Theory of radiative corrections to Auger and fluorescence yields and dielectronic satellite line intensities

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The excitation and decay of autoionizing states of multiply charged ions in plasmas can play an important role in the determination of both the distribution of the various charge states and the spectrum of the emitted radiation. All previous theoretical treatments for low-density plasmas are based on the conventional expressions for the Auger and fluorescence branching ratios, in which the autoionization and radiative decay rates are assumed to be additive. Using a two-level atom model, Armstrong, Theodosiou, and Wall and also Haan and Cooper have investigated the effects of the interaction between the final continuum states which result from the autoionization and radiative decay modes. They have shown that this interaction produces an interference which can alter the relative probabilities for decay into the two alternative continuum channels. In the present investigation, the general properties of angular momentum and spherical tensor operators are employed to extend the theory of this interference to the case in which each of the atomic levels consists of a set of $2J+1$ degenerate magnetic sublevels, where J is the total electronic angular momentum. In the case where this interference involves only a single term in the partial-wave expansion for the electron-continuum state, the expressions obtained for the Auger and fluorescence branching ratios are in agreement with those derived in the previous investigations. When several terms in the electron-continuum partial-wave expansion are involved, the Auger and fluorescence branching ratios contain terms corresponding to the interference between different partial-wave components analogous to those which occur in the expression for the photoelectron angular distribution asymmetry parameter. The electromagnetic interaction between the final continuum states of the combined atom plus quantized radiation field system may be expressed in terms of the matrix elements for either the photoelectric transition or the inverse radiative recombination process connecting the final atomic states. Finally, the corrected expressions are obtained for the resonant electron-impact excitation rates and the intensities of the dielectronic satellite lines resulting from the decay of autoionizing states of multiply charged ions in plasmas. The modifications to the conventional expression for the satellite line intensities may be interpreted as terms corresponding to the interference between the direct radiative recombination and dielectronic recombination processes together with radiative corrections to the dielectronic recombination process.

I. INTRODUCTION

The process of dielectronic recombination has been the subject of intense theoretical interest ever since the discovery by Burgess[1] that this process is often the dominant recombination mechanism for multiply charged atomic ions in low-density high-temperature laboratory and astrophysical plasmas. Several attempts have been made to determine experimentally both the dielectronic recombination cross sections describing crossed electron-ion beam interactions[2–4] and the effective recombination rates of impurity ions in hydrogen plasmas.[5–7] The satellite lines (associated with the resonance lines) which are produced by the dielectronic recombination process are often prominent features in the far-ultraviolet and x-ray emission spectra of both low-density[8–10] and high-density[11,12] plasmas, and the analysis of the satellite spectra has been found to be of great value in the spectroscopic determination of temperatures, densities, and departures from ionization equilibrium.

In order to provide a complete description of both the electron-ion beam interaction and the effective recombination process which occurs in a plasma, it has been necessary to generalize the conventional theory of dielectronic recombination[1] to include the effects of electrostatic fields[13,14] and charged-particle collisions.[15–17] The traditional theory of dielectronic satellite line intensities[18] has also been extended[19–22] to include the effects of electron-induced collisional transitions between the autoionizing states. Motivated by the various possibilities for making a reliable experimental determination of the cross section or rate coefficient for a dielectronic recombination process, the present investigation is an attempt to develop a rigorous quantum-mechanical theory of this resonant electron-ion recombination process. In order to accomplish this objective, it is necessary to employ the methods of multichannel collision theory[23,24] and quantum electrodynamics.[25,26]

The basic theoretical description of the autoionization process in the absence of spontaneous radiative decay has been firmly established as a result of the asymmetric lineshape theory developed by Fano[27–29] together with the

projection-operator formalism introduced by Feshbach.[30,31] The first quantum-mechanical theory for the spontaneous radiative decay of excited atomic states was presented by Weisskopf and Wigner,[32,33] who showed that the spectral line shape can be represented in the Lorentzian form. The theory of spontaneous radiative emission has been reformulated in very general terms by Heitler[25] and by Goldberger and Watson.[23] To these formulations must be added the important work of Low,[34] who applied the S-matrix theory developed by Feynman[35,36] and Dyson[37,38] to obtain radiative corrections to the spectral line shape. Only a very limited number of theoretical analyses have been reported in which the autoionization and radiative decay processes are treated in a unified manner. The methods of multichannel collision theory and quantum electrodynamics have been employed by Shore[39] in a quantum-mechanical treatment of the effects of autoionizing levels on spectral line shapes. However, only the lowest-order nonvanishing terms were retained in his results for the T matrix, leading to expressions in which the autoionization and radiative decay rates are additive.

Although the interference between the autoionization and radiative decay processes is in principle included in the theories of dielectronic recombination presented by Davies and Seaton[40] and by Trefftz,[41] Armstrong, Theodosiou, and Wall[42] were the first to report a comprehensive investigation of the effects of the electromagnetic interaction between the final continuum states resulting from these two alternative decay modes. Using a two-level atom model, they demonstrated that the final-state interaction can alter the relative probabilities for decay into the two alternative continuum channels. They also obtained expressions for the Auger and fluorescence branching ratios for the case in which spontaneous radiative transitions can occur into a set of lower atomic states or when autoionization into a set of states of the residual ion is permissible. However, the realistic case of angular momentum degeneracy was not considered, and their analysis was presented using the energy and angular momentum representation of the continuum states, which does not yield results for the angular distributions of the emitted electrons and photons. Finally, they derived approximate expressions for the Auger and fluorescence yields which they believed were valid only when the final-state continuum-continuum coupling is weak.

The results obtained by Armstrong, Theodosiou, and Wall[42] have been recently rederived by Haan and Cooper[43] using elegant techniques from multichannel scattering theory.[23,24] By taking advantage of the separable form of the final-state continuum-continuum coupling, they were able to obtain an exact closed-form solution for the two-level atom problem. In addition, they demonstrated that the approximate expressions for the Auger and fluorescence branching ratios first obtained by Armstrong, Theodosiou, and Wall[42] remain valid even when the continuum-continuum coupling is strong and, therefore, have a wider region of validity than originally recognized.

The remainder of this paper is organized in the following manner. In Sec. II the two-level atom model is extended to the case in which each of the atomic levels consists of a set of degenerate magnetic sublevels and the electron- and photon-continuum states are specified by giving the momentum in the propagation direction and the spin projection or polarization. This extension is facilitated by using the general angular momentum and spherical-tensor-operator techniques[44,45] which have been employed in the theory of the angular distribution and spin polarization of photoelectrons.[46] Assuming that the limited number of eigenstates of the unperturbed Hamiltonian without the atom field interaction forms an adequate basis set, an exact diagonalization of the complete Hamiltonian for the combined atom plus quantized radiation field system is carried out by utilizing a method based on the Møller scattering operator.[24] Assuming that the atom field system has been initially prepared in the autoionizing state, closed-form expressions are obtained for the exact autoionization and radiative decay amplitudes. In Sec. III approximate expressions are derived for the Auger and fluorescence branching ratios, and the cases of multichannel electron and photon continua are treated. The resonant electron-impact excitation and dielectronic recombination rates in the presence of the final-state continuum-continuum interaction are obtained for low-density plasmas. Finally, the conclusions are presented in Sec. IV.

II. THE TWO-LEVEL ATOM MODEL

A. Unperturbed eigenstates and their interactions

As a first step in generalizing the theory of the interference between autoionization and spontaneous radiative decay to realistic atomic problems, the two-level atom model employed by Armstrong, Theodosiou, and Wall[42] and by Haan and Cooper[43] must be extended to the case in which each of the atomic levels consists of a set of degenerate magnetic sublevels. In order to make possible the inclusion of relativistic effects in the atomic wave functions, the atomic states will be assumed to be eigenstates of the total electronic angular momentum J only rather than of both the orbital and spin angular momenta L and S. In this paper the word "atom" will be used to indicate either a neutral atom or a positive ion, whereas the word "ion" will be used to refer to the residual atomic system resulting from autoionization. Finally, both the emitted electron and photon states will be specified by giving the momentum in the propagation direction and by giving the spin projection or polarization. This allows the determination of the angular distributions if desired.

