Strong-field autoionization by smooth laser pulses

Kazimierz Rzążewski and Jakub Zakrzewski

Institute for Theoretical Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, PL-02-668 Warsaw, Poland

Maciej Lewenstein*

Fachbereich Physik, Universität Essen, D-4300 Essen 1, Federal Republic of Germany

Joseph W. Haus†

Research Directorate, U.S. Army Missile Laboratory, U.S. Army Missile Command, Redstone Arsenal, Alabama 35898
(Received 17 December 1984)

We consider pulses of coherent electromagnetic radiation impinging upon an autoionizing atom. Analytic solutions in the cases of hyperbolic-secant- and exponential-pulse envelopes are given for the ground-state atomic population and for the photoelectron spectrum. The sensitivity of these quantities to the pulse area and to the interference between the autoionizing state and the continuum is studied.

I. INTRODUCTION

Most recent studies of strong-field photoionization have assumed that the laser light is suddenly switched on and its intensity remains constant during the interaction time.[1,2] These studies predicted a number of new coherence phenomena, especially noteworthy are those pertaining to trapping of the electronic population in the ground state,[2] and to spectral features appearing in the photoelectron and photoemission spectra. All of these properties can be adequately discussed within the dressed-atom picture.[3]

The term "dressed atom" was coined in the theory of bound-bound transitions. A composite system consisting of a two-level atom and a single mode of the electromagnetic field has a time-independent Hamiltonian whose eigenstates are the "dressed states." The inclusion of any relaxation mechanism, such as, spontaneous emission or collisional dephasing of the atomic dipole or in the case of ionization, the very existence of the continuous atomic spectrum gives the dressed-atomic states a finite width; that is, they become unstable.

The photoelectron spectrum in strong-field ionization, the absorption spectrum in double-optical resonance experiments, and the fluorescence spectrum for a transition from the excited state to a third level are all probing the population of the excited state; hence, they have a number of maxima equal to the number of dressed states which have a nonvanishing overlap with the probed state. In the corresponding semiclassical description, there is an obvious simplicity of the cw signal in the rotating-wave approximation. It allows the elimination of time dependence in the coefficients of the linear-evolution equation, and thus, their solution is amenable to solution by Laplace transform methods.[4] The atomic structure of the model determines the number of poles whose locations depend on the strength of the field. These poles manifest themselves as maxima of various spectra.

A completely new situation arises if the laser light is a smooth pulse. The Hamiltonian is no longer time independent and the dressed-atom picture is no longer applicable to this situation. Also, since the coefficients of the evolution equations contain an explicit nontrivial time dependence, their solution by resolvent methods must be abandoned. However, a few analytically soluble examples have been studied. In contrast to the cw signal case, it becomes a more complicated matter now to anticipate how many maxima will appear in the spectra.[5]

The remainder of this paper is devoted to two analytically soluble pulse forms driving the atom from the ground state to an autoionizing state.[5] In Sec. II we derive the basis evolution equations for our study of strong-field ionization. The density of continuum states is dressed by the autoionizing state embedded in the continuum, the result being a Fano profile[6] with an asymmetry characterized by a single parameter. Section III presents results for the ground-state population and photoelectron spectrum when the pulse form has a hyperbolic-secant shape. In Sec. IV corresponding results are provided for exponential pulses and the conclusions are given in Sec. V.

II. GENERAL FRAMEWORK

In this section we derive the basic formulas relevant to the study of strong-field autoionization by a smooth pulse of coherent radiation. Our model atom has only one bound state and a single narrow autoionizing resonance located far away from the ionization edge. This atom is irradiated by a strong laser pulse with envelope $E(t) = E_0 f(t)$, where E_0 is its typical strength and $f(t)$ is a dimensionless function characterizing the pulse shape. The Hamiltonian for our system is

$$H = \int d\omega \, \hbar\omega \, |\omega\rangle\langle\omega|$$
$$+ \int d\omega [\hbar\Omega(\omega)f(t)e^{i\omega_L t}|0\rangle\langle\omega| + \text{H.c.}] , \quad (2.1)$$

where for convenience the ground-state energy has been chosen to be zero, and the integrals extend over the entire

real axis (i.e., we neglect the ionization threshold), and the radiative matrix element $\Omega(\omega)$ is a double Lorentzian extension of Fano's parametrization of the autoionization resonance as discussed in Ref. 6:

$$\Omega(\omega) = \frac{\Omega_0}{(4\pi\gamma_0)^{1/2}}$$
$$\times \left[\frac{\gamma_0}{\omega - \omega_e - i\gamma_0} - \frac{i\gamma_1}{(q+i)(\omega - \omega_e - i\gamma_1)} \right] . \quad (2.2)$$

The energy of the autoionization resonance is $\hbar\omega_e$ and γ_0 is its width. Ω_0 is the Rabi frequency containing the transition-dipole moment and the electric field amplitude E_0; q is the Fano asymmetry parameter determining the shape of the resonance. The second term in Eq. (2.2) contains the background contribution; in Fano's theory the background is flat and it remains very broad in the present model ($\gamma_1 \gg \gamma_0$). The limit $\gamma_1 \to \infty$ is taken whenever it does not lead to a divergence.

