Comments

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Comment on "Microscopic derivation of nonlinear hydrodynamics in ordered systems with applications to nematic liquid crystals"

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We comment on comparisons made by Ben-Mizrachi and Procaccia [Phys. Rev. A **27**, 2126 (1983)] between their and our works. We clarify the reported discrepancies and point out a real difference with respect to flow alignment.

Recently, a projector-operator technique was employed[1] to derive nonlinear hydrodynamic equations for uniaxial nematic liquid crystals. Differences with previous work[2–8] were discussed. Here we wish to comment on comparisons of the work of Ben-Mizrachi and Procaccia with the work of the present authors.[2–4]

After Eq. (3.7) and in the summary of Ref. 1 it was stated that we "assumed" that the director **n** transforms like a vector under rigid rotations, and that this was one reason for the different equations obtained. This statement is incorrect. In fact, when a term inadvertently omitted by Ben-Mizrachi and Procaccia is included, their equation reduces to ours. Only the notation is different.

In Refs. 2–4, the local (normalized) director **n** was chosen as a variable—in accordance with previous descriptions (cf. Ref. 7). It is related to the tensor order parameter Q_{ij} of nematics by $Q_{ij}=n_in_j-\frac{1}{3}\delta_{ij}$ (apart from an overall prefactor). From the behavior of tensors under rigid rotations ($\boldsymbol{\omega}=\text{const}$) one obtains

$$\dot{Q}_{ij}=\epsilon_{ikl}\omega_k Q_{lj}+\epsilon_{jkl}\omega_k Q_{il} \tag{1}$$

and thus for **n**

$$\dot{n}_i=\epsilon_{ijk}\omega_j n_k \quad ; \tag{2}$$

i.e., **n** *transforms* like a vector under rigid rotations. On the other hand, Ben-Mizrachi and Procaccia have used the microscopic variable $N_i\equiv(\delta_{ij}-n_i^0n_j^0)Q_{jk}^{\text{mic}}n_k^0$ where n_i^0 is the normalized equilibrium director (assumed to be constant in Ref. 1) and Q_{ij}^{mic} is the microscopic local order parameter tensor. They find, when their missing term is inserted,

$$\langle \dot{N}_i\rangle_L=(\boldsymbol{\omega}\times\mathbf{n})_j(\delta_{ij}-n_i^0n_j^0)(1-\tfrac{1}{2}\langle\mathbf{N}\rangle_{\text{NE}}^2) + \langle N_i\rangle_{\text{NE}}n_j^0(\boldsymbol{\omega}\times\langle\mathbf{N}\rangle_{\text{NE}})_j+O(\omega\langle N\rangle_{\text{NE}}^3) \quad , \tag{3}$$

where $\langle\mathbf{N}\rangle_{\text{NE}}$ is the nonequilibrium average of **N**, and related to **n** and $\mathbf{n}^0$ via $\mathbf{n}\equiv(\mathbf{n}^0+\langle\mathbf{N}\rangle_{\text{NE}})/(1+\langle\mathbf{N}\rangle_{\text{NE}}^2)^{1/2}$ [Eq. (2.16) of Ref. 1, where $\langle\mathbf{N}\rangle_{\text{NE}}$ was abbreviated by **n** and the usual **n** was called $\boldsymbol{\nu}$]. In addition, the average $\langle\mathbf{N}\rangle_L$ can be inferred from $\langle Q_{ij}^{\text{mic}}\rangle_L\sim n_in_j-\frac{1}{3}\delta_{ij}$ [Eq. (A5) of Ref. 1]. With these definitions, which imply $\langle\mathbf{N}\rangle_L=\langle\mathbf{N}\rangle_{\text{NE}}(1+\langle\mathbf{N}\rangle_{\text{NE}}^2)^{-1}$, one finds that Eq. (3) and Eq. (2) differ only in notation. Thus, although N_i itself is not a vector, n_i *is* a vector—a result obtained in Ref. 2 by requiring rotational invariance of the free-energy density.

A second difference according to Ref. 1 is that we treat $\nabla_i n_j$ as "an additional independent hydrodynamic variable." This statement is also irrelevant. As can easily be seen from Ref. 2, we have only one dynamic quantity $\dot{n}_i$ [Eq. (3.4)] characterizing a spontaneously broken continuous symmetry. Of course, $\nabla_j n_i$ enters the Gibbs relation—and in the linearized theory only $\nabla_j n_i$ enters—because ϵ depends on this quantity (Frank free energy). In the nonlinear regime

$$d\epsilon\sim h_i\,dn_i+\phi_{ij}\,d\nabla_j n_i \quad . \tag{4}$$

Thus, one obtains for the thermodynamic conjugate of δn_i, $H_i\equiv h_i-\nabla_j\phi_{ij}$ (cf. also Refs. 7 and 8). In the nonlinear part of the stress tensor there occurs a term $\phi_{kj}\nabla_i n_k$ additionally. However, the fact that a rapidly decaying variable appears in the current for a conserved quantity does not imply that $\nabla_j n_k$ is an independent hydrodynamic variable.

A third difference, commented upon by Ben-Mizrachi and Procaccia relates to some terms which we obtained but they did not. This difference probably results from a misprint in Ref. 2. There, Eq. (3.9) is obviously just the sum of Eqs. (3.5) and (3.6) and thus reads[2]

$$Z_i=\delta_{il}^{\text{tr}}n_k(\nabla_l v_k-\nabla_k v_l)+\alpha\delta_{il}^{\text{tr}}n_k(\nabla_l v_k+\nabla_k v_l) \quad . \tag{5}$$

On the other hand, Ben-Mizrachi and Procaccia have presented new terms not obtained before. Especially they give four flow alignment terms for uniaxial nematic liquid crystals. This result differs from results previously obtained for uniaxial nematics[2,7–9] that obtain only one flow alignment parameter. The difference comes from the simultaneous use of $\mathbf{n}^0$ and $\mathbf{n}$ in Ref. 1. In the usual treatments of nonlinear hydrodynamics which describe also situations far from equilibrium, the equilibrium parameter $\mathbf{n}^0$ is not included. If the complete hydrodynamics is given in terms of $\mathbf{n}$ only, there is precisely one flow alignment parameter. If $\mathbf{n}^0$ together with $\mathbf{n}$ is allowed to occur in the equations, one can construct four different terms with (probably) four different flow alignment parameters. We believe, however, that the use of $\mathbf{n}^0$ (which is constant in Ref. 1) violates the postulate of rotational covariance: Homogeneous rotations of the reference state $\mathbf{n}^0$, which are physically irrelevant, must not change the equations.

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