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# Evolution of electromagnetic instability in a hot two-component magnetic plasma stream anisotropy: Ion fusion mode

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We derive precisely the results of streaming ($V_0 < c$) dispersion and electromagnetic growth term of instability for whistler's ($\omega > \Omega_e$) and Alfvén's ($\omega < \Omega_I$) mode. The electron cyclotron resonance reveals that the phase velocity ($\omega/k$) is of the order of the velocity of light, and the refractive index is unity for moderate streaming velocity. However, for ion fusion mode, the corresponding $\omega/k$ approaches $10^2$ in a dense refractive medium for low ion cyclotron frequency. Moreover, with $\Omega_I = 10^4$ rad/sec, luminous waves are generated. Early isotropic results are also recovered in my analysis.

## I. INTRODUCTION

The study of streaming anisotropic plasma in an external magnetic field is of interest both from the point of view of laboratory experiments as well as from the magnetospheric space research connected with satellite probes. It is known that the presence of magnetic field has a substantial effect particularly when cyclotron resonance and magnetic fusion results are derived.

Circularly polarized waves in a magnetoplasma were studied by Pradhan[1] following the technique of Vankampen[2] in solving the singularity in the complex analytic plane for resonant whistler modes in isotropic plasmas. Scarf[3] studied the cyclotron Landau damping for similar electrostatic waves. It is known that for low temperatures, the magnetized electron stream can gyrate in helicons and the dispersion results follow the Appleton-Hartree (AH) formula of Bell and Buneman.[4] Sudan[5] also investigated the electromagnetic instability for low and non-Maxwellian thermal whistlers and outlined the criteria of dispersion and instability, which do not include streaming. But the above results are inadequate for explaining the two component thermal plasma species and their dissipative modes containing streaming loss mechanism ($\mathbf{v}_0 \cdot \mathbf{p}_j$) in the nonrelativistic frame.[6]

With regard to the study of anisotropic ion plasmas,[7,8] Grewal analyzed the electrostatic instability driven in the Bernstein[9] mode magnetoplasma only and indicated stability criteria in terms of ($\Omega_I^2/\omega_{0I}^2 \sim \theta_\perp/\theta_\parallel$) cyclotron to local plasma frequencies versus perpendicular to parallel temperature components. However, Rosenbluth and Post[10] dealt with the electron dispersion having inductive contribution ($\omega/kV_\parallel > 1$). The work of Dory, Guest, and Harris[11] reviewed the results of a wide variety of distribution functions of ($V_1$) which also included the works of Grewal.[8] Along the same line, Busuardo-Neto, Dawson, Kamimura, and Lin[7] reported the numerically simulated results of ion cyclotron resonance heating taking into account the temperature anisotropy in a loss cone distribution of plasma heating [for $\sim \exp(-V_I^2/a_I^2 - V_{II}^2/a_{II}^2)$]. However, such investigations do not reveal any streaming anisotropic results having beam plasma and wave plasma interactions for the species.

Here, we choose a two component warm magnetized plasma beam consisting of both the species suitable for radio frequency heating mechanisms in the laboratory space plasma and magnetospheric plasma. In order to study the parallel field ($k \parallel B$) current driven microinstability and heating of the plasma, it is necessary to choose collisionless Vlasov plasma with a set of Maxwell's electromagnetic equations. The particles are in the high-energy regime, and the temperature region lies in the nonrelativistic limit ($m_j c^2 >> k_B T_j$); the electrons and ions become relativistic particles only at $10^8$ and $10^{13}$ K, respectively. The nonrelativistic distribution function anisotropic in streaming beam velocity ($v_0 < c$) is chosen to explain the formalism. The dissipation in the beam energy lies in the nonrelativistic frame ($\mathbf{v}_0 \cdot \mathbf{p}_j$), and within the limit, all kinds of thermal modes and streaming velocities are explained in the results. Both nonisothermal ($T_i \neq T_e$) and isothermal ($T_i = T_e$) modes are possible, and ($T_i = 0$) frozen in lines of ion plasma dissipative stream can occur in the formulation to explain the heating of the plasma species in the linear or quasilinear regime. We clear up the main issue of calculating the modified frequency spectrum in a streaming magnetized plasma beam valid for varying thermal modes and varying streaming velocities for both the constituent species.