The time-dependent Schrödinger equation

$$-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi = H\psi \quad (2.3)$$

splits into a set of c-number equations upon introducing the state vector $|\psi(t)\rangle$ in the form

$$|\psi(t)\rangle = \alpha(t)|0\rangle + e^{-i\omega_L t}\int d\omega\,\beta(\omega,t)|\omega\rangle . \quad (2.4)$$

For the amplitudes, we obtain the following integrodifferential equations:

$$\dot{\alpha}(t) = -i\int d\omega\,\Omega(\omega)f(t)\beta(\omega,t) , \quad (2.5a)$$

and

$$\dot{\beta}(\omega,t) = -i(\omega - \omega_L)\beta(\omega,t) - i\Omega^*(\omega)f(t)\alpha(t) . \quad (2.5b)$$

The second equation (2.5b) can be formally solved for the density amplitudes $\beta(\omega,t)$; we shall assume that the atom was in the bound state before the pulse arrived:

$$\lim_{t\to-\infty}\alpha(t) = 1, \quad \lim_{t\to-\infty}\beta(\omega,t) = 0 ,$$

and

$$\lim_{t\to-\infty}f(t) = 0 .$$

The solution is

$$\beta(\omega,t) = -i\Omega^*(\omega)\int_{-\infty}^{t}d\tau\,f(\tau)\alpha(\tau)e^{-i(\omega-\omega_L)(t-\tau)} . \quad (2.6)$$

Substituting Eq. (2.6) into Eq. (2.5a) we obtain an integrodifferential equation for $\alpha(t)$:

$$\dot{\alpha}(t) = -f(t)\int d\omega\,|\Omega(\omega)|^2\int_{-\infty}^{t}d\tau\,f(\tau)\alpha(\tau)$$
$$\times e^{-i(\omega-\omega_L)(t-\tau)} . \quad (2.7)$$

This equation can be transformed into a second-order differential equation using our parameterization of the radiative matrix element $\Omega(\omega)$ and taking the limit $\gamma_1 \to \infty$. The important simplification comes from the simple exponential form of the kernel in Eq. (2.7) after the ω integrals have been performed. The basic differential equation is

$$\ddot{\alpha} + \left[\xi - \frac{\dot{f}}{f} + bf^2\right]\dot{\alpha} + (\eta f + b\xi f + b\dot{f})f\alpha = 0 , \quad (2.8)$$

where

$$\xi = \gamma_0 + i(\omega_e - \omega_L) = \gamma_0 + i\Delta , \quad (2.9a)$$

$$\eta = \frac{\Omega_0^2}{4}\frac{(q-i)}{(q+i)} , \quad (2.9b)$$

and

$$b = \frac{\Omega_0^2}{4\gamma_0(1+q^2)} . \quad (2.9c)$$

Equation (2.8) is supplemented by the boundary conditions $\alpha(-\infty) = 1$ and $\dot{\alpha}(-\infty) = 0$; the last condition being easily obtained from Eq. (2.7).

The differential equation (2.8) contains an important special case: When $q \to \infty$, the terms proportional to b vanish in the equation; furthermore, if $\gamma_0 \to 0$ it reduces to the equation established for the two-level system in the thirties.[7]

The photoelectron spectrum is an important physical quantity which naturally arises in the present case. This spectrum is defined as

$$W(\omega,t) = |\beta(\omega,t)|^2 , \quad (2.10)$$

and can be expressed by the solution of Eq. (2.8):

$$W(\omega,t) = |\Omega(\omega)|^2\left|\int_{-\infty}^{t}d\tau\,e^{i(\omega-\omega_L)\tau}f(\tau)\alpha(\tau)\right|^2 . \quad (2.11)$$

The spectrum expressed in Eq. (2.11) is easiest to compute for $t \to \infty$. It is then related to the Fourier transform of $\alpha(t)f(t)$. In the following sections we present specific examples.

III. HYPERBOLIC-SECANT PULSE

The hyperbolic-secant pulses,

$$f(t) = \text{sech}(\gamma t) , \quad (3.1)$$

play an important role in quantum optics. They appear naturally in the study of the propagation of very short pulses through a medium of two-level atoms.[4] In fact, the evolution of a lossless two-level atom driven by a hyperbolic-secant pulse was found in 1932 by Rosen and Zener.[7] The solution for this case may be expressed as a hypergeometric function;[8] this function is also the solution for the amplitude $\alpha(t)$ in the case of a symmetric Fano profile ($q \to \infty$):

$$\alpha(t) = {}_2F_1\left[\frac{\Omega_0}{2\gamma}, \frac{-\Omega_0}{2\gamma}, \frac{\xi+\gamma}{2\gamma}; \frac{\tanh(\gamma t)+1}{2}\right] . \quad (3.2)$$

