Electron-atom elastic shadow scattering

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The interference between scattered and unscattered elastic waves in beam-beam scattering is investigated in the small-angle limit. It is shown that the scattered intensity averaged over the scattering volume leads to a *diminution* of the forward scattering proportional to the total scattering cross section or imaginary part of the elastic scattered amplitude with angular dependence determined by the shape of the projectile and target-beam-intensity distributions. This effect *cannot* explain earlier observations of forward scattering *enhancement* observed by Geiger and Morón-León [Phys. Rev. Lett. **42**, 1336 (1979)].

It is well known that the intensity of elastic scattering of a single projectile particle by a single target particle in a plane-wave formalism is proportional to[1]

$$I \propto 1+\frac{2}{r}\cos[kr(1-\cos\theta)+\eta(\theta)]|f(\theta)|+\frac{|f(\theta)|^2}{r^2}\ , \qquad (1)$$

where r is the distance between the scattering point and the detector, k is the incident projectile momentum, θ is the scattering angle, and $\eta(\theta)$ is the phase of the scattered amplitude $f(\theta)$. The average over all angles of the interference term in Eq. (1) was shown by Schiff to lead to the optical theorem.[1] Hence, the average of the interference term over all angles is given by

$$\int d\Omega \frac{2}{r}\cos[kr(1-\cos\theta)+\eta(\theta)]|f(\theta)| \cong -\frac{4\pi}{kr^2}\,\mathrm{Im}f(0)\ , \qquad (2)$$

which points out that the major angular effect of this term is to decrease the forward scattering which results in the formation of a true shadow. For single-projectile–single-target scattering the zero-angle contribution from the interference term is $(2/r)\,\mathrm{Re}f(0)$ which is positive. However, this term does not contribute significantly to the result obtained by averaging over some angular range about zero due to the presence of the $\sin\theta$ term in the differential solid angle $d\Omega$.

Normally the contribution from the interference term to the differential cross section is not considered. The reason is that the product kr for electron scattering is of the order 10^8–10^{12} and, hence, the interference is highly oscillatory and averages to zero over any finite-sized projectile detector.

On the other hand, even if $kr \sim 10^{12}$, this term is still significant for sufficiently small scattering angles (μrad range). In fact the interference term is generally on the order of 10^8–10^9 times larger in the small-angle limit than the usually observed $[|f(\theta)|^2/r^2]$ scattering term. Hence, if one attempts to measure the differential cross section close to the incident beam there may always be some scattering events taking place close to the edge of the projectile beam for which the effective scattering angle into the detector is in the μrad range. The probability for such events to contribute significantly to the scattering need only be as large as 10^{-8} times the probability for scattering events in the projectile-beam center. This observation suggests that the effect of projectile-beam and target-beam sizes may play a significant role in the nature of the contribution of the interference term to the elastic differential scattering cross section. The purpose of this Brief Report is to investigate this possibility.

A beam-beam scattering geometry is assumed with projectile and target beams with respective beam distributions $p(x,y,z)$ and $t(x,y,z)$.[2] Only p is normalized to unity. The z axis is parallel to the incident projectile direction and the detector is placed at the point (x_0,y_0) at a distance r_0 from the origin in the center of the interaction region. We consider the two possible extreme orientations of the target-beam axis $t_x(x,y,z)$ and $t_y(x,y,z)$ for the beam parallel to the x and y directions, respectively. These distributions lead to inequivalent results due to lack of rotational symmetry about the z axis of the product $p(x,y,z)t_y(x,y,z)$ or $p(x,y,z)t_x(x,y,z)$. The average of the interference term can be written as

$$\left\langle \frac{2}{r}\cos[kr(1-\cos\theta)+\eta(\theta)]|f(\theta)|\right\rangle = \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_{-(L<r_0)}^{r_0}dz\,p(x,y,z)\begin{Bmatrix} t_x(x,y,z)\\ t_y(x,y,z)\end{Bmatrix}$$
$$\times\{\exp[ikr(1-\cos\theta)+i\eta(\theta)]|f(\theta)|+\text{c.c.}\}/r\ , \qquad (3)$$

where both r and θ are functions of x, y, and z as

$$r(1-\cos\theta)=\frac{r_0}{2}[(u_0-u)^2+(v_0-v)^2]/(1-\omega)\ , \qquad (4)$$

with

$$u_0=\frac{x_0}{r_0},\quad v_0=\frac{y_0}{r_0},\quad u=\frac{x}{r_0},\quad v=\frac{y}{r_0},\quad \omega=\frac{z}{r_0}\ .$$

For the $1/r$ term the approximation

$$\frac{1}{r}=\frac{1}{r_0(1-\omega)}\left\{1-\frac{(u_0-u)^2+(v_0-v)^2}{2(1-\omega)^2}+\cdots\right\} \qquad (5)$$

is used with the z-direction integration limited by the detector (r_0) in the positive z direction and the position of the projectile source in the negative z direction $(L<r_0)$. It is

assumed that the distribution p vanishes in the x and y directions and t vanishes in the z direction before any experimental impediments are encountered and that both distributions are slowly varying in comparison to the term involving $\cos[kr(1-\cos\theta)+\eta(\theta)]$. The effective scattering angle θ_s, is given as $(x_0^2+y_0^2)^{1/2}/r_0$ in radians.