Further, our results identify the plasmas occurring in space, earth's magnetospheres, ionospheres, and equatorial electrojets, etc., and explain the two component streaming effects, such as streaming whistler's ($\omega < \Omega_e + kV_c/2$) and Alfvén's ($\omega > \Omega_1 + kV_0/2$) for both resonant ($\omega \simeq \Omega_j$) and nonresonant ($\omega \simeq \Omega_j$) cyclotron oscillations as well. In addition, the results indicate the fusion mode in the warm beam at ion plasma temperature, with waves and instabilities therein. Under suitable modifications, the results fit into the $\epsilon = 0$ (absence of inhomogeneity) in the case of toroidal confinement devices[12,13] for nonuniform and anti-loss-cone plasmas, where both electromagnetic and electrostatic drift waves propagate.

The plan of this paper is as follows. In Sec. II we analytically derived the dispersion formula. In Secs. III and IV we arrive at cold plasma and finite-temperature approximations. In Sec. V the damping term is evaluated and later Sec. VI deals with the discussions and applications of the results in detail.

## II. BASIC EQUATIONS AND DISPERSION RELATIONS

Following the Vankampen's technique[1,2] of solving the singularity, as is done for magnetized electron plasma, the problem of streaming dissipative or drift modes of instability in a high $\beta$ $[\beta = nk_BT/(B_0^2/8\pi)]$, two component plasma species is qualitatively solved for the linear and quasilinear evolution of waves. The phase velocity of the species, evidently, is $[V_0 \pm (\omega \mp \Omega)/k]$ whereas, the equilibrium Maxwellian distribution function describing the velocity anisotropy with the streaming in the nonrelativistic frame[6] $(\mathbf{v}_0 \cdot \mathbf{p}_j)$ is as follows:

$$f_0(u) = \sum_{j=+,-} \left(\frac{m_j}{2\pi k_B T_j}\right)^{3/2} \exp\left[-\frac{1}{k_B T_j}\left(\frac{p_j^2}{2m_j} - V_0 \cdot \mathbf{p}_j\right)\right] . \tag{1}$$

$V_0$ represents the streaming velocity of the species as a whole, and the beam velocity of the plasma and the thermal velocities are given by $(k_B T_j/m_j)^{1/2}$.

We need the following terms to be evaluated in order to explain the streaming dispersion and damping in the growth term of instability for a two component collisionless nonrelativistic magnetoplasma.

$$\frac{u_R^2 - c^2}{u_R} = -i\pi u_0^2 F^*(u_R - u_L) \tag{2}$$

and

$$\gamma = \frac{1}{\tau(\omega)} = \tfrac{1}{2}\pi k u_0^2 F(u_R - u_L) . \tag{3}$$

The notations explain the following parameters having the units and dimensions of phase velocity, viz., $u_R = \omega/k$, $u_L = \Omega_j/k$, and $u_0 = \omega_{0j}/k$. $F$ and $F^*$ denote the sum and differences of the respective positive and negative frequency parts of the Fourier transform. For such constituent species, the Fourier representations in terms of the cyclotron and incident phase velocity are given by

$$i\pi F^*(u_R - u_L) = \exp\left[\frac{1}{16\alpha}\left(\frac{m_j V_0^2}{k_B T_j}\right)^2\right] \int_0^\infty \exp\left(-p_j^2 - \frac{m_j V_0}{k_B T_j} i p_j\right) \Big/ 4\alpha \sin p_j (u_R - u_L) dp_j . \tag{4}$$

