Exact solution to the degenerate four-wave mixing in reflection geometry in photorefractive media

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A method for exact solution of the stationary holographic degenerate four-wave mixing in reflection geometry in photorefractive media is presented. The effects of pump depletion and light absorption in the nonlinear dynamic medium are rigorously taken into account. Numerical steps in the procedure are reduced to the solution of one simple nonlinear first-order differential equation, or are absent altogether when the equation is linearized, or for the case of no absorption. The method of solution is applicable to other wave-mixing geometries as well.

I. INTRODUCTION

Recent progress made in experiments on real-time holography and four-wave mixing[1-4] (FWM) in reflection or transmission geometry placed a more urgent need for solutions of the theories of wave mixing in which pump depletion and light absorption in the nonlinear medium are accounted for. Purely numerical accounts[5] or accounts with undepleted pumps[1,2,6] have been available for some time, but there are very few completely analytic treatments.[7-10] The closest in relevance to the work reported here appears to be that of Cronin-Golomb *et al.*,[8] who solved analytically (and by a different method) the degenerate FWM in transmission and reflection geometry without absorption. In one of his recent articles,[5] Ja states that it seems impossible to obtain an analytic solution to the system of nonlinear coupled-wave equations for the degenerate FWM when considering either light absorption or different types of grating. We believe that we have obtained such a solution to the problem.

The standard reflection geometry configuration for FWM is depicted in Fig. 1. Two pump waves A_1 and A_2 impinge on a dynamic nonlinear medium situated in between the planes $z=0$ and $z=d$ from the opposite sides. Because of the nonlinear coupling of waves in the medium, a signal wave A_3 incident from the left causes generation of the counter-propagating phase-conjugated wave A_4. Reflection gratings are being formed between the signal wave A_3 and the pump A_2, and the other pump is diffracted off the grating into the reconstruction A_4. The alternative possibility of the signal making predominant grating with the pump A_1 corresponds to the transmission geometry of FWM. Variation of the waves is described by a set of nonlinear coupled-wave equations.

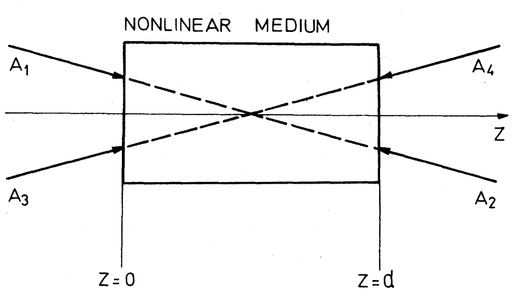

FIG. 1. Four-wave-mixing configuration in a reflection geometry. A_1 and A_2 are the pump waves, A_3 is the signal wave, and A_4 the phase-conjugated wave generated in the medium.

There exist two similar models, and two similar sets of nonlinear coupled-wave equations describing the degenerate FWM. In one of the models[2,5] the intensities of the waves are used as the dependent variables; in the other[6-8] the complex wave amplitudes are used. In consistency with our previous work on two-wave mixing in reflection geometry[10] we use intensities as the variables. Also, we will be concerned with the stationary energy transfer and generation of the phase-conjugated wave, and not with the phase transfer. In this case the nonlinear coupled-wave equations in slowly varying amplitude approximation for FWM in reflection geometry are of the form[5]

$$I_1' = -\alpha I_1 - 2g\frac{I_1 I_4 \pm (I_1 I_2 I_3 I_4)^{1/2}}{I_0}\,, \tag{1.1a}$$

$$I_2' = \alpha I_2 - 2g\frac{I_2 I_3 \pm (I_1 I_2 I_3 I_4)^{1/2}}{I_0}\,, \tag{1.1b}$$

$$I_3' = -\alpha I_3 - 2g\frac{I_3 I_2 \pm (I_1 I_2 I_3 I_4)^{1/2}}{I_0}\,, \tag{1.1c}$$

$$I_4' = \alpha I_4 - 2g\frac{I_4 I_1 \pm (I_1 I_2 I_3 I_4)^{1/2}}{I_0}\,, \tag{1.1d}$$