Consider an excited state $|a\rangle$ of an atom which can decay either by an autoionization process to the state $|i\rangle$ of the ion, emitting an electron with momentum $\vec{p}$, or by a spontaneous radiative transition to a state $|f\rangle$, emitting a photon with momentum $\vec{k}$. The direct-product states denoted by $|a,0\rangle$, $|i\vec{p},0\rangle$, and $|f,\vec{k}\rangle$ may then be treated as eigenstates of the Hamiltonian

$$H^0=H_A+H_F\ , \tag{1}$$

consisting of the unperturbed atomic Hamiltonian H_A and the Hamiltonian H_F for the free radiation field. Using this set of states as a basis, the solution of the model

problem involves the diagonalization of the complete Hamiltonian

$$H = H_A + H_F + H_{AF} \tag{2}$$

for the combined atom plus radiation field system, including the atom field interaction H_{AF}. The relevant atomic levels are schematically illustrated in Fig. 1, which shows the interactions which occur between the eigenstates of H^0. The interference between the autoionization and radiative decay processes is a consequence of the electromagnetic interaction coupling the final continuum states which result from the two alternative decay modes. The corresponding interaction matrix element occurs in the cross section for photoionization from the state $|f\rangle$ and also for the inverse radiative recombination process $|i\vec{p},0\rangle \rightarrow |f,\vec{k}\rangle$.

Before describing the diagonalization of the complete Hamiltonian H, it is necessary to specify the atomic states in greater detail and to evaluate the matrix elements of the interactions. The autoionizing state $|a\rangle$ and the final atomic state $|f\rangle$ will be specified by

$$|a\rangle = |\gamma_a J_a M_a\rangle \tag{3}$$

and

$$|f\rangle = |\gamma_f J_f M_f\rangle \ , \tag{4}$$

where J_a and J_f are the total electronic angular momenta, M_a and M_f are the angular momentum components along the direction of quantization, and all additional quantum numbers are represented by γ_a and γ_f. A rigorous description of the autoionization process is provided by the Feshbach projection-operator formalism,[30,31] according to which the autoionizing state $|a\rangle$ is not an exact eigenstate of the full atomic Hamiltonian H_A but may instead be taken to be an eigenstate of the projected Hamiltonian $(1-P)H_A(1-P)$, where P is the projection operator onto the subspace of the open electron-continuum channel $|i,\vec{p}\rangle$. The interaction which gives rise to autoionization is precisely given by the projected interaction $(1-P)H_AP$, but the projection operators will not be explicitly indicated in the following discussion.

Denoting the electron-spin projection quantum number by m_s, the continuum state of the ejected-electron residual-ion system will be specified by

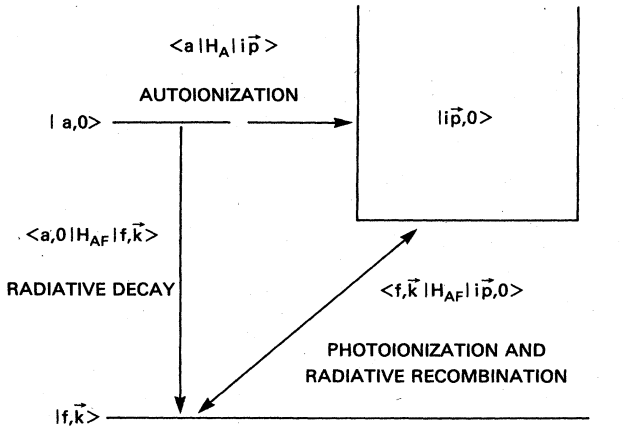

FIG. 1. The eigenstates of the Hamiltonian $H^0 = H_A + H_F$ and the interactions between them which occur in the diagonalization of the complete Hamiltonian $H = H_A + H_F + H_{AF}$ for the two-level atom model.

$$|i\vec{p}m_s\rangle = |\gamma_i J_i M_i, \vec{p}m_s\rangle \tag{5}$$

and will be assumed to have the asymptotic form corresponding to a Coulomb-modified plane wave and an incoming spherical wave.[47] The electron-continuum states will also be assumed to satisfy the δ-function normalization and orthogonality property

$$\langle i\vec{p}m_s | i\vec{p}\,'m_s'\rangle = \delta^3(\vec{p}-\vec{p}\,')\delta_{m_s,m_s'} \ . \tag{6}$$

In order to evaluate the matrix elements of the interaction, it will be necessary to make a transformation to a representation in which the ejected-electron states are specified by the relative orbital angular momentum quantum numbers l and m. The ejected-electron angular momenta l and s are then coupled to the total angular momentum J_i of the residual ion to form the total electronic angular momentum J of the electron-ion system. Using the angular momentum coupling scheme introduced by Blatt and Biedenharn[48] and the Wigner 3-j symbols, whose properties may be found in the book by deShalit and Talmi,[45] the electron-continuum partial-wave expansion can be expressed in the form[46]

$$|i\vec{p}m_s\rangle = \sum_{l,m} \exp[i(l\pi/2-\sigma_l)]Y_{lm}^*(\hat{p}) \times \sum_{K,N}\sum_{J,M}(-1)^{s-J_i-N+K-l-M}[(2K+1)(2J+1)]^{1/2}\begin{pmatrix} J & s & K \\ M_i & m_s & -N\end{pmatrix}\begin{pmatrix} l & K & J \\ m & N & -M\end{pmatrix}|\gamma_i J_i, Kpl; JM\rangle \ , \tag{7}$$

where σ_l denotes the Coulomb phase shift. The continuum states represented by this partial-wave expansion describe electron-ion scattering in the absence of the electron-electron interaction which gives rise to autoionization and in the absence of the electromagnetic interaction which leads to spontaneous radiative decay. In the terminology of the Feshbach projection-operator formalism,[30,31] these states correspond to the nonresonant scattering states which are the exact eigenstates of the projected Hamiltonian PH_AP, defined within the open electron-continuum channel subspace. The 3-j selection rules[45] together with the Wigner-Eckart[44,45] theorem, which will be employed in the evaluation of the various interaction matrix elements, restrict the range of the angu-

lar momentum summations in Eq. (7) to a very small number of terms.

In the diagonalization of the complete Hamiltonian H, the eigenstates of H^0 will be assumed to satisfy the conditions

$$\langle a,0 \mid H \mid a,0\rangle = E_a \ , \tag{8}$$

$$\langle i\vec{p}m_s,\ 0 \mid H \mid i\vec{p}\,'m_s',\ 0\rangle = (E_i+\epsilon_p)\delta^3(\vec{p}-\vec{p}\,')\delta_{m_s,m_s'} \equiv E_p\delta^3(\vec{p}-\vec{p}\,')\delta_{m_s,m_s'} \ , \tag{9}$$

$$\langle f,\vec{k}\lambda \mid H \mid f,\vec{k}'\lambda'\rangle = (E_f+\hbar\omega_k)\delta^3(\vec{k}-\vec{k}')\delta_{\lambda,\lambda'} \equiv E_k\delta^3(\vec{k}-\vec{k}')\delta_{\lambda,\lambda'} \ , \tag{10}$$

where λ denotes the photon polarization ($\lambda=1,2$ for each value of $\vec{k}$). The photon-continuum state is specified in the occupation number representation by the direct-product states $|f,\vec{k}\lambda\rangle$ which, according to Eq. (10), have an orthonormality property analogous to that for the electron-continuum state given by Eq. (6). Note that only the two Fock-space state vectors $|0\rangle$ and $|\vec{k}\lambda\rangle$ of the quantized radiation field are considered in the present investigation. They correspond to the vacuum state and the state with one emitted photon, respectively.