With $\beta = 1/4\alpha = k_B T_j/4m_j$, $\gamma = iV_0/4$, and $b = (u_R - u_L)$, the above integral is reduced to the form, when $m_j V_0^2 << k_B T_j$, as follows:[14]

$$\int_0^\infty \exp(-\beta x^2 - \gamma x)\sin bx\, dx = -\frac{i}{4}\left(\frac{\pi}{\beta}\right)\left\{\exp\left(\frac{\gamma - ib}{4\beta}\right)^2\left[1 - \phi\left(\frac{\gamma - ib}{2\sqrt{\beta}}\right)\right] - \exp\left(\frac{\gamma - ib}{4\beta}\right)^2\left[1 - \phi\left(\frac{\gamma + ib}{2\sqrt{\beta}}\right)\right]\right\} . \tag{5}$$

The function $\mathrm{erfc}(\gamma \mp ib/2\sqrt{\beta})$ is given by $1 - \phi(Z)$ with $\phi(Z)$ being the probability integral which can be expressed in terms of the Kummer function as

$$\phi(Z) = \frac{2}{\pi^{1/2}} \int_0^Z \exp(-t^2) dt = \frac{2}{\pi^{1/2}} Z\, {}_1F_1\{\tfrac{1}{2}, \tfrac{3}{2}, -Z^2\} , \tag{6}$$

where $Z = i\alpha^{1/2}(V_0/2 \pm \omega \mp \Omega_j/k)$ and $\alpha = m_j/k_B T_j = 1/\lambda_{Dj}^2 \omega_{cj}^2$. $\lambda_{Dj}$ and $\omega_{0j}$ denote the Debye length and plasma frequencies of the species. Using (5) in (2), one can write, after some lengthy simplifications, the following relation:

$$u_R - \frac{c^2}{u_R} = -u_c^2 \frac{i}{2}(\pi\alpha)^{1/2}\left\{\exp\left[-\left(\frac{V_0}{2} \pm \frac{\omega \mp \Omega_j}{k}\right)^2 \alpha\right]\left\{1 - \phi\left[i\alpha^{1/2}\left(\frac{V_0}{2} \pm \frac{\omega \mp \Omega_j}{k}\right)\right]\right\}\right.$$
$$\left. - \exp\left[-\left(\frac{V_0}{2} \mp \left(\frac{\omega \pm \Omega_j}{k}\right)\right)^2 \alpha\right]\left\{1 - \phi\left[i\alpha^{1/2}\left(\frac{V_0}{2} \mp \frac{\omega \pm \Omega_j}{k}\right)\right]\right\}\right\} . \tag{7}$$

## III. COLD PLASMA STREAMING DISPERSION

One can make use of the asymptotic relations $(Z \to \infty)$ of the Kummer function to explain the nonthermal Appleton-Hartree modified streaming formula for $T_j \to 0$ and $\lambda_{Dj} \to 0$ (Debye wavelength); and also the finite-temperature effects to signify the various thermal modes like isothermal $T_j = T_e = T_i$, and nonisothermal $(T_e \neq T_i)$ and frozen-in lines for ion plasma $(T_i = 0)$, etc., which are common in streaming magnetoplasma of space and intergalactic region.

Using the relation[15]

$$\lim_{Z \to \infty} {}_1F_1(a, C, Z) = (Z)^{-a} \sum_{n=0}^{\infty} \frac{an(a - C + 1)n}{n} (-Z)^n , \tag{8}$$

in Eqs. (6) and (7) one can obtain with $n = 0$ terms only the dispersion relation for the species as

$$n^2 - 1 = \frac{\omega_{0j}^2}{\omega(\Omega_j - \omega + kV_0/2)} . \tag{9}$$

For electronplasma, with negligible motion of ions, the modified Appleton-Hartree formula is as follows:

$$n^2 - 1 = \frac{\omega_{0e}^2}{\omega(\Omega_e - \omega + kV_0/2)} , \tag{10}$$