The presently discussed case is nontrivially richer than the case of a two-level system, as it contains the notion of the photoelectron spectrum. As has been shown in Ref. 9, this spectrum is multipeaked, the maximum number of peaks being determined by the value of the pulses' area defined as

$$A = \Omega_0\int_{-\infty}^{\infty}d\tau\,f(\tau) . \quad (3.3)$$

For sufficiently short pulses of area $A = 2\pi n$, the maximum number of peaks is equal to n. Thus, we see here the breakdown of the dressed-atom picture mentioned in the Introduction. The number of peaks depends on the dynamics; no longer is it determined only by the number of atomic levels participating in the interaction. There is

also no pole structure corresponding to the peaks in the spectrum. The characteristic frequencies appearing in the Fourier transform of $f(t)\alpha(t)$ reveal the positions of the maxima. An explicit formula for the electron spectrum under the condition $q = \infty$, but otherwise arbitrary pulse area detuning and pulse duration, takes the form

$$W(\omega) = \frac{|\Omega(\omega)|^2}{\gamma^2} \left| \frac{\Gamma((\gamma - i\delta)/2\gamma)\Gamma((\gamma + i\delta)/2\gamma)}{\Gamma((3\gamma + i\delta)/2\gamma)} \, _3F_2 \left[\frac{\Omega}{2\gamma}, \frac{-\Omega}{2\gamma}, \frac{\gamma + i\delta}{\gamma}, \frac{\xi + \gamma}{2\gamma}, \frac{3\gamma + i\delta}{2\gamma}; 1 \right] \right|^2 ,$$

$$\text{where } \delta = \omega - \omega_L . \quad (3.4)$$

To complete the brief but fully analytic presentation in Ref. 9, we present and discuss some numerical results obtained for the hyperbolic-secant pulse.

In Fig. 1 we show the time dependence of the photoelectron spectrum based upon the substitution of Eq. (3.2) in Eq. (2.11). To avoid a contradiction with the time-energy uncertainty principle, the physical meaning of this time-dependent spectrum should be as follows: The spectrum $W(\omega, t)$ corresponds to the pulse truncated (switched off) at time t, the observation time is actually much longer.[10] The evolution of the spectrum in Fig. 1 corresponds to a pulse evolving with a total area $A = 4 \times 2\pi$. We clearly see the development of the four-peak structure out of a single peak through double and triple peaks as the evolution of the population unfolds its nonperiodic oscillations.

We have also studied hyperbolic-secant-pulse autoionization when the atom possesses an asymmetric Fano resonance. The solution of the evolution equation (2.8) cannot be expressed by known special functions. After substitution of the new variable

$$z(t) = \frac{\tanh(\gamma t) + 1}{2} \quad (3.5)$$

into Eq. (2.8), it takes the form

$$z(1-z)\frac{d^2\alpha}{dz^2} + \left[\frac{\xi}{2\gamma} + \frac{(1-2z)}{2} + \frac{2bz}{\gamma}(1-z) \right] \frac{d\alpha}{dz}$$
$$+ \left[\frac{\eta}{\gamma^2} + \frac{b\xi}{\gamma^2} + \frac{b}{\gamma}(1-2z) \right] \alpha = 0 . \quad (3.6)$$

Equation (3.6) can be solved by the power series method:

$$\alpha = \sum_{n=0}^{\infty} a_n z^n , \quad (3.7a)$$

where

$$a_0 = 1 \text{ and } a_1 = -2[\eta + b(1+\xi)]/(1+\xi) . \quad (3.7b)$$

The ansatz (3.7) leads to a three-term recurrence formula, which can be trivially solved numerically:

$$(n+1)\left[n + \frac{\xi}{2\gamma} + \frac{1}{2} \right] a_{n+1}$$
$$+ \left[-n^2 + \frac{b(2n+1)}{\gamma} + \frac{\eta}{\gamma^2} + \frac{b\xi}{\gamma^2} \right] a_n$$
$$- \frac{2bn}{\gamma} a_{n-1} = 0 . \quad (3.8)$$

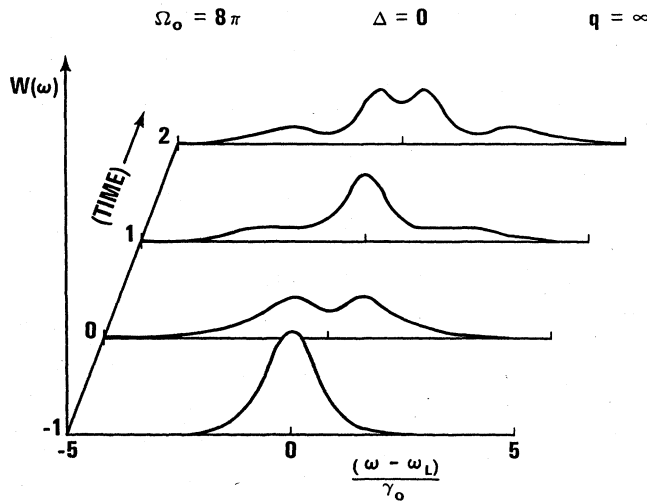$$\Omega_0 = 8\pi \qquad \Delta = 0 \qquad q = \infty$$

FIG. 1. Electron-energy spectrum $W(\omega, t)$ for a sech-pulse of area $A = 8\pi$ and $q = \infty$, $t = -1, 0, 1,$ and 2 are shown ($\Delta = 0$, $\gamma = \gamma_0$).
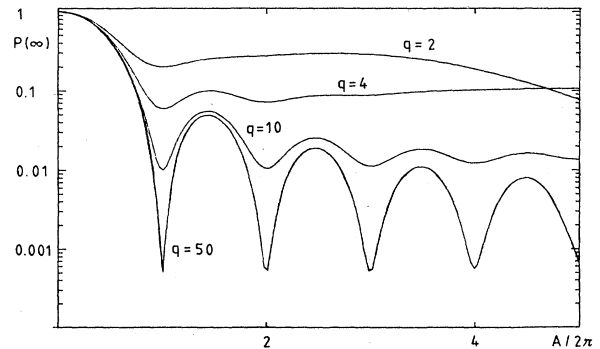