The partial-wave expansion (7) for the electron-continuum state together with the Wigner-Eckart theorem[44,45] allows the interaction matrix element responsible for autoionization to be expressed in the form

$$\langle a \mid H_A \mid i\vec{p}m_s\rangle = \sum_{l,m}\exp[i(l\pi/2-\sigma_l)]Y^*_{lm}(\hat{p}) \times \sum_{K,N}(-1)^{s-J_i-N+K-l-M_a}[(2K+1)]^{1/2}\begin{pmatrix} J_i & s & K \\ M_i & m_s & -N\end{pmatrix}\begin{pmatrix} l & K & J_a \\ m & N & -M_a\end{pmatrix}\langle\gamma_aJ_a||H_A||\gamma_iJ_i,Kpl;J_a\rangle \ , \tag{11}$$

where the double bars denote the reduced matrix element of H_A.

Using temporarily the finite-volume quantization for the radiation field, the electromagnetic interaction in the electric dipole approximation can be written as[49]

$$H_{AF} = -e\vec{D}\cdot\vec{E} = -\vec{D}\cdot\sum_{\vec{k},\lambda} i\left[\frac{2\pi e^2\hbar\omega_k}{V}\right]^{1/2}(a_{\vec{k}\lambda}\hat{\epsilon}_{\vec{k}\lambda}-a^\dagger_{\vec{k}\lambda}\hat{\epsilon}^*_{\vec{k}\lambda}) \ , \tag{12}$$

where $\hat{\epsilon}_{\vec{k}\lambda}$ are the unit polarization vectors and $a_{\vec{k}\lambda}$ and $a^\dagger_{\vec{k}\lambda}$ are the photon annihilation and creation operators, respectively. The atomic dipole-moment operator denoted by $\vec{D}$ has the dimensions of length.

The interaction matrix element describing the spontaneous radiative decay process is given by

$$\langle a,0 \mid H_{AF} \mid f,\vec{k}\lambda\rangle = -i\left[\frac{2\pi e^2\hbar\omega_k}{V}\right]^{1/2}\sum_\mu(\hat{\epsilon}_{\vec{k}\lambda})^*_\mu(-1)^{J_a-M_a}\begin{pmatrix} J_a & 1 & J_f \\ -M_a & \mu & M_f\end{pmatrix}\langle\gamma_aJ_a||\vec{D}||\gamma_fJ_f\rangle \ , \tag{13}$$

where μ denotes the spherical tensor components of the unit polarization vector $\epsilon_{\vec{k}\lambda}$. Note that the complex conjugation arises from the definition of the scalar product in spherical components and not from the creation-operator term in Eq. (12).

Finally, the final-state continuum-continuum interaction matrix element is obtained in the separable form[50]

$$\langle i\vec{p}m_s,0 \mid H_{AF} \mid f,\vec{k}\lambda\rangle = \sum_\mu f^*_\mu(\vec{p},m_s)g_\mu(\vec{k},\lambda) \ , \tag{14}$$

where

$$f^*_\mu(\vec{p},m_s) = \langle i\vec{p}m_s \mid \vec{D}_\mu \mid f\rangle \equiv f^*_\mu(M_i\vec{p}m_s,M_f) = \sum_{l,m}\exp[-i(l\pi/2-\sigma_l)]Y_{lm}(\hat{p}) \times \sum_{K,N}\sum_{J,M}(-1)^{s-J_i-N+K-l-2M+J}[(2K+1)(2J+1)]^{1/2} \times \begin{pmatrix} J_i & s & K \\ M_i & m_s & -N\end{pmatrix}\begin{pmatrix} l & K & J \\ m & N & -M\end{pmatrix}\begin{pmatrix} J & 1 & J_f \\ -M & \mu & M_f\end{pmatrix}\langle\gamma_iJ_i,Kpl;J||\vec{D}||\gamma_fJ_f\rangle \tag{15}$$

and

$$g_\mu(\vec{k},\lambda) = -i\left[\frac{2\pi e^2\hbar\omega}{V}\right]^{1/2}(\hat{\epsilon}_{\vec{k}\lambda})^*_\mu \ . \tag{16}$$

B. Diagonalization of the complete Hamiltonian H

The transition amplitudes describing autoionization and spontaneous radiative decay will be obtained in Sec. II C by employing the method of diagonalization involving the Møller scattering operator[24] Ω_- [defined in Eqs. (35) and (36) of Sec. II C], which provides a transformation from the continuum eigenstates of the unperturbed Hamiltonian H^0 to the continuum eigenstates of the complete Hamiltonian H satisfying the required asymptotic boundary conditions. The Møller scattering operator Ω_- can in turn be expressed in terms of the resolvent or Green's operator $G(z)$ which satisfies the equation

$$(z-H^0)G(z)=1+VG(z)\ , \tag{17}$$

where V represents both the electromagnetic interaction H_{AF} and the projected electron-electron interaction. Taking the matrix elements of Eq. (17) between the eigenstates of H^0 yields a set of coupled equations for the components of $G(z)$. The components which are required for the evaluation of the electron and photon transition amplitudes carried out in Sec. II C will be denoted by $\langle a,0\,|\,G(z)\,|\,a,0\rangle$, $\langle i\vec{p}m_s,0\,|\,G(z)\,|\,a,0\rangle$, and $\langle f,\vec{k}\lambda\,|\,G(z)\,|\,a,0\rangle$. Because $G(z)$ is a spherical tensor operator of rank 0, it must be diagonal in M_a.

The expression obtained for $(z-E_a)\langle a,0\,|\,G(a)\,|\,a,0\rangle$ can be written in the form

$$(z-E_a)\langle a,0\,|\,G(z)\,|\,a,0\rangle=1+\sum_{M_i}\sum_{M_s}\int d^3p\langle a,0\,|\,V\,|\,i\vec{p}m_s,0\rangle\langle i\vec{p}m_s,0\,|\,G(z)\,|\,a,0\rangle$$
$$+\sum_{M_f}\sum_{\lambda}\sum_{\vec{k}}\langle a,0\,|\,V\,|\,f,\vec{k}\lambda\rangle\langle f,\vec{k}\lambda\,|\,G(z)\,|\,a,0\rangle\ . \tag{18}$$

Analogous expressions are obtained for $(z-E_p)\langle i\vec{p}m_s,0\,|\,G(z)\,|\,a,0\rangle$ and $(z-E_k)\langle f,\vec{k}\lambda\,|\,G(z)\,|\,a,0\rangle$. An equation relating $\langle f,\vec{k}\lambda\,|\,G(z)\,|\,a,0\rangle$ and $\langle a,0\,|\,G(z)\,|\,a,0\rangle$ can be derived by eliminating $\langle i\vec{p}m_s,0\,|\,G(z)\,|\,a,0\rangle$. To extract a solution, this equation must be multiplied by

$$\begin{pmatrix} J_a & 1 & J_f \\ -M_a & \mu & M_f \end{pmatrix} g_\mu(\vec{k},\lambda)$$

for some fixed μ and then summed over M_f, μ, λ, and $\vec{k}$. The resulting equation can be expressed in the form

$$\sum_{M_f}\sum_{\mu}\sum_{\lambda}\sum_{\vec{k}}\begin{pmatrix} J_a & 1 & J_f \\ -M_a & \mu & M_f \end{pmatrix} g_\mu(\vec{k},\lambda)\langle f,\vec{k}\lambda\,|\,G(z)\,|\,a,0\rangle$$
$$=\sum_{\mu}\sum_{M_f}\begin{pmatrix} J_a & 1 & J_f \\ -M_a & \mu & M_f \end{pmatrix}\sigma_\mu^{ga}(M_fM_a,z)\langle a,0\,|\,G(z)\,|\,a,0\rangle$$
$$+\sum_{\mu}\sum_{\nu}\sum_{M_f}\sigma_{\mu\nu}^{gg}(z)\begin{pmatrix} J_a & 1 & J_f \\ -M_a & \mu & M_f \end{pmatrix}\sigma_\nu^{fa}(M_fM_a,z)\langle a,0\,|\,G(z)\,|\,a,0\rangle$$
$$+\sum_{\mu}\sum_{\mu'}\sigma_{uu'}^{gg}(z)\sum_{\nu}\sum_{M_f}\sum_{M_f'}\begin{pmatrix} J_a & 1 & J_f \\ -M_a & \mu & M_f \end{pmatrix}\sigma_{\mu'\nu}^{ff}(M_fM_f',z)\sum_{\lambda'}\sum_{\vec{k}'}g_\nu(\vec{k}',\lambda')\langle f',\vec{k}'\lambda'\,|\,G(z)\,|\,a,0\rangle\ . \tag{19}$$