$$n = \frac{kc}{\omega} . \tag{11}$$

Equation (10) tends to give the nonstreaming dispersion and AH formula for isotropic case in the limit of vanishing drift velocity $(V_0 \to 0)$. For resonant conditions $(\omega = \Omega_e)$, therefore, it does not yield $n^2 \to \infty$, unlike the nonstreaming or isotropic results of Pradhan. On the contrary, it shows that

$$n^2 \simeq 1 + \frac{\omega_{0e}^2 \lambda}{\Omega_e V_0} . \tag{12}$$

Let us discuss the numerical estimates using the following values for the parameters $\omega_{0e} = 10^5$ rad/sec. $\Omega_e = 10^6$ rad/sec, $\lambda = 10$ cm, $V_0 = 10^{-3}$ C, $= 3 \times 10^7$ cm/sec, the cyclotron resonant oscillation show that $n^2 \simeq 1.001$ for streaming electron plasmas and $\omega/k \simeq c$.

However, Hasegawa[16] has shown an extra term $(kV_0/\omega - 1)$ in the numerator, because the simplified drifted Maxwellian distribution has been chosen for the beam plasma. Moreover, nonrelativistic temperature limits $(m_j c^2 >> k_B T_j)$ and the low streaming energies of the species $(m_j V_0^2 << k_B T_j)$ have not been considered in the physical sense. Note that the term $(kV_0/\omega - 1)$ tends to make the value of $n^2$ or higher, i.e., 1.017 and therefore it is missing in our Eq. (10).

In addition to the modified Appleton-Hartree theory, it explains the growth of electron whistler waves and ion Alfvén waves with necessary cutoff values for varying thermal modes. However, for the ion plasma waves, the resonant condition $\omega = \Omega_I$ displays that the corresponding dielectric function ( ) or refractive index $(n^2)$ diminishes to 1.01. Equations (12a) and (12b) of Sudan,[5] however, do not explain streaming and isotropic temperature $(T_\perp = T_\parallel)$ results. The waves do not propagate at all with varying $V_0$.

## IV. FINITE-TEMPERATURE EFFECT, WARM PLASMA DISPERSION: ION FUSION MODE

To account for the finite-temperature effect, one can retain terms up to the second order in the asymptotic expansion of Eq. (8) for Kummer function, i.e., using the approximations and simplifying Eqs. (6) and (7) one easily finds the expression for dielectric function or refractive index as follows:

$$\epsilon = n^2 = 1 + \frac{\omega_{0j}^2 \omega(\pm\Omega_j - \omega + kV_0/2)}{1 - [2\omega_{0j}^2 k_B T_j / m_j c^2 (\pm\Omega_j - \omega + kV_0/2)^2]} . \tag{13}$$

Equivalently, with the dimensionless quantity and substituting $Z_j$ as $m_j c^2/k_B T_j$ one can express Eq. (13) for the refractive index and phase velocity of the waves in terms of $\Omega_j$ and $V_0$ as

$$n^2 = 1 + \sum_{j=+,-} \frac{\omega_{0j}^2 \omega(\pm\Omega_j - \omega + kV_0/2)}{[Z_j(\pm\Omega_j - \omega + kV_0/2)^2 - 2\omega_{0j}^2]/Z_j(\pm\Omega_j - \omega + kV_0/2)^2} , \tag{14}$$

i.e.,

$$\frac{\omega^2}{k^2} = c^2 - \frac{\omega^2}{k^2} \frac{\omega_{0j}^2 \omega(\pm\Omega_j - \omega + kV_0/2)}{[Z_j(\pm\Omega_j - \omega + kV_0/2) - 2\omega_{0j}^2]/Z_j(\pm\Omega_j - \omega + kV_0/2)^2} . \tag{15}$$