FIG. 2. Population of the ground state at $t = \infty$ as a function of the pulse areas. Three values of q are shown ($\Delta = 0$, $\gamma = \gamma_0$).
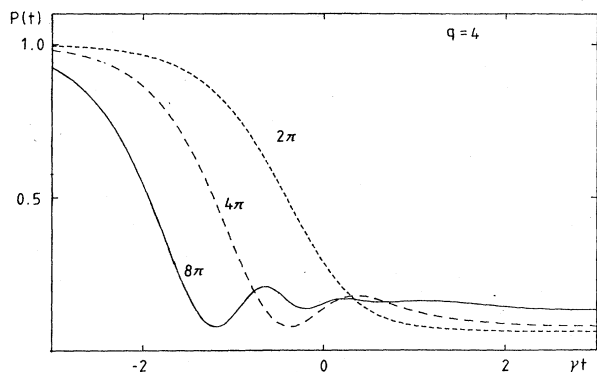
FIG. 3. Population of the ground state as a function of scaled time γt for $A = 2\pi$, 4π, and 8π ($q=4$, $\Delta=0$, $\gamma=\gamma_0$).
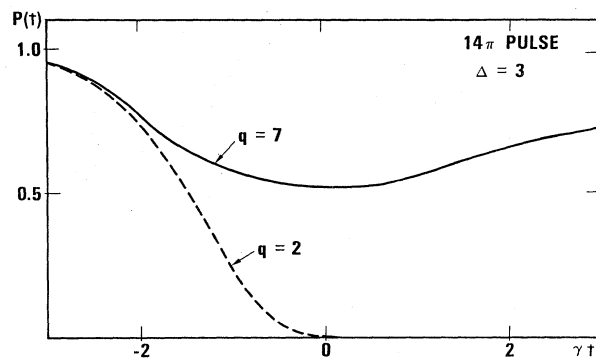

FIG. 5. Population of the ground state $P(t)$ for $A=14\pi$ and $q=2$ and 7 and detuning $\Delta=3$ ($\gamma=\gamma_0$).

The electronic spectrum $W(\omega,t)$ is given for $t \to \infty$ by the expression

$$W(\omega) = |\Omega(\omega)|^2 \left| \sum_{n=0}^{\infty} a_n I_n(\omega - \omega_L) \right|^2 , \quad (3.9)$$

where

$$I_{n+1}^{(\omega)} = \int_{-\infty}^{\infty} dt\, e^{i\omega t}[z(t)]^n f(t) . \quad (3.10)$$

The integrals $I_n(\omega)$ also fulfill a simple recursion formula:

$$I_{n+1} = \left[\frac{(2n+1)\gamma + i\omega}{2(n+1)\gamma} \right] I_n . \quad (3.11)$$

The formulas (3.8) and (3.11) are sufficient to obtain the spectrum (3.9). It is worth stressing that with the help of the finite-time version of the recurrence relation (3.11), we may also investigate $W(\omega,t)$ for finite times.

In Fig. 2 we have plotted the population of the bound state long after the pulse has passed versus the pulse area. The most important observation is that the distinguished role played by the $2\pi n$ pulses is washed out as q decreases. However, we note that the 2π pulse is distinguished even for q as small as 2.

The typical time dependence of the population for finite

q is displayed in Fig. 3. As in the $q = \infty$ case, the number of Rabi oscillations increases with the area of the pulse. In Fig. 4, a pulse with area $A = 7 \times 2\pi$ is used to demonstrate the dependence of the ground-state population on the asymmetry parameter q. As in Fig. 2, there is a clear tendency toward the suppression of oscillations as q decreases. This is easily understood, since as the low-q resonances arise in the Fano theory when the direct radiative coupling between the bound state and the continuum is strong. A similar effect of suppressing Rabi oscillations has been noticed earlier for the cw excitations.[11] The addition of detuning between the autoionizing state and the laser photon energy is shown in Fig. 5. The detuning weakens the effective coherent field strength and also suppresses the Rabi oscillations.

The photoelectron spectra have a complicated structure and are not easy to analyze. In Fig. 6, the spectra for $q=2$ and areas 2π, 4π, and 8π are shown. The general feature, noted already for $q = \infty$, that the number of peaks changes with the area of the pulse, is retained. However, due to the direct coupling of the ground state to the continuum, which suppressed the oscillations in Fig. 4, the maximal number of peaks is limited and saturates with growing area of the pulse. An additional feature of the finite q spectra is the presence of the exact Fano
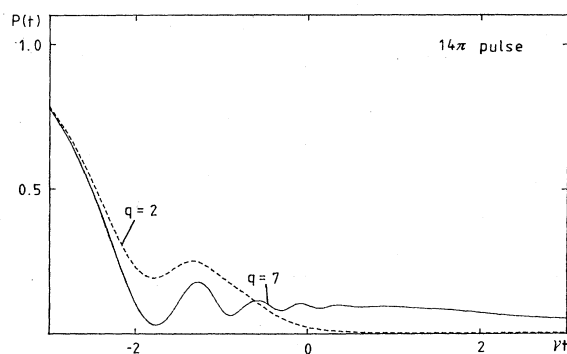