The vector and tensor self-energies in Eq. (19), which are generalizations of those introduced by Haan[50] to include the magnetic quantum numbers, are given by the following definitions:

$$\sigma_\mu^{fa}(M_fM_a,z)=\sum_{M_i}\sum_{m_s}\int d^3p\frac{f_\mu(M_i\vec{p}m_s,M_f)}{z-E_p}\times\langle i\vec{p}m_s,0\,|\,V\,|\,a,0\rangle\ , \tag{20}$$

$$\sigma_{\mu\nu}^{ff}(M_fM_f',z)=\sum_{M_i}\sum_{m_s}\int d^3p\frac{f_\mu(M_i\vec{p}m_s,M_f)}{z-E_p}\times f_\nu^*(M_i\vec{p}m_s,M'_f)\ , \tag{21}$$

$$\sigma_\mu^{ga}(M_fM_a,z)=\sum_{\lambda}\sum_{\vec{k}}\frac{g_\mu(\vec{k},\lambda)\langle f,\vec{k}\lambda\,|\,V\,|\,a,0\rangle}{z-E_k}\ , \tag{22}$$

$$\sigma_{\mu\nu}^{gg}(z)=\sum_{\lambda}\sum_{\vec{k}}\frac{g_\mu(\vec{k},\lambda)g_\nu^*(\vec{k},\lambda)}{z-E_k}\ . \tag{23}$$

The transpose elements are defined by relations such as the following:

$$\sigma_\mu^{af}(M_aM_f,z)=\sum_{M_i}\sum_{m_s}\int d^3p\langle a,0\,|\,V\,|\,i\vec{p}m_s,0\rangle\times f_\mu^*(M_i\vec{p}m_s,M_f)$$
$$=[\sigma_\mu^{fa}(M_fM_a,z^*)]^*\ . \tag{24}$$

A substantial simplification of Eq. (19) can be achieved by taking advantage of the properties of the tensor self-energies $\sigma^{ff}(z)$ and $\sigma^{gg}(z)$. The tensor self-energy $\sigma^{gg}(z)$ can be shown[50] to be expressible in the diagonal form

$$\sigma^{gg}_{\mu\nu}(z)=\Sigma^{gg}(z)\delta_{\mu\nu}\,, \tag{25}$$

where the scalar self-energy $\Sigma^{gg}(z)$ is given by

$$\Sigma^{gg}(z)=\frac{2e^2\hbar}{3\pi c^3}\int_0^\infty \frac{d\omega_k\omega_k^3}{z-E_k}\,. \tag{26}$$

In arriving at the final expression for $\Sigma^{gg}(z)$, the discrete sum over $\vec{k}$ has been converted into an integral by making the transformation

$$\sum_{\vec{k}}\rightarrow\frac{V}{(2\pi)^3}\int\frac{\omega_k^2}{c^3}d\omega_k\int d\Omega_k\,. \tag{27}$$

The scalar self-energy $\Sigma^{gg}(z)$ can be evaluated as a sum of a principal-value integral and a δ-function integral. The principal-value integral is well known to be divergent and the technique of mass renormalization[51] must be employed to obtain a finite result. In this paper the final expressions for the transition amplitudes will be presented in the pole approximation, in which the principal-value integral is ignored.

Using the expansion (15) for $f^*_\mu(M_i\vec{p}m_s,M_f)$, the tensor self-energy $\sigma^{ff}(z)$ can be reduced to the result

$$\sigma^{ff}_{\mu\nu}(M_fM_f',z)=\sum_{J,M}\begin{pmatrix}J&1&J_f\\-M&\mu&M_f\end{pmatrix}\times\begin{pmatrix}J&1&J_f\\-M&\nu&M_f'\end{pmatrix}(2J+1)\Sigma^{ff}(z)\,, \tag{28}$$

where

$$\Sigma^{ff}(z)=\frac{1}{(2J+1)}\int_0^\infty\frac{p^2dp}{(z-E_p)}\times\sum_K\sum_l|\langle\gamma_iJ_i,Kpl;J||\vec{D}||\gamma_fJ_f\rangle|^2. \tag{29}$$

The tensor self-energy $\sigma^{ff}(z)$ is seen to be diagonal only in the case where f is a $J_f=0$ state, for which $M_f=M_f'=0$.

The properties of the tensor self-energies given by Eqs. (25) and (28) allow Eq. (19) to be solved for $\langle f,\vec{k}\lambda\,|\,G(z)\,|\,a,0\rangle$ to give

$$\langle f,\vec{k}\lambda\,|\,G(z)\,|\,a,0\rangle=\left[\langle f,\vec{k}\lambda\,|\,V\,|\,a,0\rangle+\sum_\mu\frac{g^*_\mu(\vec{k},\lambda)}{\Psi(z)}[\sigma^{fa}_\mu(M_fM_a,z)+\Sigma^{ff}(z)\sigma^{ga}_\mu(M_fM_a,z)]\right]\frac{\langle a,0\,|\,G(z)\,|\,a,0\rangle}{z-E_k}\,, \tag{30}$$

where

$$\Psi(z)=1-\Sigma^{gg}(z)\Sigma^{ff}(z)\,. \tag{31}$$

The analogous result for $\langle i\vec{p}m_s,0\,|\,G(z)\,|\,a,0\rangle$ is given by the expression

$$\langle i\vec{p}m_s,0\,|\,G(z)\,|\,a,0\rangle=\left[\langle i\vec{p}m_s,0\,|\,V\,|\,a,0\rangle+\sum_\mu\sum_{M_f}\frac{f^*_\mu(M_i\vec{p}m_s,M_f)}{\Psi(z)}[\sigma^{ga}_\mu(M_fM_a,z)+\Sigma^{gg}(z)\sigma^{fa}_\mu(M_fM_a,z)]\right]\frac{\langle a,0\,|\,G(z)\,|\,a,0\rangle}{z-E_p}\,. \tag{32}$$

Finally, the expression for $\langle a,0\,|\,G(z)\,|\,a,0\rangle$ is obtained in the form

$$[\langle a,0\,|\,G(z)\,|\,a,0\rangle]^{-1}=z-E_a-\Sigma^{aa}(z)-\sum_\mu\sum_{M_f}\frac{\sigma^{af}_\mu(M_aM_f,z)}{\Psi(z)}[\sigma^{ga}_\mu(M_fM_a,z)+\Sigma^{gg}(z)\sigma^{fa}_\mu(M_fM_a,z)]-\sum_\mu\sum_{M_f}\frac{\sigma^{ag}_\mu(M_aM_f,z)}{\Psi(z)}[\sigma^{fa}_\mu(M_fM_a,z)+\Sigma^{ff}(z)\sigma^{ga}_\mu(M_fM_a,z)]\,, \tag{33}$$

where

$$\Sigma^{aa}(z)=\sum_{M_i}\sum_{m_s}\int d^3p\frac{|\langle a,0\,|\,V\,|\,i\vec{p}m_s,0\rangle|^2}{z-E_p}+\sum_{M_f}\sum_\lambda\sum_{\vec{k}}\frac{|\langle a,0\,|\,V\,|\,f,\vec{k}\lambda\rangle|^2}{z-E_k}\,. \tag{34}$$

Equations (30)—(34) together with the definitions of the vector and tensor self-energies provide the generalization of the solution obtained by Haan[50] to the case in which each of the atomic levels consists of a set of degenerate magnetic sublevels.

