New electrostatic resonance driven by laser radiation at perpendicular incidence in superdense plasmas

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Contrary to the well-known (Denisov) resonance absorption of obliquely incident p-polarized laser light on plasma, we derive and evaluate a second-harmonic resonance for perpendicular incidence at four times the critical density, from hydrodynamics, including unrestricted electric fields.

I. INTRODUCTION

In laser-plasma interactions, numerous parametric excitations occur[1,2] and a p-polarized *obliquely* incident radiation (on an inhomogemeous plasma along the depth x) causes a resonance maximum of the E_x field in the evanescent wave field (resonance absorption) derived by a magnetic field treatment by Denisov,[3] the electric field treatment without collisions in the plasmas by White and Chen,[4] and with collision in Chapter 12 of Ref. 5 where the quiverdrift of the electrons was the reason for the absorption. An alternative absorption process is wave breaking.[6] The earlier microscopic computation basically covers all these details.[1]

As essentially distinct from this case we derived a resonance process at *perpendicular* incidence of the laser radiation. Following a basically new hydrocode for two genuine electron and ion fluids with unlimited coupling by Poisson's equation[7] we arrived at an undamped oscillation with twice the laser frequency of the laser-driven electrostatic field, where a typical resonance factor results in high amplitudes at densities four times the critical density. In view of the very high profile steepening measured[8] or calculated,[9] the evaluation of the field by Airy functions in the superdense (evanescent wave) range nevertheless arrives at significant effects. This type of resonance—contrary to the usual resonance absorption at oblique incidence—appears at perpendicular incidence of the laser pulse.

It should be noted that the longitudinal oscillations of the electrons in a laser field with the frequency 2ω (where ω is the laser radian frequency), are known as a basic property of nonlinear laser-plasma interaction.[10]

Based on the results of the new hydrocode,[7] this paper considers the marcoscopic result of 2ω oscillations in the plasma produced by gradients of the gas dynamic and electrodynamic (nonlinear force produced) pressure. We are not considering here the 2ω pumping that causes wavebreaking superthermal electrons,[11] as this cannot be described hydrodynamically with equilibrium energy distributions but requires kinetic models or multiparticle simulation.

II. THEORY

We consider the model of fully ionized plasma of two genuine fluids in which the electron and ion motions are coupled due to collisions and due to the electrostatic interactions of charge separation. The corresponding momentum balance equations discussed in detail by Lalousis and Hora[6] permit the study of general electrostatic fields, contrary to the earlier theory with a quasineutral plasma. The equations of motion are

$$\frac{\partial(n_e v_e)}{\partial t} = -\frac{\partial}{\partial x}(n_e v_e^2) - \frac{1}{m_e}\frac{\partial P_e}{\partial x} - \frac{e n_e}{m_e}E - n_e \nu (v_e - v_i) + \frac{1}{m_e} f_{\rm nl} \tag{1}$$

and

$$\frac{\partial(n_i v_i)}{\partial t} = -\frac{\partial}{\partial x}(n_i v_i^2) - \frac{1}{m_i}\frac{\partial P_i}{\partial x} + \frac{e n_i}{m_i}E + n_e \nu (v_e - v_i) + \frac{m_e}{m_i^2} f_{\rm nl} \ , \tag{2}$$

where e is the electronic charge; n_e, n_i are the number densities of the electrons and ions; m_e, m_i are the masses of electrons and ions; v_e, v_i are the velocities of electrons and ions; P_e, P_i are the electron and ion pressures; E is the electric field due to charge separation; ν is the phenomenological electron-ion collision frequency; and $f_{\rm nl}$ is the (ponderomotive) nonlinear force due to the external laser field. We are considering one-dimensional fluids with a dependence of n_e, n_i, T_e, T_i, v_e, and v_i on time t and the spatial variable x. The electric field is then directed along x only and can be determined from the following equation:

$$\frac{\partial^2 E}{\partial t^2} = 4\pi e \left[\frac{\partial}{\partial t}(n_e v_e) - \frac{\partial}{\partial t}(n_i v_i) \right] \ , \tag{3}$$

where use has been made of the Poisson equation

$$\frac{\partial E}{\partial x} = -4\pi e (n_e - n_i) \tag{4}$$

and we have assumed the ionic charge (Z) to be unity. Now, the nonlinear force $\mathbf{f}_{\rm nl}$ (acting on the electrons) due to the external laser field is given by[4]

$$\mathbf{f}_{\rm nl} = -\hat{\mathbf{x}} \frac{1}{8\pi} \frac{\partial}{\partial x}(E_{yL}^2 + H_{zL}^2) \ , \tag{5}$$