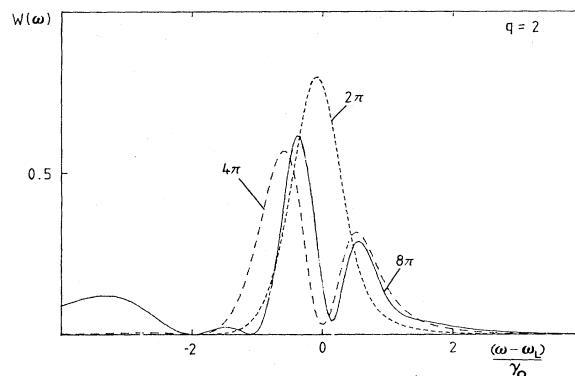FIG. 4. Population of the ground state $P(t)$ for $A = 14\pi$ and $q=2$ and 7 ($\Delta=0$, $\gamma=\gamma_0$).

FIG. 6. Electron-energy spectrum for $A=2\pi$, 4π, and 8π ($q=2$, $\Delta=0$, $\gamma=\gamma_0$).
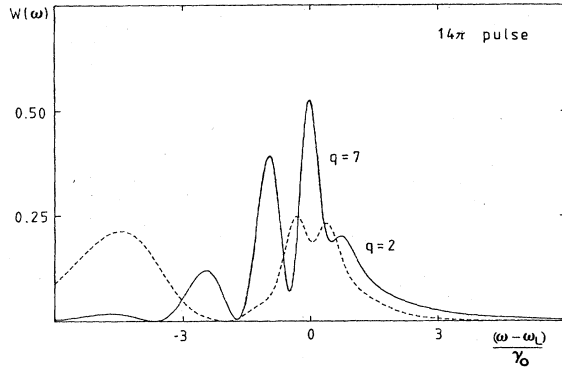
FIG. 7. Electron-energy spectrum for $A = 14\pi$ and $q = 2$ and 7 ($\Delta = 0$, $\gamma = \gamma_0$).

minimum at $\Delta\omega = -q\gamma_0$ in all the spectra. The degree of complication grows even further when we go to pulses of larger areas. The $A = 14\pi$ case is illustrated for $q = 2$ and 7 in Fig. 7. The number of maxima for $q = 2$ is three and corresponds to the above mentioned saturation. The number of peaks for $q = 7$ is five and there is an extra structure far in the wing at $\Delta\omega = -7\gamma_0$ left out of the figure.

IV. EXPONENTIAL PULSES

In this section we discuss various cases of the pulses composed of exponentially time-dependent fields. The nice feature of such pulses is that explicit analytic formulas are available, both for the evolution of the population and the photoelectron spectrum even for finite q. We present the results for the following specific cases.

(i) Exponentially growing pulse:

$$f(t) = \begin{cases} e^{\gamma t}, & \text{for } t \leq 0, \\ 0, & \text{otherwise}. \end{cases} \tag{4.1}$$

(ii) Suddenly switched-on, exponentially decaying pulse:

$$f(t) = \begin{cases} 0, & \text{for } t \leq 0, \\ e^{-\gamma t}, & \text{otherwise}. \end{cases} \tag{4.2}$$

(iii) Symmetric exponential pulse:

$$f(t) = e^{-\gamma |t|}. \tag{4.3}$$

(iv) Exponentially switched-on CW signal:

$$f(t) = \begin{cases} e^{\gamma t}, & \text{for} + t \leq 0, \\ 1, & \text{otherwise}. \end{cases} \tag{4.4}$$

The fundamental equation (2.8) is transformed into one for the confluent hypergeometric function upon change of variables ($e^{\gamma t}$ is the variable). The solutions for the cases (i) to (iv) can be all expressed in terms of the confluent hypergeometric function.

Consider the exponentially growing pulse; for this case the amplitude $\alpha(t)$ is given by

$$\alpha(t) = {}_1F_1(a, c; -(b/2\gamma)e^{2\gamma t}), \tag{4.5}$$

where ${}_1F_1$ is the hypergeometric function[7] and

$$a = \frac{1}{2} + \frac{1}{2\gamma}(\gamma_0 + i\Delta) + \frac{\gamma_0}{2\gamma}(q - i)^2 = \gamma + \xi + \frac{\gamma_0}{2\gamma}(q - i)^2, \tag{4.6}$$

$$c = \frac{1}{2} + \frac{1}{2\gamma}(\gamma_0 + i\Delta) = \frac{\gamma + \xi}{2\gamma}.$$

For the symmetric Fano profile ($q \to \infty$), the solution (4.1) reduces to[11]

$$\alpha(t) = [(\Omega_0/2\gamma)e^{\gamma t}]^{(\gamma - \xi)/2\gamma}$$
$$\times \Gamma((\gamma + \xi)/2\gamma)J_{(\xi - \gamma)/2\gamma}((\Omega_0/2\gamma)e^{\gamma t}), \tag{4.7}$$

where $J_\nu(x)$ is the Bessel function and $\Gamma(x)$ is the Euler gamma function.