C. Autoionization and radiative decay amplitudes

Assuming that the atom field system has been initially prepared in the eigenstate $|a,0\rangle$ of H^0, the spectral amplitudes describing autoionization and spontaneous radiative decay are given by the projections of the initial state onto the eigenstates $|i\vec{p},0^-\rangle$ and $|f,\vec{k}^-\rangle$ of H satisfying the incoming spherical-wave boundary conditions. In terms of the Møller scattering operator Ω_-, the transition amplitudes for the two alternative continuum channels are given by[50]

$$\langle a,0|i\vec{p},0^-\rangle=\langle a,0|\Omega_-|i\vec{p},0\rangle=\langle a,0|1+G(E_p-i\epsilon)V|i\vec{p},0\rangle \tag{35}$$

and

$$\langle a,0|f,\vec{k}^-\rangle=\langle a,0|\Omega_-|f,\vec{k}\rangle=\langle a,0|1+G(E_k-i\epsilon)V|f,\vec{k}\rangle\ , \tag{36}$$

which involve matrix elements between the eigenstates of H^0. Introducing the magnetic quantum numbers and the photon polarization, the electron and photon amplitudes may be related to the components of G derived in Sec. II B by means of the expressions

$$\begin{aligned}\langle a,0|i\vec{p}m_s,0^-\rangle^*&=\langle i\vec{p}m_s,0^-|a,0\rangle\\&=\langle i\vec{p}m_s,0|V|a,0\rangle\langle a,0|G(E_p+i\epsilon)|a,0\rangle\\&\quad+\sum_{M_f}\sum_{\lambda}\sum_{\vec{k}}\langle i\vec{p}m_s,0|V|f,\vec{k}\lambda\rangle\langle f,\vec{k}\lambda|G(E_p+i\epsilon)|a,0\rangle\end{aligned} \tag{37}$$

and

$$\begin{aligned}\langle a,0|f,\vec{k}\lambda^-\rangle^*&=\langle f,\vec{k}\lambda^-|a,0\rangle\\&=\langle f,\vec{k}\lambda|V|a,0\rangle\langle a,0|G(E_k+i\epsilon)|a,0\rangle\\&\quad+\sum_{M_i}\sum_{m_s}\int d^3p\langle f,\vec{k}\lambda|V|i\vec{p}m_s,0\rangle\langle i\vec{p}m_s,0|G(E_k+i\epsilon)|a,0\rangle\ .\end{aligned} \tag{38}$$

Using the expressions for the components of G given by Eqs. (30) and (32) together with the separable form (14) of the continuum-continuum interaction matrix elements, the expressions for the electron and photon amplitudes can be reduced to the forms

$$\begin{aligned}\langle i\vec{p}m_s,0^-|a,0\rangle=\Bigg\{&\langle i\vec{p}m_s,0|V|a,0\rangle\\&+\sum_{\mu}\sum_{M_f}\frac{f_\mu^*(M_i\vec{p}m_s,M_f)}{\Psi(z)}[\sigma_\mu^{ga}(M_fM_a,z)+\Sigma^{gg}(z)\sigma_\mu^{fa}(M_fM_a,z)]\Bigg\}\langle a,0|G(z)|a,0\rangle\end{aligned} \tag{39}$$

and

$$\begin{aligned}\langle f,\vec{k}\lambda^-|a,0\rangle=\Bigg\{&\langle f,\vec{k}\lambda|V|a,0\rangle+\sum_{\mu}g_\mu^*(\vec{k},\lambda)\sigma_\mu^{fa}(M_fM_a,z)\\&+\sum_{\mu}\sum_{\nu}\sum_{M_f'}\frac{g_\mu^*(\vec{k},\lambda)\sigma_{\mu\nu}^{ff}(M_fM_f',z)}{\Psi(z)}[\sigma_\nu^{ga}(M_f'M_a,z)+\Sigma^{gg}(z)\sigma_\nu^{fa}(M_f'M_a,z)]\Bigg\}\langle a,0|G(z)|a,0\rangle\ ,\end{aligned} \tag{40}$$

which are to be evaluated at $z=E_p+i\epsilon$ and $z=E_k+i\epsilon$, respectively.

Although the electron-emission amplitude given by Eq. (39) can be seen to be in the same form as the result obtained by Haan[50] for the nondegenerate case, the photon-emission amplitude expressed by (40) may be reduced to the corresponding amplitude obtained by Haan only when the tensor self-energy σ^{ff} is diagonal. As discussed in connection with Eq. (28), this is the case only when f is a $J_f=0$ state.

III. AUGER AND FLUORESCENCE YIELDS AND DIELECTRONIC SATELLITE LINE INTENSITIES

A. Approximate branching ratios and effective decay rates for the two-level atom model

In this subsection the approximate expressions for the Auger and fluorescence branching ratios obtained by Armstrong, Theodosiou, and Wall[42] and by Haan and

Cooper[43] are derived more generally to incorporate the angular momentum degeneracy of the atomic levels and the partial-wave expansion (7) for the electron-continuum state. The density-matrix formalism which has been applied in the theory of photoionization[46] allows the fractional probabilities for producing the two alternative pairs of decay products, with specific values of the magnetic quantum numbers and photon polarization, to be presented as functions of the energy and angular variables of the emitted particles for arbitrary distributions of the initial autoionizing state among its magnetic substates. For the case in which the magnetic sublevels of the initial autoionizing state are uniformly populated and the magnetic quantum numbers and polarizations of the final decay products are not of interest, the total electron and photon probabilities are obtained after performing the appropriate averages, summations, and three-dimensional integrations:

$$P_a(a\to i\epsilon_p)=\sum_{M_a}\sum_{M_i}\sum_{m_s}\int d^3p\frac{|\langle i\vec{p}m_s,0^-|a,0\rangle|^2}{2J_a+1}\ . \tag{41}$$

and

$$P_r(a\to f)=\sum_{M_a}\sum_{M_f}\sum_{\lambda}\int d^3k\frac{|\langle f,\vec{k}\lambda^-|a,0\rangle|^2}{2J_a+1}\ . \tag{42}$$

The explicit expressions for the probabilities of autoionization and radiative decay $P_a(a\to i\epsilon_p)$ and $P_r(a\to f)$ are most easily obtained when the vector and tensor self-energy occurring in the expressions for the electron and photon amplitudes are evaluated in the pole approximation

$$\frac{1}{z+i\epsilon-E}=\mathrm{P}\left[\frac{1}{z-E}\right]-i\pi\delta(z-E)$$

$$\cong -i\pi\delta(z-E)\ , \tag{43}$$

in which the principal-value term denoted by P is neglected. The accuracy of this approximation must be carefully considered in each separate case.

The total autoionization and spontaneous radiative decay rates $A_a(a\to i\epsilon_p)$ and $A_r(a\to f)$ in the absence of the final-state continuum-continuum interaction are given by

$$A_a(a\to i\epsilon_p)=\sum_{K,l}A_a(a\to i,K\epsilon_p l)$$

$$=\frac{2\pi}{\hbar(2J_a+1)}\times\sum_{K,l}|\langle\gamma_aJ_a||H_A||\gamma_iJ_i,K\epsilon_pl;J_a\rangle|^2 \tag{44}$$

and

$$A_r(a\to f)=\frac{4e^2\omega^3}{3\hbar c^3}\frac{|\langle\gamma_aJ_a||\vec{\mathbf{D}}||\gamma_fJ_f\rangle|^2}{2J_a+1}\ , \tag{45}$$

where the electron-continuum wave function is now normalized to a δ function in the electron energy ϵ_p. It is also convenient to introduce the partial photoionization cross section $\sigma_p(f\to i\epsilon_p,J_a)$ associated with the J_a total-angular-momentum component of the electron-continuum state[46]

$$\sigma_p(f\to i\epsilon_p;J_a)=\sum_{K,l}\sigma_p(f\to i,K\epsilon_pl;J_a)$$

$$=\frac{4\pi^2\alpha\hbar\omega}{3(2J_f+1)}\times\sum_{K,l}|\langle\gamma_iJ_i,K\epsilon_pl;J_a||\vec{\mathbf{D}}||\gamma_fJ_f\rangle|^2 \tag{46}$$

and the multichannel Fano line-profile parameter[28]

$$Q_f=\frac{(2J_a+1)\langle\gamma_aJ_a||\vec{\mathbf{D}}||\gamma_fJ_f\rangle}{\pi\sum_{K,l}\langle\gamma_aJ_a||H_A||\gamma_iJ_i,K\epsilon_pl;J_a\rangle\langle\gamma_iJ_i,K\epsilon_pl;J_a||\vec{\mathbf{D}}||\gamma_fJ_f\rangle}\ . \tag{47}$$

The reduced matrix elements in the denominator of Eq. (47) determine the partial-wave components $A_a(a\to i,K\epsilon_pl)$ and $\sigma_p(f\to i,K\epsilon_pl;J_a)$, respectively.