where E_{yL} and H_{zL} are the y and z components of the electric and magnetic fields, respectively, associated with the (linearly polarized) laser beam propagating perpendicularly in a one-dimensional inhomogeneous plasma with spatial density variation in the x direction. In Eq. (5), the additional subscript L signifies that the fields correspond to the laser beam. In the WKB approximation, Eq. (5) is consistent with the following equation (see, e.g., Sec. 8.1 of Ref. 4):

$$\mathbf{f}_{\rm nl} = -\frac{\omega_p^2}{8\pi\omega^2} \sin^2(\omega t) \frac{\partial E_y^2}{\partial x} \hat{\mathbf{x}} \ , \tag{6}$$

where

$$E_{yL}=E_y\sin(\omega t),\quad \omega_p=\left(\frac{4\pi n_e^2}{m_e}\right)^{1/2} \tag{7}$$

represents the plasma frequency and we have assumed the time dependence of the laser field to be of the form of $\sin(\omega t)$. We should mention that $\mathbf{f}_{\rm nl}$ in Eq. (6) represents the *unaveraged* nonlinear force. If we substitute Eqs. (1) and (2) into Eq. (3), use Eq. (6), and carry out some approximations (see Ref. 6), we would obtain

$$\frac{d^2E}{dt^2}+2\gamma\frac{dE}{dt}+\omega_p^2E=A+B\cos(2\omega t)\ , \tag{8}$$

where

$$A=-\frac{4\pi e}{m_e}\left(\frac{\partial P_e}{\partial x}+\frac{\omega_p^2}{16\pi\omega^2}\frac{\partial E_y^2}{\partial x}\right)\ , \tag{9}$$

$$B=\frac{e\omega_p^2}{4m_e\omega^2}\frac{\partial E_y^2}{\partial x}\ , \tag{10}$$

and

$$\gamma=\frac{\nu}{2}\left(1+\frac{m_e}{m_i}\right)\ . \tag{11}$$

The solution of Eq. (8) can be readily obtained and is given by

$$\begin{aligned}E=\frac{A}{\omega_p^2}&+\exp(-\gamma t)\{C_1\cos[(\omega_p^2-\gamma^2)^{1/2}t]\\&+C_2\sin[(\omega_p^2-\gamma^2)^{1/2}t]\}\\&+G\cos(2\omega t)+H\sin(2\omega t)\ ,\end{aligned} \tag{12}$$

where C_1 and C_2 represent the integration constants and

$$G=\frac{(\omega_p^2-4\omega^2)B}{(\omega_p^2-4\omega^2)^2+4\gamma^2\omega^2}\ , \tag{13}$$

$$H=\frac{4\gamma\omega B}{(\omega_p^2-4\omega^2)^2+4\gamma^2\omega^2}\ . \tag{14}$$

III. EVALUATION OF THE RESONANCE TERM

The last two terms in Eq. (12) are of considerable interest as they represent the driving of oscillations at twice the laser frequency. The term proportional to $\sin(2\omega t)$ would resonantly dominate at $\omega_p=2\omega$ and its coefficient H can be written in the form

$$H=\left(\frac{e}{m_e\omega^2}\right)\frac{\epsilon\Omega}{[(\Omega-4)^2+4\epsilon^2]^2}\frac{\partial}{\partial x}(\overline{E_{yL}^2}+\overline{H_{zL}^2})\ , \tag{15}$$

where $\epsilon=\gamma/\omega$ and $\Omega=(\omega_p/\omega)^2$ and the overbar means averaging over one laser period. The maximum of the function

$$f(\Omega)=\frac{\epsilon\Omega}{(\Omega-4)^2+4\epsilon^2} \tag{16}$$

occurs at

$$\Omega=4[1+(\epsilon^2/4)]^{1/2}\cong 4+\tfrac{1}{2}\epsilon^2\ , \tag{17}$$

where its value is

$$\frac{\epsilon}{8}\left[\left(1+\frac{\epsilon^2}{4}\right)^{1/2}-1\right]^{-1}\cong\frac{1}{\epsilon}\left(1+\frac{\epsilon^2}{8}\right)\ . \tag{18}$$

The last terms in Eqs. (17) and (18) are valid when ϵ is small compared to unity. The full width at half maximum is approximately 2ϵ; thus, for $\epsilon \ll 1$ (which would be usually the case), the function is very sharply peaked at $\Omega\cong 4+\frac{1}{2}\epsilon^2$.