Substituting (4.1) into the formula for the photoelectron spectrum, we get

$$W(\omega, t > 0) = W(\omega, t = 0)$$
$$= \left| \frac{\Omega(\omega)}{2\gamma} \frac{{}_2F_2(a, \mathcal{X}, c, \mathcal{X} + 1; -b/2\gamma)}{\mathcal{X}} \right|^2, \tag{4.8}$$

where

$$\mathcal{X} = \frac{1}{2} + i\frac{(\omega - \omega_L)}{2\gamma}, \tag{4.9}$$

and ${}_2F_2$ is the generalized hypergeometric function.[8] It is worth noting, that the spectrum for $t = 0$ and a definite value of the Rabi frequency Ω_0 is the same as the spectrum for $t = \tau$ and $\tilde{\Omega}_0 = \Omega_0 e^{-\gamma \tau}$. Therefore, the discussion of the $t = 0$ spectra dependence on Ω_0 is equivalent to the discussion of the time dependence of the spectra of the fixed value of Ω_0.

For the exponentially decaying pulse[9] [case (ii)] we proceed in a similar manner. The proper initial conditions in this case are $\alpha(0) = 1$, $\dot{\alpha}(0) = 0$. The solution for $\alpha(t)$ reads:[12]

$$\alpha(t) = D_1 \, {}_1F_1(1 - a, 1 - c; (b/2\gamma)e^{-2\gamma t}) + D_2[(b/2\gamma)e^{-2\gamma t}]^c \, {}_1F_1(c + 1 - a, c + 1; (b/2\gamma)e^{-2\gamma t}). \tag{4.10}$$

Using the initial condition and the formula for the Wronskian, the coefficients D_1 and D_2 are

$$D_1 = {}_1F_1(a - 1, c; -b/2\gamma), \tag{4.11a}$$

$$D_2 = \frac{a - 1}{c(1 - c)} \left[\frac{b}{2\gamma} \right]^{1 - c} {}_1F_1(a - c, 2 - c; -b/2\gamma). \tag{4.11b}$$

The spectrum of outgoing electrons as $t \to \infty$ is given by

$$W(\omega) = \left| \frac{\Omega(\omega)}{2\gamma} \right|^2 \left| \frac{D_1}{\chi^*} \, _2F_2(1-a,\chi^*,1-c,\chi^*+1;b/2\gamma) \right.$$

$$\left. + D_2 \left[\frac{b}{2\gamma} \right]^c \frac{1}{(\chi^*+c)} \, _2F_2(c+1-a,\chi^*+c,c+1,\chi^*+c+1;b/2\gamma) \right|^2 . \tag{4.12}$$

The symmetric pulse [case (iii)] is interesting for two reasons: (a) It can be compared with our earlier results for the hyperbolic-secant pulse; and (b) the exponential symmetric pulse has a Lorentzian-Fourier spectrum. The same form of the spectrum is implied by the widely used phase diffusion model of the single-mode cw laser. This model was used in the context of strong-field autoionization. Our present results allow for the comparison with that case as well.

The solution in the present case is obtained by combining the results of (i) and (ii). The evolution (4.1) determines the initial values $\alpha(0)$ and $\dot{\alpha}(0)$ used for the evolution (4.6). The long-time photoelectron spectrum reads

$$W(\omega) = \left| \frac{\Omega(\omega)}{2\gamma} \right|^2 \left| \frac{1}{\chi} \, _2F_2(a,\chi,c,\chi+1;-b/2\gamma) + \frac{\widetilde{D}_1}{\chi^*} \, _2F_2(1-a,\chi^*,1-c,\chi^*+1;b/2\gamma) \right.$$

$$\left. + \frac{\widetilde{D}_2}{\chi^*+c} \left[\frac{b}{2\gamma} \right]^c \, _2F_2(c+1-a,\chi^*+c,c+1,\chi^*+c+1;b/2\gamma) \right|^2 . \tag{4.13}$$

The new constants $\widetilde{D}_1$ and $\widetilde{D}_2$ are given by

$$\widetilde{D}_1 = D_1 \, _1F_1(a,c;-b/2\gamma) + \frac{ab^2}{2\gamma c^2} \, _1F_1(a+1,c+1;-b/2\gamma) \, _1F_1(a,c+1;-b/2\gamma) , \tag{4.14a}$$

$$\widetilde{D}_2 = D_2 \, _1F_1(a,c;-b/\alpha\gamma) + \frac{ab^2}{2\gamma c^2} \, _1F_1(a+1,c+1;-b/2\gamma) \, _1F_1(c-a,1-c;-b/2\gamma) . \tag{4.14b}$$

Finally, we consider case (iv) of an exponentially switched-on cw signal. In contrast to the previous cases, the notion of the pulse area is not applicable. The assumed form of $f(t)$ allows for proper treatment of the switch-on problem, which has been neglected in the earlier treatments. The solutions of (i) can now be used to determine the initial conditions $\alpha(0)$ and $\beta(\omega,0)$ for the constant laser signal which then is solved in the usual manner in Laplace transform domain.