The diagonal component of the Green's operator expressed by Eq. (33) reduces in the pole approximation to

$$[\langle a,0|G(z)|a,0\rangle]^{-1}=z-E_a+\frac{\hbar A_r(a\to f)}{\Psi(z)Q_f}+\frac{i\hbar}{2}A_a(a\to i\epsilon_p)+\frac{i\hbar}{2\Psi(z)}A_r(a\to f)\left[1-\frac{1}{Q_f^2}\right], \tag{48}$$

where the continuum-continuum coupling parameter is now given by

$$\Psi(z)=1+\Omega(a,f)\sigma_p(f\to i\epsilon_p;J_a)=1+\frac{1}{a_0^2}\left[\frac{\alpha^2}{2\pi}\right]\left[\frac{\hbar\omega}{E_H}\right]^2\frac{2J_f+1}{2J_a+1}\sigma_p(f\to i\epsilon_p;J_a)\ . \tag{49}$$

It is now advantageous to express the multichannel Fano line-profile parameter Q_f as a sum of partial-wave contributions according to the relationship

$$\frac{1}{Q_f}=\sum_{K,l}\frac{1}{Q_f(K,l)}\ , \tag{50}$$

because the squares of the partial-wave contributions are simply given by

$$Q_f^2(K,l)=\frac{A_r(a\to f)}{A_a(a\to i,K\epsilon_pl)\Omega(a,f)\sigma_p(f\to i,K\epsilon_pl;J_a)}\ . \tag{51}$$

The shift Δ_a and width Γ_a of the autoionizing level produced by the interaction Hamiltonian may be defined by

$$[\langle a,0\,|\,G(z)\,|\,a,0\rangle]^{-1}=z-E_a-\Delta_a+i\Gamma_a/2\ . \tag{52}$$

The autoionization and radiative decay probabilities can now be expressed in terms of one-dimensional energy integrations in the forms

$$P_a(a\to i\epsilon_p)=\sum_{K,l}\frac{\hbar A_a(Kl)}{2\pi\Psi^2}\left[\frac{A_r^2}{A_a^2(K,l)Q_f^2(K,l)}\left[1+\sum_{K',l'}\frac{A_r}{A_a(K',l')Q_f^2(K',l')}-\frac{A_r}{A_a(K,l)Q_f(K,l)Q_f}\right]^2\right]$$
$$\times\int\frac{dE_p}{(E_p-E_a-\Delta_a+i\Gamma_a/2)(E_p-E_a-\Delta_a-i\Gamma_a/2)}\ , \tag{53}$$

and

$$P_r(a\to f)=\frac{\hbar A_r}{2\pi\Psi^2}\left[1+\frac{1}{Q_f^2}\right]\int\frac{dE_k}{(E_k-E_a-\Delta_a+i\Gamma_a/2)(E_k-E_a-\Delta_a-i\Gamma_a/2)}\ , \tag{54}$$

where only the quantum numbers specifying the partial-wave components are indicated explicitly in the decay rates.

With the reasonable assumption that $E_a+\Delta_a$ is much greater than $\Gamma_a/2$, the energy integrations in Eqs. (53) and (54) can be evaluated by completing the corresponding complex integration along the contour shown in Fig. 2. The integral along the semicircle vanishes in the limit $R\to\infty$ and the integral from $-R$ to the appropriate threshold energy can be neglected in comparison with the contribution passing near the poles at $z=E_a+\Delta_a\pm i\Gamma_a/2$. Using the Cauchy integral formula for the residue in the upper half of the complex plane, the energy integrations are given by the contour integral which is evaluated as

$$\oint\frac{dz}{(z-E_a-\Delta_a+i\Gamma_a/2)(z-E_a-\Delta_a-i\Gamma_a/2)}=\frac{2\pi}{\hbar A_a(a\to i\epsilon_p)+\dfrac{\hbar A_r(a\to f)}{\Psi}\left[1-\dfrac{1}{Q_f^2}\right]}\ . \tag{55}$$

All previous treatments of the effects of autoionizing levels in low-density plasmas are based on the traditional expressions for the Auger and fluorescence branching ratios, which are given for the two-level atom model by

$$P_a(a\to i\epsilon_p)=\frac{A_a(a\to i\epsilon_p)}{A_a(a\to i\epsilon_p)+A_r(a\to f)} \tag{56}$$

and

$$P_r(a\to f)=\frac{A_r(a\to f)}{A_a(a\to i\epsilon_p)+A_r(a\to f)}\ . \tag{57}$$

After introducing the effective decay rates

$$\tilde{A}_a(a\to i\epsilon_p)=\frac{A_a(a\to i\epsilon_p)}{\Psi^2}\left[\Psi^2-\frac{2\Psi A_r(a\to f)}{Q_f^2A_a(a\to i\epsilon_p)}+(\Psi-1)\left[1+\frac{1}{Q_f^2}\right]\frac{A_r(a\to f)}{A_a(a\to i\epsilon_p)}\right] \tag{58}$$

and

$$\tilde{A}_r(a\to f)=\frac{A_r(a\to f)}{\Psi^2}\left[1+\frac{1}{Q_f^2}\right], \tag{59}$$

the branching ratios in the presence of the final-state interaction, which will now be denoted by $\tilde{P}_a(a\to i\epsilon_p)$ and $\tilde{P}_r(a\to f)$, may be expressed in the identical forms as

$$\tilde{P}_a(a\to i\epsilon_p)=\frac{\tilde{A}_a(a\to i\epsilon_p)}{\tilde{A}_a(a\to i\epsilon_p)+\tilde{A}_r(a\to f)} \tag{60}$$

and

$$\tilde{P}_r(a\to f)=\frac{\tilde{A}_r(a\to f)}{\tilde{A}_a(a\to i\epsilon_p)+\tilde{A}_r(a\to f)}\ . \tag{61}$$

For the case in which the continuum-continuum cou-

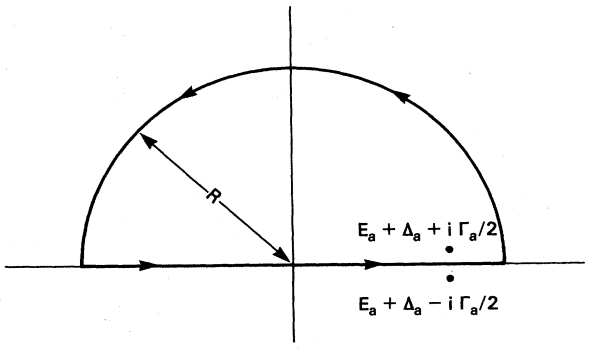

FIG. 2. The contour used for the evaluation of the energy integrations occurring in the expressions for the autoionization and radiative decay amplitudes.

pling involves only a single term in the partial-wave expansion for the electron-continuum state, the expressions [(58) and (59)] for the effective decay rates in the presence of the final-state interaction may be reduced to the results obtained by Armstrong, Theodosiou, and Wall[42] and by Haan and Cooper.[43] In the more general case described by Eqs. (50) and (51) for the multichannel Fano line-profile parameter, the Auger and fluorescence branching ratios will contain terms corresponding to the interference between different partial-wave components in the electron-continuum expansion. An analogous interference phenomenon occurs in the expression for the asymmetry parameter which describes the angular distribution of electrons ejected in electric dipole photoionization processes.[46] Finally, the total width Γ_a, which is given by twice the imaginary part in Eq. (48), reduces to $1/\Psi$ times the total width obtained in the absence of the continuum-continuum coupling, in agreement with the result obtained by Haan and Cooper.[43] They demonstrate that this reduction in the total width is not an indication of any violation of the unitary condition, which involves integrations over the entire range of the continuous energy variables. They emphasize that the reduction applies only in the energy range in which the pole approximation is valid and not over the entire continuum.