Next, in order to determine $\partial(E_{yL}^2+H_{zL}^2)/\partial x$ we assume a linear variation of the refractive index and neglect collisions as a small perturbation for this procedure for solving Maxwell's equations so that

$$n^2(x)=1-\frac{\omega_p^2}{\omega^2}=1-\frac{N(x)}{N_c}=-\alpha^2x\ , \tag{19}$$

where $N(x)$ represents the electron density, N_c is the critical density, and we have assumed

$$N(x)=N_c(1+\alpha^2x)\ . \tag{20}$$

The plane $x=0$ corresponds to the critical layer. Thus, the wave equation

$$\frac{d^2E_y}{dx^2}+n^2(x)\frac{\omega^2}{c^2}E_y=0 \tag{21}$$

becomes

$$\frac{d^2E_y}{d\xi^2}-\xi E_y=0\ , \tag{22}$$

where

$$\xi=(\alpha\omega/c)^{2/3}x\ . \tag{23}$$

The solution of Eq. (21) which decays as $\xi\to\infty$ is the Airy function Ai whose (here applicable) asymptotic forms are as follows:

$$E_y(\xi)=a\ \mathrm{Ai}(\xi) \tag{24}$$

$$\to\begin{cases}\dfrac{a}{2\sqrt{\pi}}\xi^{-1/4}e^{-\zeta}\ \text{ for }\xi\to\infty\ , & (25)\\[2ex] \dfrac{a}{\sqrt{\pi}}|\xi|^{-1/4}\sin\left(\zeta+\dfrac{\pi}{4}\right)\ \text{ for }\xi\to-\infty\ , & (26)\end{cases}$$

where

$$\zeta=\tfrac{2}{3}|\xi|^{3/2}=\tfrac{2}{3}(\alpha\omega/c)|x|^{3/2}\ . \tag{27}$$

At $x=-1/\alpha^2$, $N(x)=0$ beyond which (i.e., for $x<-1/\alpha^2$) we assume the refractive index to take a constant value (equal to unity). Now, at $x=-1/\alpha^2$ we find

$$\xi=(\omega/c\alpha^2)^{2/3}(\equiv\xi_0) \tag{28}$$

and we may set

$$\frac{a}{\sqrt{\pi}}\xi_0{}^{-1/4}=E_0\ , \tag{29}$$

where E_0 is the amplitude of the electric field associated with the incident laser beam. Thus,

$$a=\sqrt{\pi}(\omega/c\alpha^2)^{1/6}E_0 \tag{30}$$

and for large positive values of x we have [see Eq. (25)]

$$E_y(x) \cong \frac{E_0}{2\eta^{1/4}} \exp\left[-\frac{2}{3}\left(\frac{\omega}{c\alpha^2}\right)\eta^{3/2}\right] , \quad (31)$$

where

$$\eta \cong \alpha^2 x . \quad (32)$$

Thus,

$$\frac{\partial E_y^2}{\partial x} \cong -\frac{\alpha^2}{8\eta^{3/2}} E_0^2 \left[1 + 4\left(\frac{\omega}{c\alpha^2}\right)\eta^{3/2}\right] \times \exp\left[-\frac{2}{3}\left(\frac{\omega}{c\alpha^2}\right)\eta^{3/2}\right] . \quad (33)$$

Substituting Eq. (33) into Eq. (12) we get

$$H \cong \frac{eE_0^2}{m_e c\omega} \frac{1}{2}\left(1 + \frac{1}{6\Gamma}\right) e^{-\Gamma} f(\Omega) , \quad (34)$$

where

$$\Gamma \cong \frac{2}{3} \frac{\omega}{c\alpha^2} \eta^{3/2} . \quad (35)$$

Now, when $f(\Omega)$ attains its maximum value [$\cong 1/\epsilon$, see Eq. (18)], $\Omega \cong 4$, i.e., $\Omega_p^2 \cong 4\omega^2$ which occurs when $N = 4N_c$; the corresponding values of x and η are $3/4\alpha^2$ and 3, respectively.

IV. RESULTS

In Fig. 1 we have plotted H as a function of η $(=\alpha^2 x)$ for $\omega/c\alpha^2 = 1.0$ and 5.0. Assuming the frequency of neodymium glass lasers $\omega \cong 1.7\times 10^{15}$ Hz, these will correspond to

$$\alpha^2 = 5.67\times 10^4 \text{ and } 1.13\times 10^4 \text{ cm}^{-1} , \quad (36)$$

respectively, which are of the same order as has been measured[7] and calculated[8] for the steepened density profiles at laser-plasma interaction for laser intensities near 10^{16} W/cm^2. In Fig. 1 for each value of $\omega/c\alpha^2$ there are two sets of curves corresponding to

$$\epsilon \cong \frac{\gamma}{2\omega} = 0.025 \text{ and } 0.001 . \quad (37)$$