where I_1, I_2, I_3, I_4 represent the beam intensities, $I_0 = I_1 + I_2 + I_3 + I_4$ is the total intensity, and the prime denotes the derivative with respect to z. Further, α is the linear intensity absorption coefficient, and g is the effective wave-coupling constant, assumed to be real. Thus we assume a $\pi/2$ phase shift between the interference fringes and the index grating, so that the theory in this form applies to the physically and experimentally interesting case of photorefractive media. The upper sign in Eqs. (1.1) applies when $g>0$, the lower when $g<0$. We will consider the case $g>0$. Note that the system of equations (1.1) is concerned only with the fundamental components of the phase grating.[2,5-8] The corresponding boundary conditions are given at two end points, that is $I_1(0)$, $I_2(d)$, $I_3(0)$, and $I_4(d)$ are known.

The physics and processes occurring in FWM and real-time (dynamic) holography are described at length elsewhere,[1,2,8] and will not be of major concern here. In Ref. 5 Ja considered different computational methods for solution of the system (1.1), and obtained various numerical solutions (which we will use for comparison later). The main result of this paper is a procedure for analytic solution of the system of equations (1.1) with or without absorption, and for different types of grating. The paper is organized in the following way. Section II deals with the procedure for solution of Eqs. (1.1) when no absorption is present. In Sec. III the procedure is generalized to account for the absorption. The generalization requires numerical integration of a simple first-order differential equation, and some manipulation of implicitly given functions. All the numerical steps in this paper are performed on a Texas Instruments TI 59 handheld calculator. In Sec. IV we present results and compare them with the numerical two-point shooting method of Ref. 5.

II. ANALYSIS OF THE EQUATIONS WITHOUT ABSORPTION

It is convenient to proceed awhile with the general case $\alpha \neq 0$, and at a certain point switch to the case $\alpha=0$. First, new dependent variables are introduced: $u_1=I_1+I_4$, $v_1=I_1-I_4$, $u_2=I_2+I_3$, $v_2=I_2-I_3$. In terms of these variables and two more auxiliary functions $f_1^2=u_1^2-v_1^2$ and $f_2^2=u_2^2-v_2^2$, Eqs. (1.1) get the form

$$u_1'=-\alpha v_1-g f_1\frac{f_2+f_1}{u_2+u_1}, \tag{2.1a}$$

$$u_2'=\alpha v_2-g f_2\frac{f_2+f_1}{u_2+u_1}, \tag{2.1b}$$

$$v_1'=-\alpha u_1,\quad v_2'=\alpha u_2 . \tag{2.1c}$$

The functions f_1 and f_2 obey their own differential equations, naturally not independent of Eqs. (2.1):

$$f_1'=-g u_1\frac{f_2+f_1}{u_2+u_1},\quad f_2'=-g u_2\frac{f_2+f_1}{u_2+u_1} . \tag{2.2}$$

With another change of dependent variables, $U_1=u_2+u_1$, $U_2=u_2-u_1$, $V_1=v_2-v_1$, $V_2=v_2+v_1$, and a new set of auxiliary functions $F_1=f_2+f_1$, $F_2=f_2-f_1$, a further simplification is achieved. Equations (2.1) and (2.2) become

$$U_1'=\alpha V_1-g\frac{F_1^2}{U_1}, \tag{2.3a}$$

$$U_2'=\alpha V_2-g\frac{F_2F_1}{U_1}, \tag{2.3b}$$

$$V_1'=\alpha U_1,\quad V_2'=\alpha U_2, \tag{2.3c}$$

$$F_1'=-gF_1,\quad F_2'=-g\frac{U_2F_1}{U_1}, \tag{2.4}$$

and three of these equations are integrated without difficulty:

$$U_1^2-V_1^2=E^2+F_1^2, \tag{2.5}$$

$$V_2^2-U_2^2=E^2-F_2^2, \tag{2.6}$$

$$F_1=\exp[f_d+g(d-z)], \tag{2.7}$$

giving two relations among the four intensities,

$$A_1A_2-A_3A_4=\frac{E}{2}, \tag{2.8}$$

$$A_1A_4+A_2A_3=\frac{F_1}{2}, \tag{2.9}$$

where $E=2\sqrt{I_1(d)C_2}$ and $A_i=\sqrt{I_i}$. The remaining three equations (for V_1, V_2, and F_2) should either give two extra relations among the intensities, or sufficient information about some auxiliary functions which would allow unique determination of the intensities. The first of the two strategies will be followed in the case $\alpha=0$, the second in the case $\alpha\neq0$. Indeed, if $\alpha=0$, equations for V_1 and V_2 are easily integrated, to yield

$$I_2-I_3=\text{const}=C_2-I_{3d}, \tag{2.10}$$

$$I_1-I_4=\text{const}=I_{1d} . \tag{2.11}$$

We rewrite Eqs. (2.8)–(2.11) for convenience in the following form:

$$A_2^2-A_3^2=A^2, \tag{2.12a}$$

$$A_1^2-A_4^2=B^2, \tag{2.12b}$$

$$A_1A_2-A_3A_4=C^2, \tag{2.12c}$$

$$A_1A_4+A_2A_3=D^2, \tag{2.12d}$$

with the obvious notation. Then the intensities are given by

$$A_2=A\cosh x,\quad A_3=A\sinh x, \tag{2.13a}$$

$$A_1=B\cosh y,\quad A_4=B\sinh y, \tag{2.13b}$$

where x and y obey

$$x-y=Y=\cosh^{-1}\frac{C^2}{AB}, \tag{2.14}$$

$$\exp(2y)=\frac{1+(1+a^2-b^2)^{1/2}}{a+b}, \tag{2.15}$$

and $a=[A^2\cosh(2Y)+B^2]/2D^2$, $b=A^2\sinh(2Y)/2D^2$. This completes the writing of the solution for the case $\alpha=0$.

The connection of the prescribed two-point boundary values with the values of the constants A,B,C,f_d used thus far in description of the intensities is most easily established by evaluation of the missing boundary values $I_{1d},I_{20},I_{3d},I_{40}$ in terms of the given ones $I_{10}=1$, $I_{2d}=C_2$, $I_{30}=C_3$, and $I_{4d}=0$. This is achieved by solving the following system of four algebraic equations:

$$A_{20}^2+A_{3d}^2=C_2+C_3, \tag{2.16a}$$

$$A_{1d}^2+A_{40}^2=1, \tag{2.16b}$$

$$A_{20}-\sqrt{C_3}A_{40}=A_{1d}\sqrt{C_2}, \tag{2.16c}$$

$$\sqrt{C_3}A_{20}+A_{40}=\sqrt{C_2}A_{3d}e^{gd}, \tag{2.16d}$$

obtained when boundary conditions are applied to Eqs. (2.12). Here $A_{20}=\sqrt{I_2(0)}$, etc. The solution is given in the form

$$A_{1d}=\cos\xi \ , \quad A_{40}=\sin\xi \ , \tag{2.17a}$$

$$A_{20}=\sqrt{C_2+C_3}\cos\eta \ , \quad A_{3d}=\sqrt{C_2+C_3}\sin\eta \ , \tag{2.17b}$$

where

$$\cot\xi=\frac{1+C_3+C_2e^{gd}}{\sqrt{C_2C_3}(e^{gd}-1)} \ , \tag{2.18}$$

and

$$\sqrt{C_2}e^{gd}\tan\eta=\sqrt{C_3}+\frac{1}{\sqrt{C_2}\cot\xi+\sqrt{C_3}} \ . \tag{2.19}$$