Further, to demonstrate the ion fusion mode in the magnetized beam plasma, one can quantitatively take $T_i \simeq 10^8$ K and $\Omega_I = +eB/m_I c \simeq 0.5$ rad/sec and $\omega_{0I} \sim 10^4$ rad/sec using the above parameters; the numerical estimates for $\omega/k$ in (15) is evaluated as $\omega/k \sim 10^2$. It generates subluminous waves in an extremely dense plasma beam of ions, whereas for $\Omega_I \sim 10^4$ rad/sec, $\omega/k$ approaches $10^9$; with $n \geqslant 3$. It generates near luminous waves. Further, the results agree with the isotropic finite-temperature effects as discussed in Pradhan[1] for electron whistlers and those of the electron streaming instability.[4,5] Sudan[5] chose a simple drifted $f_0$, without having streaming dissipation in the $(V_0 p_j)$ frame. Moreover, it found no instability for $T_\perp = T_\parallel$ electrons, i.e., isotropic temperatures. In (13), $\pm\Omega_j$ denote the electron on ion cyclotron frequency for right-handed or left-handed circularly polarized waves, respectively. Our results conform to the nonstreaming or isotropic results as discussed in Sec. III of Misra and Mohanty.[17] Physically, the linear or quasilinear waves evolve in the magnetosphere or ionosphere and the space region for varying thermal modes $(T_j)$; the laboratory simulations for magnetized plasma beam can indicate magnetic fusion at $10^8$ K ion temperatures as we have shown earlier.

## V. DAMPING AND GROWTH TERM OF INSTABILITY

Likewise, the damping or growth term of instability for finite-temperature and frequency limits is calculated using (3), and assuming that streaming kinetic energy of the plasma beam is much less than the thermal energies, one obtains

$$\gamma = \sum_{j=+,-} \frac{\pi\omega_{0j}^2}{2k} \left(\frac{m_j}{2\pi k_B T_j}\right)^{3/2} \times \exp\left[-\frac{m_j}{2k_B T_j}\left(\frac{\omega \mp \Omega_j}{k}\right)^2 + \frac{m_j}{k_B T_j} V_0 \left(\frac{\omega \mp \Omega_j}{k}\right)\right] , \tag{16}$$

where the terms in the exponent like $m_j V_0^2/k_B T_j$ are neglected.

In the linear analysis, the trapping of plasma particles as suggested by Denavit and Sudan[18] does not occur for the resonant particles. Usually it occurs in tokamak-type devices.

For no thermal oscillations, it is obvious that the exponent vanishes and damping is negligibly small $(\gamma \to 0)$. However, for very high temperature limits in the regime $(m_j c^2 > k_B T_j)$, i.e., $T_e \sim 10^8$ K and $T_i \sim 10^{13}$ K and for large wave number $(k)$ or small wavelength $(\lambda)$, the growth term does not vanish as it approaches the relativistic regime.

Alternatively if $T_j >> 1$ and $k << 1$, the damping term reduces to

$$\gamma = \sum_{j=+,-} \frac{\pi\omega_{0j}^2}{2k} \left(\frac{m_j}{2\pi k_B T_j}\right)^{1/2} . \tag{17}$$

The exponent in Eq. (16) approaches unity in the above approximations. However, for vanishing cyclotron motion $\Omega_j \to 0$, the early work of Mohanty and Misra[19] reveals that $\gamma$ is negligibly small because the propagation characteristics are valid for $k << 1$, i.e., long wavelength limit and low streaming kinetic energies ($mV_0^2 << k_B T_j$). Sudan,[5] in fact, obtained the growth rate for nonstreaming electron only [Eq. (13f)], which vanishes in the limit of temperature isotropy.