The long time spectrum is given by

$$W(\omega) = \frac{|\Omega(\omega)|^2}{\gamma_0^2} \left| \frac{[\gamma_0 \, _1F_1(a,c;-b/2\gamma) + \gamma_0 \eta \, _1F_1(a,c+1;-b/2\gamma)/2\gamma c(i\omega-\gamma_0)]}{\mathscr{H}(-i(\omega-\omega_L))} \right.$$

$$\left. + \frac{\gamma_0}{2\gamma\chi} \, _2F_2(a,\chi,c,\chi+1;-b/2\gamma) \right|^2 , \tag{4.15}$$
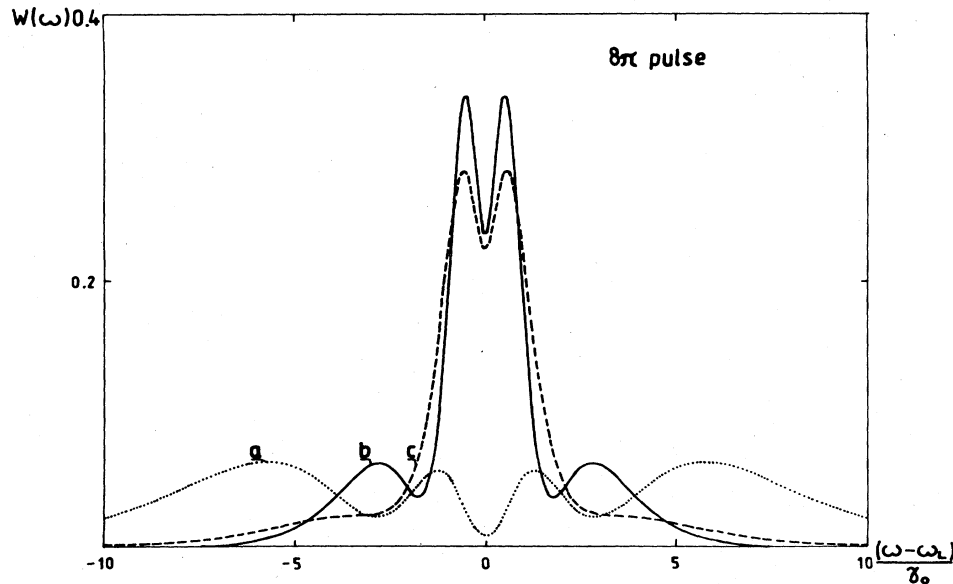

FIG. 8. Electron-energy spectrum for $A=8\pi$ ($q=\infty$, $\Delta=0$, $\gamma=\gamma_0$): curve a, decreasing-exponential pulse; b, cusp-shaped pulse; c, increasing-exponential pulse.
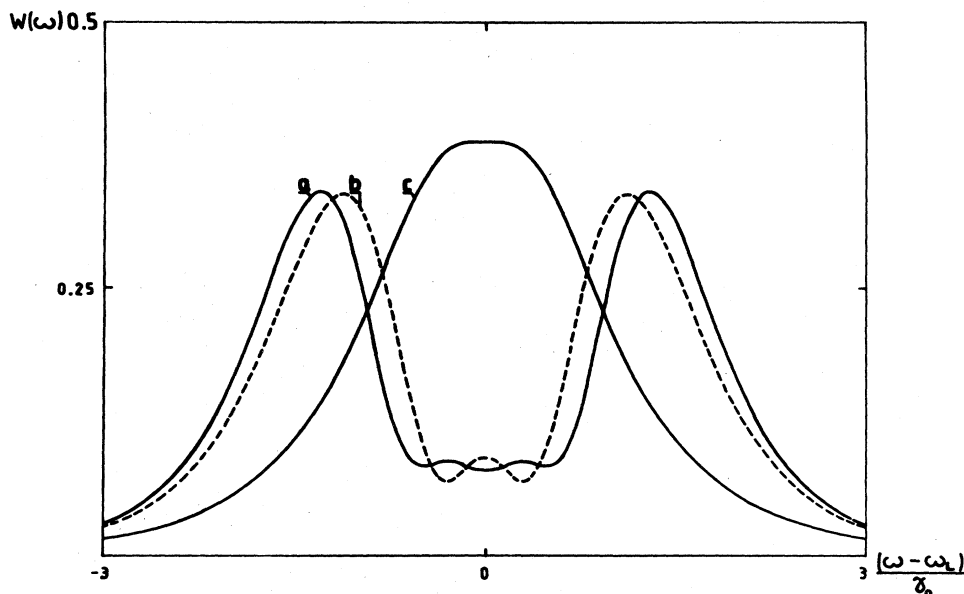
FIG. 9. Electron-energy spectrum for decreasing-exponential pulses with $\Omega_0 = 2\pi\gamma_0$ and three values of the pulse width corresponding to curve a, $A = 12\pi$; b, $A = 8\pi$; c, $A = 2\pi$.

where the resolvent of the cw signal problem is given by

$$\mathcal{H}(z) = z + b + \frac{\eta}{z - i\Delta + \gamma_0} . \tag{4.16}$$

Of course, in the limit of the instantaneous switch-on ($\gamma \to \infty$), we get back the formula from Ref. 11.