B. Multichannel electron and photon continua

Before investigating resonant contributions to inelastic electron collisions and dielectronic recombination processes in plasmas, it will be necessary to further extend the results obtained for the Auger and fluorescence yields to the most general case in which the autoionizing state $|a\rangle$ can undergo spontaneous radiative transitions into a set of final atomic states $|f\rangle$ or can autoionize into a set of states $|i\rangle$ of the residual ion. All these states may consist of degenerate magnetic substates. Since these generalizations have been considered by Armstrong, Theodosiou, and Wall[42] for the case of a single electron-continuum partial-wave component, it will be sufficient to briefly present the results which are obtained by the appropriate extensions of the theory described in Sec. II.

For the case of several photon-continuum states $|f,\vec{k}_f\lambda\rangle$, the continuum-continuum coupling parameter is given in the pole approximation by

$$\Psi=1+\sum_f \Omega(a,f)\sigma_p(f\rightarrow i\epsilon_p;J_a)\ , \tag{62}$$

and the Fano line-profile parameter Q_f for each final atomic state $|f\rangle$ is given by Eq. (47). The Auger and fluorescence branching ratios in the presence of the final-state interactions are now obtained in the forms

$$\tilde{P}_a(a\rightarrow i\epsilon_p)=\frac{\tilde{A}_a(a\rightarrow i\epsilon_p)}{\tilde{A}_a(a\rightarrow i\epsilon_p)+\sum_f \tilde{A}_r(a\rightarrow f)}\ , \tag{63}$$

$$\tilde{P}_r(a\rightarrow f)=\frac{\tilde{A}_r(a\rightarrow f)}{\tilde{A}_a(a\rightarrow i\epsilon_p)+\sum_{f'} \tilde{A}_r(a\rightarrow f')}\ . \tag{64}$$

When the continuum-continuum coupling involves only a single partial-wave component in the expansion of the electron-continuum state, the effective decay rates are given by[43]

$$\tilde{A}_a(a\rightarrow i\epsilon_p)=\frac{A_a(a\rightarrow i\epsilon_p)}{\Psi^2}\times\left[1+\left[\frac{1}{A_a(a\rightarrow i\epsilon_p)}\sum_f\frac{A_r(a\rightarrow f)}{Q_f}\right]^2\right] \tag{65}$$

and

$$\tilde{A}_r(a\rightarrow f)=\frac{A_r(a\rightarrow f)}{\Psi^2}\left\{\frac{1}{Q_f^2}+\left[1+\frac{1}{A_a(a\rightarrow i\epsilon_p)}\sum_{f'}\left[\frac{A_r(a\rightarrow f')}{Q_{f'}}\frac{Q_f-Q_{f'}}{Q_{f'}}\right]\right]^2\right\}. \tag{66}$$

For the case of several electron-continuum states $|i\vec{p}_i m_s,0\rangle$, the pole approximation for the continuum-continuum coupling parameter becomes

$$\Psi=1+\Omega(a,f)\sum_i \sigma_p(f\rightarrow i\epsilon_i;J_a)\ , \tag{67}$$

which involves a summation over all open electron-continuum channels with total angular momentum J_a. The corrected expressions for the Auger and fluorescence yields now have the forms

$$\tilde{P}_a(a\rightarrow i\epsilon_i)=\frac{\tilde{A}_a(a\rightarrow i\epsilon_i)}{\sum_{i'}\tilde{A}_a(a\rightarrow i'\epsilon_{i'})+\tilde{A}_r(a\rightarrow f)} \tag{68}$$

and

$$\tilde{P}_r(a\rightarrow f)=\frac{\tilde{A}_r(a\rightarrow f)}{\sum_i \tilde{A}_a(a\rightarrow i\epsilon_i)+\tilde{A}_r(a\rightarrow f)}\ . \tag{69}$$

For the case in which the continuum-continuum coupling involves only a single term in the partial-wave expansion for each electron-continuum state $|i\vec{p}_i m_s,0\rangle$, the effective decay rates are given by[43]

$$\tilde{A}_a(a\rightarrow i\epsilon_i)=\frac{A_a(a\rightarrow i\epsilon_i)}{\Psi^2}\times\left[\left[1+\sum_{j\neq i}\Omega(a,f)\sigma_p(f\rightarrow j\epsilon_j;J_a)\right]^2+\frac{A_r(a\rightarrow f)^2}{Q_f^2 A_a(a\rightarrow i\epsilon_i)^2}\right] \tag{70}$$

$$\tilde{A}_a(a\rightarrow j\epsilon_j)=\frac{A_r(a\rightarrow f)}{\Psi^2}\Omega(a,f)\sigma_p(f\rightarrow j\epsilon_j)\left[1+\frac{1}{Q_f^2}\right]\quad \text{for } j\neq i\ , \tag{71}$$

$$\widetilde{A}_r(a\rightarrow f)=\frac{A_r(a\rightarrow f)}{\Psi^2}\left[1+\frac{1}{Q_f^2}\right], \tag{72}$$

where i now refers to the resonant continuum channel obtained from the prediagonalization of the electron continua described by Fano.[27]

C. Resonant electron-impact excitation and dielectronic recombination rates

The branching ratios obtained in Secs. III A and III B describe the autoionization and spontaneous radiative decay of an atomic system prepared in an unstable state. In order to obtain the collision rates and spectral line intensities associated with the decay of virtual autoionizing states of atomic systems in plasmas, it is necessary to take into account the scattering mechanisms by which these states are prepared. Although photoexcitation can become important in intense radiation fields, autoionizing states in plasmas are excited predominantly by electron collisions. In the isolated-resonance approximation, in which the width is assumed to be small compared with the energy difference between adjacent levels, the evaluation of the scattering matrix for the resonant electron-collision processes according to the procedure described by Goldberger and Watson[23] leads to the Breit-Wigner[52,53] one-level resonance formula. When the predominant contribution to the resonant scattering cross section is associated with a single, long-lived, virtual state, the resonant cross section can be expressed as the product of the excitation cross section and the branching ratio for the appropriate decay channel.

In the dielectronic recombination process the autoionizing state $|a\rangle$ is excited by the radiationless electron-capture process which corresponds to the inverse of autoionization. If the rate coefficient describing the radiationless electron capture is denoted by $\widetilde{C}_{\rm cap}(i\epsilon_i\rightarrow a)$, the dielectronic-recombination rate coefficient, which gives the photoemission rate per unit volume and electron density, is simply given in the isolated resonance approximation by

$$\widetilde{\alpha}_{\rm DR}(i\epsilon_i\rightarrow a\rightarrow f)=\widetilde{C}_{\rm cap}(i\epsilon_i\rightarrow a)\widetilde{P}_r(a\rightarrow f), \tag{73}$$

where $\widetilde{P}_r(a\rightarrow f)$ is the fluorescence yield in the presence of the final-state interaction. The resonant contribution to the electron-impact excitation rate coefficient is obtained from the Auger branching ratio $\widetilde{P}_a(a\rightarrow j\epsilon_j)$ for $j>i$ by

$$\widetilde{C}_R(i\epsilon_i\rightarrow a\rightarrow j\epsilon_j)=\widetilde{C}_{\rm cap}(i\epsilon_i\rightarrow a)\widetilde{P}_a(a\rightarrow j\epsilon_j). \tag{74}$$