In the quadratic equation for $\cot\xi$ we have retained the always positive solution. Then the unknown constants A,B,C,f_d are given in terms of the known ones:

$$A=(C_2-A_{3d}^2)^{1/2}=(A_{20}^2-C_3)^{1/2} \ , \tag{2.20a}$$

$$B=(A_{1d}^2)^{1/2}=(1-A_{40}^2)^{1/2} \ , \tag{2.20b}$$

$$C^2=\sqrt{C_2}B \ , \quad f_d=\ln(2\sqrt{C_2}A_{3d}) \ , \tag{2.20c}$$

and the intensities are uniquely determined by Eqs. (2.13). This completes the analysis of the case $\alpha=0$. In Fig. 4 the analytic solution $\alpha=0$ is compared with the numerical solution of Ref. 5. As expected, no noticeable difference is evident. In Ref. 8 Cronin-Golomb *et al.* also reported an analytic solution to the degenerate FWM in reflection geometry without absorption, obtained by a different method. However, a comparison is difficult to establish, since in their solution the procedure for connecting the constants of integration with the prescribed boundary values is not given.

III. THE GENERAL CASE $\alpha\neq 0$

For treatment of the general case we have to go back to Eqs. (2.3) and (2.4). Actually two relations among four intensities are already known [Eqs. (2.8) and (2.9)], and the additional information is obtained when U_1 is represented as $e^{\phi}\cosh u$, and correspondingly V_1 as $e^{\phi}\sinh u$. Then $\phi=\frac{1}{2}\ln(E^2+F_1^2)$, and u is found to satisfy the following differential equation:

$$u'-gF_1^2e^{-2\phi}\tanh u=\alpha \ . \tag{3.1}$$

In terms of the intensities we have

$$A_1^2+A_3^2=\tfrac{1}{2}e^{\phi-u}=G^2 \ , \tag{3.2}$$

$$A_2^2+A_4^2=\tfrac{1}{2}e^{\phi+u}=H^2 \ . \tag{3.3}$$

Together with Eqs. (2.8) and (2.9) a system of four equations is formed. However, this system is not closed, as can be seen if a solution of the form

$$A_1=G\cos\lambda \ , \quad A_3=G\sin\lambda \ , \tag{3.4a}$$

$$A_2=H\cos\mu \ , \quad A_4=H\sin\mu \tag{3.4b}$$

is assumed. Then Eqs. (3.2) and (3.3) are satisfied identically, while Eqs. (2.8) and (2.9) provide the same information in the form of $\sin p=F_1/2GH$ and $\cos p=E/2GH$, respectively, where $p=\lambda+\mu$. The completeness is achieved when the equation for F_2 is brought into consideration. In terms of the quantities just introduced one obtains

$$q'=-gEF_1e^{-2\phi}\tanh u \ , \tag{3.5}$$

where $q=\lambda-\mu$. When integrated this equation yields

$$q=q_d-\frac{E}{F_1}v+\frac{E}{F_{1d}}v_d-\frac{gE}{F_{1d}}e^{gd}\int_z^d ve^{g\zeta}d\zeta \ , \tag{3.6}$$

with $v=u-\alpha z$. Thus the determination of the intensities as given by Eqs. (3.4) requires solution of a first-order differential equation, Eq. (3.1); evaluation of an integral, Eq. (3.6); and two relations among the four intensities, Eqs. (2.8) and (2.9). The numerical solution of Eq. (3.1) is simple enough to be performed on a TI 59 handheld calculator. In actual computations we solved the system of equations

$$u'=g\sin^2p\tanh u+\alpha \ , \tag{3.7a}$$

$$q'=-\frac{g}{2}\sin 2p\tanh u \ , \tag{3.7b}$$

where $p=\tan^{-1}F_1/E$. The initial values are given at $z=d$, and the integration is performed backwards to $z=0$. Some of the outputs are provided in Fig. 2.