For $\epsilon \cong 1.7\times 10^{15}$ Hz the above values of ϵ will correspond to $\nu \simeq 1.7\times 10^{14}$ s^{-1} and 6.8×10^{12} s^{-1}. We may note that for laser field amplitude $E_L \cong 10^9$ V/cm ($I \sim 10^{16}$ W/cm^2) in vacuum, the maximum value of the electric field (corresponding to the frequency 2ω) is about 10^8 V/cm for $\omega/c\alpha^2 \cong 1$ and $\epsilon \cong 10^{-3}$. For smaller values of ϵ one obtains a higher value of the electric field at resonance. We may mention here that the electron collision frequency is approximately given by (see Sec. 2.5 of Ref. 4)

$$\nu \cong 2.72\times 10^{-5} \frac{n_e}{T_e^{3/2}} \ln\Lambda ,$$

where n_e is the electron density in cm^{-3}, T_e the electron temperature in electron volts and $\ln\Lambda$ is the Coulomb logarithm factor which we may assume to be ~ 3. Further,

$$T_e = T_{\text{th}} + \tfrac{1}{2}\epsilon_{\text{osc}} ,$$

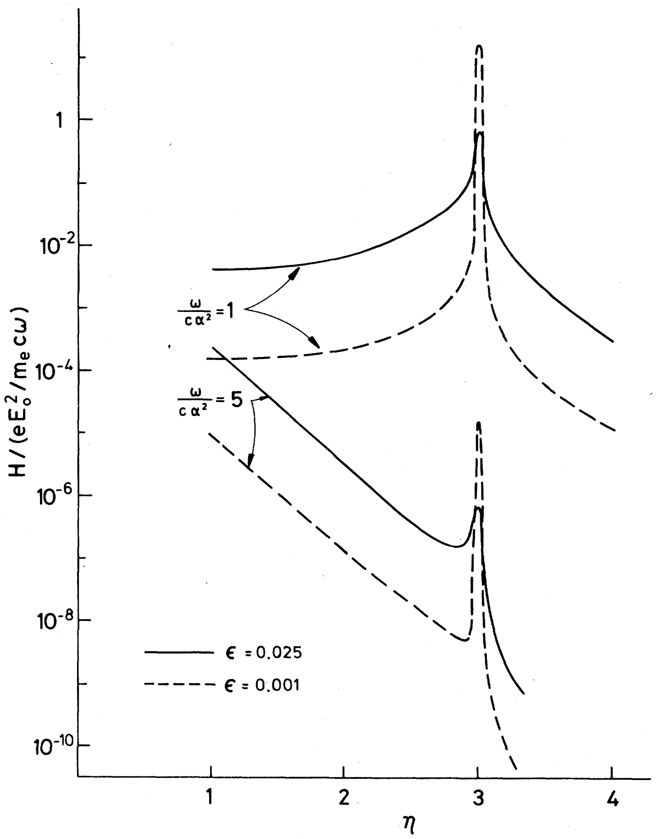

FIG. 1. Amplitude H [Eqs. (14) and (34)] for the 2ω longitudinal (electrostatic) oscillation in the laser irradiated plasma in a superdense linear density profile increasing within about one wavelength ($\omega/c\alpha^2=1$) from the critical density $n_c(\eta=1)$ to $4n_c(\eta=3)$ as expected (Refs. 8 and 9), or increasing more moderately, five times more slowly ($\omega/ca\alpha^2=5$). The resonance at $4n_c$ is higher and with smaller half-width if the collision frequency ν is smaller ($\epsilon=\nu/\omega$).

where T_{th} represents the contribution due to thermal effects and $\epsilon_{\text{osc}}^{\text{th}}$ represents the oscillation energy (in electron volts) which depends on the intensity of the laser beam. Now $T_{\text{th}} \leq 100$ eV and for $I = 10^{17}$ W/cm^2, $\epsilon_{\text{osc}} \cong 10^4$ eV. Thus, for $T_e \simeq 10^4$ eV, $n_e \cong 4\times 10^{21}$ cm^{-3} we get

$$\nu \cong 0.2\times 10^{12} \text{ s}^{-1}$$

and therefore the value of ϵ for the collisional damping is the smaller and the resonance should then be even sharper and higher. This increase of the resonance, however, may be reduced by microscopic processes, not covered by this macroscopic treatment, or by a nonlinear feedback where the usually high profile steepening may be reduced by the resonance.

From Fig. 1 for $\epsilon = 10^{-3}$ and $\omega/c\alpha^2 = 1$ one sees that the resonance at $\omega_p = 2\omega$ causes a longitudinal electrostatic oscillation which is 10^5 times higher than near the critical density. The fact that its amplitude for 10^{16} W/cm^2 laser intensity is 10% of the laser field in the vacuum indicates that a reasonable transfer of optical energy into second harmonic oscillations can be expected. As the amplitude of the field H increases by E_0^2, there will be a saturation for 10^{17} W/cm^2, if the profile steepening[8,9] is conserved.

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