For a plasma in thermal equilibrium, the radiationless capture rate coefficient, which is defined as a Maxwellian average of the corresponding capture cross section $\sigma_{\rm cap}(i\epsilon_i\rightarrow a)$, can be obtained from the autoionization rate $\widetilde{A}_a(a\rightarrow i\epsilon_i)$ in the presence of the final-state interaction by means of the detailed-balance relationship

$$\begin{aligned}\widetilde{C}_{\rm cap}(i\epsilon_i\rightarrow a)&=\langle V_e\widetilde{\sigma}_{\rm cap}(i\epsilon_i\rightarrow a)\rangle_{\rm av}\\&=2^3a_0^3\pi^{3/2}\frac{2J_a+1}{2(2J_i+1)}\left[\frac{E_H}{k_BT_e}\right]^{3/2}\\&\quad\times\exp\left[-\frac{E(a)-E(i)}{k_BT_e}\right]\widetilde{A}_a(a\rightarrow i\epsilon_i).\end{aligned} \tag{75}$$

An additional production mechanism is the electron-impact excitation of the transition $b\rightarrow a$ from the bound state $|b\rangle$ of the atomic system. If the electron-impact excitation rate coefficient is denoted by $\widetilde{C}_e(b\rightarrow a)$, the corresponding photoemission rate coefficient is obtained as

$$\widetilde{C}_e(b\rightarrow a\rightarrow f)=\widetilde{C}_e(b\rightarrow a)\widetilde{P}_r(a\rightarrow f). \tag{76}$$

If the fluorescence yield in Eq. (76) is replaced by the Auger branching ratio $\widetilde{P}_a(a\rightarrow i\epsilon_i)$, the resulting expression gives the resonant contribution to electron-impact ionization, which corresponds to autoionization following inner-shell excitation. The evaluation of the collisional excitation rate coefficient $\widetilde{C}_e(b\rightarrow a)$ in the presence of the final-state interaction is easily accomplished in the Bethe approximation[54]

$$\begin{aligned}\widetilde{C}_e(b\rightarrow a)&=\frac{32}{\sqrt{3}}\pi^{3/2}\frac{2J_a+1}{2J_b+1}\left[\frac{a_0}{\alpha}\right]^3\left[\frac{E_H}{\Delta E(a\rightarrow b)}\right]^2\\&\quad\times\widetilde{A}_r(a\rightarrow b)\left[\frac{E_H}{k_BT_e}\right]^{3/2}\frac{k_BT_e}{\Delta E(a\rightarrow b)}\\&\quad\times\frac{\sqrt{3}}{2\pi}\left[(\ln 4)\exp\left[-\frac{\Delta E(a\rightarrow b)}{k_BT_e}\right]\right.\\&\quad\left.+E_1\left[\frac{\Delta E(a\rightarrow b)}{k_BT_e}\right]\right],\end{aligned} \tag{77}$$

which involves the spontaneous emission rate $\widetilde{A}_a(a\rightarrow b)$ for the electric dipole transition $a\rightarrow b$.

If the number densities of ions in the state $|i\rangle$ and of atomic systems in the state $|b\rangle$ are denoted by $N(i)$ and $N(b)$, respectively, the total dielectronic satellite line intensity $\widetilde{I}(a\rightarrow f)$ produced by both radiationless electron captures and inner-shell excitations may be determined by the evaluation of

$$\begin{aligned}\widetilde{I}(a\rightarrow f)&=\sum_i N(i)N_e\widetilde{\alpha}_{\rm DR}(i\epsilon_i\rightarrow a\rightarrow f)\\&\quad+\sum_b N(b)N_e\widetilde{C}_e(b\rightarrow a\rightarrow f),\end{aligned} \tag{78}$$

where N_e is the electron density. It should be emphasized that the theory of the satellite line intensity in the presence of the final-state interaction has been developed in the low-density corona-model approximation, in which all excited states in a plasma are assumed to decay only by autoionization or spontaneous radiative emission and all collisional deexcitation processes are ignored. However, the traditional theory of dielectronic satellite line intensities[18] in the absence of the final-state continuum-

continuum interaction has been extended[20] to incorporate electron collisional transitions between the autoionizing levels. Finally, the total recombination rates, which enter into the determination of the corona-equilibrium charge-state distribution,[14] are obtained by summation of the first term in Eq. (78) over all states $|a\rangle$ and $|f\rangle$.

IV. CONCLUSIONS

In this investigation the traditional theory of resonant electron-impact excitation rates and dielectronic satellite line intensities in low-density plasmas has been extended within a framework of the isolated-resonance approximation to incorporate the interaction between the final continuum states which result from the autoionization and spontaneous radiative decay processes. Particular emphasis has been given to the detailed treatment of the angular momentum degeneracy of the atomic levels and to the consequences of several partial-wave components resulting from the expansion of the electron-continuum state.

To place this investigation in a proper perspective, it is necessary to identify the phenomena which cannot be treated in the isolated-resonance approximation. Strictly speaking, the autoionizing state and the adjacent nonresonant electron continuum should not be treated as separate states. Consequently, the distinction between the direct or nonresonant radiative recombination process and the dielectronic recombination process is in some respects artificial. The two recombination mechanisms are treated in a unified manner in the investigations carried out by Shore[39] and by Davies and Seaton,[40] who evaluate the scattering matrix for the complete collision process involving the capture of an incident electron by the ion accompanied by the emission of a photon. The interference between the transition amplitudes corresponding to radiative recombination and dielectronic recombination which is predicted by this unified treatment is also included in the modified expression for the dielectronic recombination rates obtained in the present investigation. The modifications to the conventional expression for the dielectronic satellite intensities may be interpreted as terms corresponding to this interference together with radiative corrections to the dielectronic recombination process. This interpretation is schematically illustrated for the two-level atom model by the Feynman-type diagrams in Fig. 3, which show how the modifications to the transition amplitude can be represented by virtual interactions between the autoionizing state and the nonresonant electron continuum together with virtual emissions and reabsorptions of a photon. For small values of the Fano line-profile parameter Q_f the nonresonant radiative recombination process is expected to play an important role, and its contribution must then be included to obtain the total recombination rates and intensities of photoemission. However, the validity of the pole approximation employed in this investigation may be questionable for small values of Q_f. In order to ensure that the interference between the resonant and nonresonant processes is taken into account completely, it may be necessary to adopt the approach based on the evaluation of the scattering matrix.

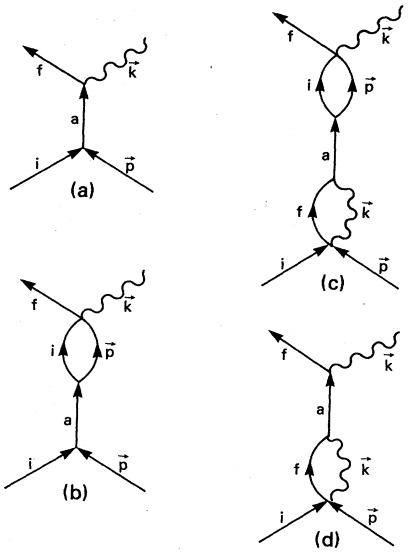

FIG. 3. Feynman-type diagrams representing the conventional amplitude for dielectronic recombination (a) and modifications [(b), (c), and (d)] due to interference with radiative recombination and radiative corrections.

Perhaps the most difficult situation, which cannot be treated by the theory developed in this investigation, arises when it becomes necessary to allow for strongly overlapping resonances, i.e., autoionizing states whose widths far exceed the energy-level separations. Such a situation must ultimately occur in the dielectronic recombination process originally described by Burgess.[1] This process is viewed as proceeding predominantly through capture of electrons into highly excited nl states, for which the autoionization rates as well as the energy-level separations decrease as n^{-3}. However, the dominant radiative decay rate, which corresponds to an inner-electron transition, is independent of n. It is therefore apparent that the spontaneous radiative emission process acting alone will eventually cause these resonances to strongly overlap. The description in terms of virtual autoionizing states in this situation does not obviously lead to any enhanced understanding of the electron-ion recombination process, and an entirely different approach may be required. Prior to the development of an entirely new theory, it appears worthwhile to evaluate the radiative corrections to the conventional expression for the dielectronic satellite line intensities and to make a comparison with the observed spectral intensities from laboratory and astrophysical plasmas. This investigation is now in progress, and the results will be reported.[55]

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