1 = \Gamma_2 \simeq 0$ we obtain the

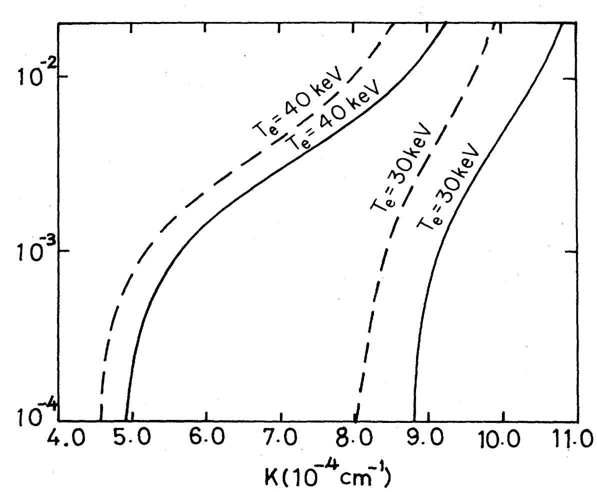

FIG. 1. Variation of γ/ω with k for $|v_0/v_e| = 0.1$ and different T_e as indicated. The solid curves represent two incident CO_2 laser beams, and the dashed curves represent the two incident Nd-glass laser beams.

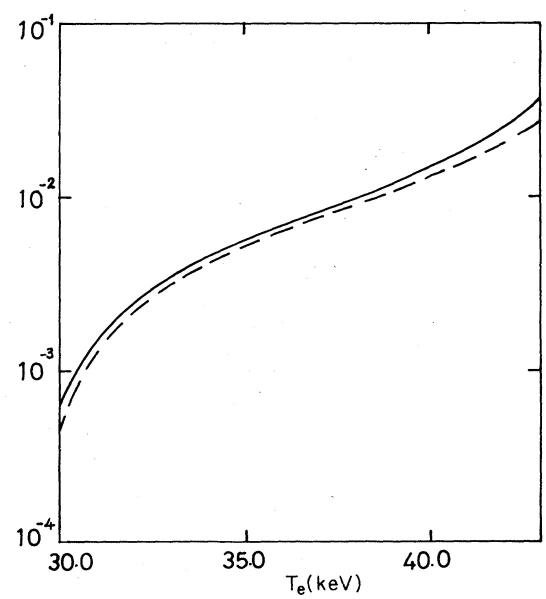

FIG. 2. Variation of γ/ω with T_e for $|v_0/v_e| = 0.1$. The solid curves represent two incident CO_2 laser beams and $k = 9.0 \times 10^4$ cm^{-1}, and the dashed curves represent two incident Nd-glass laser beams and $k = 8.2 \times 10^4$ cm^{-1}.

growth rate of the filamentation instability from the relation

$$\gamma(\gamma+\Gamma)=-\frac{1}{(\partial\epsilon/\partial\omega)}\left[\frac{\mu_1}{\partial\epsilon_1/\partial\omega_1}+\frac{\mu_2}{\partial\epsilon_2/\partial\omega_2}\right]\equiv\gamma_0^2\ , \quad (12)$$

where[12]

$$\Gamma=(\pi m/8m_i)^{1/2}kC_s+(T_e/T_i)^{3/2}\exp[-(3+T_e/T_i)/2]\ .$$

The suffix i indicates a quantity associated with plasma ions and C_s is the ion sound speed in the plasma. The growth rate of the instability in the presence of the linear damping of the ion acoustic perturbation is given by

$$\gamma=[(\Gamma^2+4\gamma_0^2)^{1/2}-\Gamma]/2\ . \quad (13)$$

For a numerical application of the results we have calculated the growth rates γ and γ_0 of the filamentation instability of the longitudinal electrostatic electron plasma wave at the difference frequency of two-laser radiation for the following parameters: $\omega'_{1,2}=1.963\times10^{14}$, 1.778×10^{14} rad s^{-1} (CO_2 laser), $\omega'_{1,2}=1.795\times10^{15}$, 1.778×10^{15} rad s^{-1} (Nd-glass laser), $T_e=50$–60 keV, $k=10^2$–10^4 cm^{-1}, $m_i/m=1836$, and $T_e/T_i=20$.

It is observed that the growth rate of the filamentation instability increases linearly with the pump-induced velocity of electrons, $|v_0/v_e|$. The undamped growth rate γ_0 is always very close to the value of the overall growth rate γ of the instability. Figure 1 shows the variation of the relative growth rate γ/ω as a function of the wave number k of the ion acoustic perturbation mode. The growth rate of the filamentation instability increases more rapidly with k for lower electron temperature. In Fig. 2 we notice that γ/ω increases with temperature for fixed values of k.

We therefore anticipate that the filamentation instability of the excited electron plasma wave at the beat frequency of two-laser beams may lead to the possible difficulty of accelerating particles uniformly. Furthermore, the application of the external transverse magnetic field, which eliminates the maximum energy gain of the particles, may suppress the undesirable filamentation instability drastically. This is yet to be seen.

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[1]B. I. Cohen, A. N. Kaufmann, and K. M. Watson, Phys. Rev. Lett. **29**, 581 (1972).

[2]M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972).

[3]B. L. Stansfield, R. Nodwell, and J. Meyer, Phys. Rev. Lett. **26**, 1219 (1971).

[4]C. J. Pawley, H. E. Huey, and N. C. Luhmann, Jr., Phys. Rev. Lett. **49**, 877 (1982), and references therein.

[5]T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).

[6]C. Joshi, T. Tajima, J. M. Dawson, H. A. Baldis, and N. A. Ebrahim, Phys. Rev. Lett. **47**, 1285 (1981); T. Tajima, in Proceedings of the ECFA-RAL meeting held at New College, Oxford, September 1982 (unpublished), p. 169.

[7]T. Katsouleas and J. M. Dawson, Phys. Rev. Lett. **51**, 392 (1983).

[8]J. D. Lawson, Proceeding of the ECFA-RAL meeting held at New College, Oxford, September 1982 (unpublished), p. 29.

[9]D. E. Baldwin, I. B. Bernstein, and M. P. H. Weenink, in *Advances in Plasma Physics,* edited by A. Simon and W. B. Thompson (Wiley, New York, 1969), Vol. 3.

[10]V. K. Tripathi, C. Grebogi, and C. S. Liu, Phys. Fluids **22**, 301 (1979).

[11]M. Y. El-Ashry, International Centre for Theoretical Physics, Trieste, Italy, Report No. IC/82/152, 1982 (unpublished).

[12]C. S. Liu and P. K. Kaw, in *Advances in Plasma Physics,* edited by A. Simon and W. B. Thompson (Wiley, New York, 1976), Vol. 6.