According to Eqs. (3.4) the intensities are given with

$$I_1=\frac{1}{2}e^{\phi-u}\cos^2\frac{p+q}{2} \ , \tag{3.8a}$$

$$I_2=\frac{1}{2}e^{\phi+u}\cos^2\frac{p-q}{2} \ , \tag{3.8b}$$

$$I_3=\frac{1}{2}e^{\phi-u}\sin^2\frac{p+q}{2} \ , \tag{3.8c}$$

$$I_4=\frac{1}{2}e^{\phi+u}\sin^2\frac{p-q}{2} \ . \tag{3.8d}$$

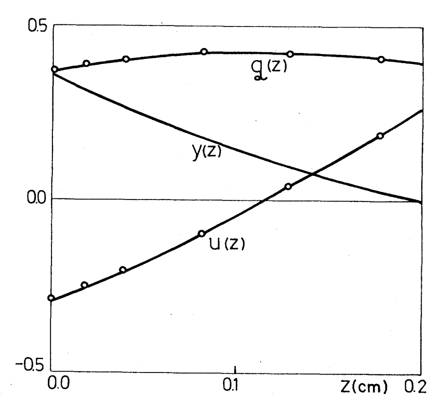

FIG. 2. Functions $u(z)$, $q(z)$, and $y(z)$, needed for specification of the intensities, and corresponding to the examples considered in Sec. IV. The points indicated atop the curves $u(z)$ and $q(z)$ are obtained from the analytic solution of the linearized Eqs. (3.7).

Therefore, the intensities are given in terms of the four quantities ϕ,p,u,q. The first two of the four, e^{ϕ} and p, are connected with E and F_1 by the polar-rectangular coordinate transformation, and thus are equivalent to them. u is the solution of a simple differential equation, and q a simple functional of u. In actual calculation of these quantities the greatest problem presented the choice of the correct initial values at $z=d$. The comparison with the all-numerical approach of Ja (Ref. 5) is depicted in Fig. 5. We postpone the discussion until the next section on results.

The problem of matching boundary conditions is now more involved, but still tractable. In our treatment it is assumed that I_{1d} (defining E), f_d (defining F_1), and u_d and q_d (being the initial values for the differential equations) are known. Our philosophy is similar to what has been done for the case $\alpha=0$. We determine the missing boundary values ϕ_0,p_0,u_0,q_0 at $z=0$ in terms of the given ones at $z=d$, and then using the split boundary conditions for intensities find the correct corresponding combination of ϕ_d,p_d,u_d,q_d. Thus looking at Eqs. (3.8) at $z=0$ and $z=d$ one finds

$$\phi_0-u_0=\ln 2(1+C_3)\ , \tag{3.9a}$$

$$\phi_d+u_d=\ln 2C_2\ , \tag{3.9b}$$

$$\tan\left[\frac{p_0+q_0}{2}\right]=\sqrt{C_3}\ , \tag{3.9c}$$

$$p_d=q_d\ . \tag{3.9d}$$

On the other hand, from the definition of p and ϕ it is

$$\tan p_0=e^{gd}\tan p_d\ , \tag{3.10a}$$

$$\phi_0=\phi_d+\tfrac{1}{2}\ln\psi\ , \tag{3.10b}$$

where

$$\psi=\frac{1+e^{2gd}\tan^2 p_d}{1+\tan^2 p_d}\ . \tag{3.10c}$$

From the numerical integration the functional dependence $u_0(u_d)$ and $q_0(q_d)$ is also known. In Fig. 3 we display these functions. Note that q_0 is also parametrically dependent on u_d, as can be seen from Fig. 3. Actually, from Eq. (3.6) it is $q_0=q_d+s(u_d)$, where s stands for a shift. Equation (3.9c) then, by the use of Eqs. (3.9d) and (3.10a), turns into a quadratic equation for $\tan q_d$:

$$e^{gd}\tan q_d+\tan(q_d+s)=\frac{2\sqrt{C_3}}{1-C_3}[1-e^{gd}\tan(q_d)\tan(q_d+s)] \tag{3.11}$$

which defines an implicitly given function $q_d(u_d)$. Likewise u_0 is parametrically dependent on q_d, and a combination of Eqs. (3.9a) and (3.10b) leads to an equation for $u_d(q_d)$:

$$u_d+u_0(u_d)=\ln\frac{C_2}{1+C_3}+\tfrac{1}{2}\ln\psi(q_d)\ . \tag{3.12}$$

The point of intersection of these two curves on the plane (u_d,q_d) defines the appropriate choice for u_d and q_d. Solution of Eqs. (3.11) and (3.12) is estimated graphically, and iterated for greater accuracy. For a realistic range of parameters and boundary values (as, for example, considered in Ref. 5) the solution is unique. Equations (3.9b) and (3.9d) then yield the values of ϕ_d and p_d. In this manner all the relevant quantities ϕ_d,p_d,u_d,q_d used in description of the intensities are evaluated in terms of the prescribed boundary values $I_{10},I_{2d},I_{30},I_{4d}$.

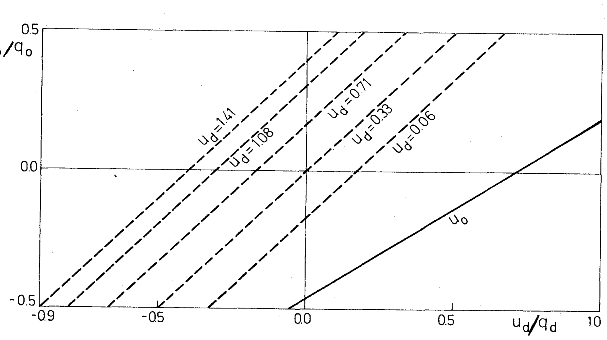

FIG. 3. Functions $u_0(u_d)$ (continuous line, for $q_d=0.392$), and $q_0(q_d)$ (dashed lines, for different values of u_d). These functions figure in the analysis of boundary conditions for the analytic procedure.

IV. RESULTS

In this section our results are summarized and compared with the numerical results of Ja.[5] For the case of no absorption the intensities are given by

$$I_1=I_{1d}\cosh^2 y\ ,\quad I_4=I_{1d}\sinh^2 y\ , \tag{4.1a}$$

$$I_2=(C_2-I_{3d})\cosh^2(y+Y)\ , \tag{4.1b}$$

$$I_3=(C_2-I_{3d})\sinh^2(y+Y)\ , \tag{4.1c}$$

where $Y=\cosh^{-1}[C_2/(C_2-I_{3d})]^{1/2}$, and y is given by Eq. (2.15). The missing end-values for intensities are determined from Eqs. (2.17):

$$I_{1d}=\cos^2\xi\ ,\quad I_{3d}=(C_2+C_3)\sin^2\eta\ , \tag{4.2}$$

where $\cot\xi$ and $\tan\eta$ are given by Eqs. (2.18) and (2.19). Using the values $g=6\text{ cm}^{-1}$, $d=0.2$ cm, $C_2=1$, and $C_3=0.6$ (in units of I_{10}), which correspond to an example considered in Ref. 5, we obtain $\xi=0.3502$, $\eta=0.3088$, $I_{1d}=0.8823$, $I_{3d}=0.1478$, $Y=0.4053$, and y as a function of z is depicted in Fig. 2. In Fig. 4 the calculated values for the intensities are plotted against the computed curves of Ja, taken from Fig. 2 in Ref. 5. As expected, the agreement is complete. In order to check the numerics of Sec. III we have also solved the case $\alpha=0$ at few z points by the method of Sec. III. These points are also displayed in Fig. 4.

For the case with absorption, the intensities are given by Eqs. (3.8). Four z-dependent functions ϕ,p,u,q are needed for specification of the intensities. The evaluation of these functions requires the knowledge of four one-side boundary values, which in turn have to be determined from the two-sided boundary conditions, as described in