The above analytic results are illustrated by several figures. In Fig. 8 we present photoelectron spectra corresponding to pulses of area 8π and different pulse shapes: the increasing exponential, the decreasing exponential, and the cusp. The last of the three cases bears a striking resemblance to the hyperbolic-secant pulses discussed in Sec. III.

The increasing-exponential pulse produces a spectrum where the central or nearly elastic part is more pronounced; whereas, the decreasing exponential pulse produces enhanced wings of the spectrum. The interpretation of this fact is rather simple. In the first case, a substantial part of the ionization process occurs while the field is weak, i.e., low Rabi frequency. In the second case, a substantial portion of the ionization process occurs at strong fields. In fact, the result for the decreasing-exponential pulse allows for a smooth transition to the well studied case of the suddenly switched-on cw field. The limiting spectrum is the Autler-Townes doublet. In Fig. 9 we exhibit this limiting behavior for the spectra with pulse areas 2π, 8π, and 12π. The Rabi frequency is held constant and the pulse width is appropriately increased. The passage to the Autler-Townes doublet is clearly visible.

V. SUMMARY

We have presented exact solutions for particular laser pulse envelopes impinging upon an autoionizing atom.

The features appearing in the ground-state population and photoelectron spectrum are dependent on several parameters. First, there is a dependence on the relation between the rates γ, γ_0, and Ω_0. Second, the physical quantities are altered by the relative coupling between the ground-continuum- and the ground-autoionizing-state transitions represented by the parameter q; and third, an important role is played by the detuning of the laser from the autoionization-state transition frequency.

Since so many parameters are involved in the theory, it is difficult to make general statements concerning the behavior of the physical quantities. However, some specific remarks are possible. For small area pulses, the duration of the pulse is insufficient to establish a multipeak structure in the photoelectron spectrum. A time of order Ω_0^{-1} is needed to dress the states in a sharp turn-on of a cw signal, so here also this time must be available for the atom to be affected by a strong field. Also, a large area pulse may not produce multiple peaks in the spectrum; this structure is absent if $\Omega_0 < \gamma_0$; in this case, the time scale for leaking population to the continuum is faster than the time scale for recycling population back to the ground state. Furthermore, the addition of a large detuning and interference effects (finite q) also washout the structure of the spectrum.

There remains still the problem of introducing spontaneous emission and computing the photoemission spectrum. This is a very difficult task for which even the two-level atom has not yet been solved in general. However, recent progress has been made on the two-level problem[13] and a solution of this interesting problem for the autoionizing atom in the limit of large radiative lifetime is presented in another publication.[14]

ACKNOWLEDGMENTS

*On leave from Institute for Theoretical Physics, Polish Academy of Sciences, PL-02-668 Warsaw, Poland.

†Present address: Physics Department, Rensselaer Polytechnic Institute, Troy, NY 12181.

[1](a) P. Lambropoulos, Appl. Opt. **19**, 3926 (1980); P. Lambropoulos and P. Zoller, Phys. Rev. A **24**, 379 (1981); (b) K. Rzążewski and J. H. Eberly, Phys. Rev. Lett. **50**, 417 (1981); Phys. Rev. A **27**, 2026 (1981).

[2](a) G. S. Agarwal, S. L. Haan, K. Burnett, and J. Cooper, Phys. Rev. A **26**, 2277 (1982); (b) J. W. Haus, M. Lewenstein, and K. Rzazweski, *ibid.* **28**, 2269 (1983) and references therein; (c) P. E. Coleman and P. L. Knight, J. Phys. B **15**, L235 (1982) and corrigendum J. Phys. B **15**, 3191 (1982).

[3]See, for instance, C. Cohen-Tannoudji, in *Laser Spectroscopy*, edited by S. Haroche (Springer, New York, 1975).

[4]L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975).

[5]A preliminary account for our results has been presented by K. Rzążewski, in *Proceedings of the Fifth Rochester Conference on Coherence and Quantum Optics, Rochester, 1983*, in *Coherence and Quantum Optics V*, edited by L. Mandel and E. Wolf (Plenum, New York, 1984), p. 187.

[6]U. Fano, Phys. Rev. **124**, 1866 (1961).

[7]N. Rosen and C. Zener, Phys. Rev. **40**, 502 (1932); see also, R. T. Robiscoe, Phys. Rev. A **17**, 247 (1978); A. Bambini and P. R. Berman, *ibid.* **23**, 2496 (1981).

[8]All special functions used in this paper and their relevant properties can be found in *Batemen Manuscript Project*, edited by A. Erdelyi (McGraw-Hill, New York, 1953), Vol. I.

[9]K. Rzążweski, Phys. Rev. A **28**, 2565 (1983).

[10]The same viewpoint was adopted for square pulses in J. W. Haus, M. Lewenstein, and K. Rzążweski, J. Opt. Soc. Am. B **1**, 641 (1984).

[11]D. Agassi, K. Rzążweski, and J. H. Eberly, Phys. Rev. A **28**, 3648 (1983).

[12]This result for the two-level system was first obtained by P. M. Radmore, Phys. Lett. **87A**, 285 (1982).

[13]K. Rzążewski and M. Florjanczyk, J. Phys. B **17**, L509 (1984).

[14]M. Lewenstein, J. Zakrzewski, and K. Rzążewski (unpublished).