## VI. DISCUSSION

We have analytically derived the results of dispersion and growth term of instability for cold and finite-temperature oscillations in a collisionless magnetoactive plasma stream having two components, i.e., electron and ion species. The electromagnetic oscillations resulting from cyclotron resonant conditions are emphasized and the nonresonant modes display streaming whistler's electrons ($\omega < \Omega_e + kV_0/2$) and Alfvén's modes ($\omega > \Omega_I + kV_0/2$). It is shown that for the electron cyclotron resonance ($\omega \simeq \Omega_e$), the refractive index for drift waves ($V_0 = 10^{-3}$ C) turns out to be slightly greater than unity. However, for nonstreaming plasmas, with the above conditions, the refractive index or dielectric function approaches infinity and the phase velocity in the dispersive medium approaches zero, showing no propagations of waves [Eqs. (10), (11), and (12)]. It agrees with earlier isotropic results. Along the same line, the finite-temperature oscillations reveal that $n = 3$ for nonstreaming resonant plasmas ($\omega \simeq \Omega_j$) and the phase velocity equals that of light propagating luminous waves only. But with streaming or drift waves in the plasma, one can use the parameters after Eq. (15) and numerically evaluate that phase velocity is of the order of $10^2$ which reveals the existence of subluminous waves, and the refractive index is much denser ($n \sim 10$). However, for cyclotron resonant ion oscillations, our numerical estimates show that $\omega/k \sim 10^9$, i.e., phase velocity of the waves approaches the velocity of light (near luminous waves).

In addition, nonresonant cyclotron oscillations ($\omega \neq \Omega_j$) and damping are also embedded in our results of dispersion. In our linear analysis it is interesting that trapping of plasma particles, as suggested by Denavit and Sudan,[18] does not occur. Usually such a phenomenon occurs in tokamak-like devices with linearity in $\omega$ and $k$ or in a nonuniform field. Moreover, the electromagnetic waves with streaming anisotropy do not manifest any loss-cone-type instability and the formalism is identical with the $\epsilon = 0$ case suitable for toroidal device.[12,13]

Equations (16) and (17) illustrate the damping of waves in the magneto active plasma for generalized and approximate cases. Following Mohanty and Misra,[19] it is assumed that streaming energy is less than the thermal energy for the propagation in the nonrelativistic region. The absorption of waves for streaming resonant interaction ($\omega = \Omega_j$) is never negligible, although ionic damping is less than its electron component owing to the heavy mass of the ion species.

We have further proved that owing to the large temperature of the species in the long wavelength limit, the damping term reduces to

$$\sum_{j=+,-} \frac{\pi\omega_{0j}^2}{2k} \left[ \frac{m_j}{2\pi k_B T_j} \right]^{1/2} ,$$

the terms in the exponent approach to unity.

Besides having significance for experimental diagnostics and simulation techniques, the dispersion results are uniquely relevant for experimental investigations of drift instabilities. The results of instabilities can be employed in the field of experimentations on "ground-based, low-frequency radio techniques," "ultra high-frequency radar techniques," and "*in situ* rocket and satellite based measurements," etc. The results illustrate the natural phenomena like streaming plasma instabilities in space and earth's magnetospheres, and thus yield valuable insight for satellite communications and geomagnetic study, etc., where excitations of frequency modes reveal electron and ion cyclotron oscillations with dissipative loss mechanism. It also indicates frequency fluctuations for different resonant or nonresonant modes. Bell and Buneman,[4] however, concluded in their Eq. (13) that only whistler's mode is possible for electron species without having the effect of streaming. Early results of Sudan[5] are of little significance to space and ionospheres, because he has adopted a non-Maxwellian equilibrium distribution, which is not a function of streaming parameter.

Lastly, the ion fusion mode is illustrated in our magnetoactive plasma stream, and we have indicated that in the temperature limit $T_i \sim 10^8$ K, extremely low ion cyclotron resonant waves have phase velocity as $10^2$ cm/sec and the plasma is very dense. Next for high cyclotron frequencies it grows up to $10^9$ cm/sec. The streaming turbulence in the magnetic plasma beam is equally relevant both for fusion analysis and space studies. We believe that the estimates can be helpful to design suitable parametric heating experiments, constructing a theory of instabilities due to anisotropy in the plasma particle momentum or magnetic field and constructing a theory of turbulence, etc.

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