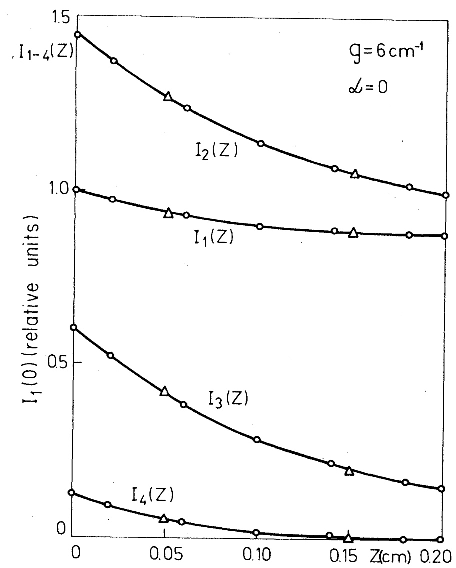

FIG. 4. Comparison of the analytic results without absorption with the numerical results of Ref. 5. The curves are taken from Fig. 2 in Ref. 5, and the points indicated are obtained by the analytic procedure described in this paper: the circles by the method of Sec. II, and the triangles by the method of Sec. III.

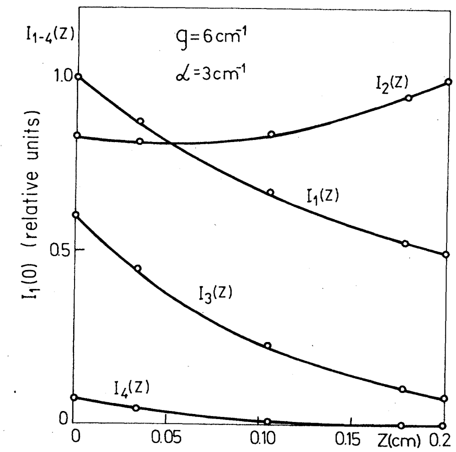

FIG. 5. Comparison of the analytic solution with absorption and the numerical solution of Ref. 5. The curves are taken from Fig. 2 in Ref. 5, and the points indicated are obtained by the method of Sec. III, Eqs. (3.8).

Sec. III. So, going backwards in Sec. III, we first solve Eqs. (3.11) and (3.12) for the values of the parameters as in the previous example, and for $\alpha=3\text{ cm}^{-1}$, to find $u_d=0.271$ and $q_d=0.392$ approximately. Using the value of u_d one finds $\phi_d=0.422$, and also $p_d=q_d=0.392$. Thus $f_d=-0.539$ and $E=1.41$, and all the necessary initial information is available. Now we are in position to solve differential equations (3.1) and (3.5) [the curves $u(z)$ and $q(z)$ are plotted in Fig. 2], to calculate the intensities, and to compare them with the numerical output of Ref. 5. This is accomplished in Fig. 5.

We note that the function $u(z)$ is smooth and monotonous in behavior, and rather small for realistic values of the parameters (in our example u varies between -0.3 and 0.3 over the whole range of z). Thus the differential equation for u can be linearized with little error, and solved exactly. The solution is of the form

$$u=u_d e^{\phi_d-\phi} - \frac{\alpha}{g}\left[1-e^{\phi_d-\phi}-Ee^{-\phi}\ln\left[\frac{E+e^{\phi}}{E+e^{\phi_d}}\frac{F_{1d}}{F_1}\right]\right]. \tag{4.3}$$

The whole procedure is now exact, free of computations, and reduced to manipulation of algebraic equations and implicitly given functions. In Fig. 2 we provide for comparison the exact solution of the linearized equation for u as well.

The whole treatment of degenerate FWM in reflection geometry as given in this paper is easily adopted and applied to other geometries, for example to the forward FWM, where all four waves propagate generally in the same direction, and impinge on the nonlinear medium from one side, or to the degenerate FWM in transmission geometry, as reported in Ref. 11. The method is also applicable to other wave-mixing processes, for example to the degenerate two-wave mixing in reflection geometry,[9] as is established in Ref. 10.

In conclusion, we propose a procedure for exact, analytic solution of a model of FWM in reflection geometry for photorefractive media when the absorption in the nonlinear medium is taken into account, and depletion of the pumps allowed for. The numerical steps in the procedure are reduced to a minimum, or absent altogether (when there is no absorption, or when the equation for u is linearized). The method is applicable equally well to other types of grating or wave-mixing geometries.

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