Because $kr_0/2(1-z/r_0)$ varies from at least 10^8 to ∞ it is clear that as long as $\eta(\theta)$ is a slowly varying function of θ in the small-angle limit or can be characterized by an expansion of the form

$$\eta(\theta)=\eta(0)+\frac{\theta^2}{2}\eta''(0) \ , \tag{6}$$

the averages over x and y in Eq. (3) are classic examples for application of the stationary phase method of integral evaluation[3] with accuracies of at least three to four significant figures. By use of the substitutions $m=1/(1-\omega)$ and $K=\frac{1}{2}[kr_0m+\eta''(0)m^2]$, Eq. (3) reduces to

$$\left\langle \frac{2}{r}\cos[kr(1-\cos\theta)+\eta(\theta)]|f(\theta)|\right\rangle = r_0^2\int_{1/(1+L/r_0)}^{\infty}\frac{dm}{m}\int_{-\infty}^{\infty}du\int_{-\infty}^{\infty}dv\,p(u,v,m)\begin{Bmatrix} t_x(u,v,m)\\ t_y(u,v,m)\end{Bmatrix}$$
$$\times\langle|f(\theta)|e^{i\eta(\theta)}\exp\{iK[(u_0-u)^2+(v_0-v)^2]\}$$
$$\times\{1-\tfrac{1}{2}m^2[(u_0-u)^2+(v_0-v)^2]+\cdots\}+\text{c.c.}\rangle \ . \tag{7}$$

Application of the stationary phase method of integral evaluation to Eq. (7) yields[3]

$$\left\langle \frac{2}{r}\cos[kr(1-\cos\theta)+\eta(\theta)]|f(\theta)|\right\rangle$$
$$= -\frac{4\pi r_0}{k}\,\mathrm{Im}f(0)\int_{1/(1+L/r_0)}^{\infty}\frac{dm}{m^2}p(x_0,y_0,m)\begin{Bmatrix} t_x(x_0,y_0,m)\\ t_y(x_0,y_0,m)\end{Bmatrix}\frac{1}{1+\eta''(0)m/kr_0}+O\left[\frac{1}{K^{3/2}}\right] \ , \tag{8}$$

where, since $\eta''(0)/kr_0$ is generally of order 10^{-9}, one has finally

$$\left\langle \frac{2}{r}\cos[kr(1-\cos\theta)+\eta(\theta)]|f(\theta)|\right\rangle \cong -r_0\frac{4\pi\,\mathrm{Im}f(0)}{k}\int_{-L/r_0}^{1}d\omega\,p(x_0,y_0,\omega)\begin{Bmatrix} t_x(x_0,y_0,\omega)\\ t_y(x_0,y_0,\omega)\end{Bmatrix} \ . \tag{9}$$

Equation (9) at first glance may seem strange since in the zero-angle limit the result does not approach the zero-angle limit of Eq. (1). The reason for this is that the average over the projectile beam can always be cast into cylindrical coordinates with origin directly over the detector at the point (x_0,y_0) which supresses the zero-angle contribution in the same way as in the case of Eq. (2) where the average is taken over scattering angles.

We note that the interference term only contributes to the scattering when the detector is in some portion of the unscattered beam since it is proportional to the unscattered beam intensity $i_0p(x_0,y_0,\omega)$ at the detector. Since the detector is inside the beam the total intensity seen by the detector will be

$$i(\eta_0)=i_0\langle p(x_0,y_0,z)\rangle_z-i_0\eta_0\frac{4\pi\,\mathrm{Im}f(0)}{k}$$
$$\times\langle p(x_0,y_0,z)t(x_0,y_0,z)\rangle_z$$
$$+i_0\eta_0\frac{|f(0)|^2}{r_0^2}\langle p(x,y,z)t(x,y,z)\rangle \ , \tag{10}$$

assuming $f(\theta)\sim f(0)$. Here, $\langle\ \rangle$ signifies integration over x,y,z, $\langle\ \rangle_z$ over z only; the direction of the target beam has not been specified; η_0 is the number density of the target gas in the scattering region and i_0; the total projectile-beam current. Note that the first two terms on the right-hand side of Eq. (10) are just the first two terms in the expansion of the Beer-Lambert law in the low-pressure limit for the transmitted intensity I written as $I=I_0e^{-nl\sigma_{\text{tot}}}$, with σ_{tot} the total scattering cross section, $n=\eta_0$, and l is the path length through the gas. The latter may be defined as

$$l=\langle p(x_0,y_0,z)t(x_0,y_0,z)\rangle_z/\langle p(x_0,y_0,z)\rangle_z \ ,$$

with the current I_0 as $i_0\langle p(x_0,y_0,z)\rangle_z$. Hence, if the current recorded in a beam trap of radius a in the absence of gas ($\eta_0=0$), $I(0)$, is divided out, the result for parallel Gaussian beams will be

$$1-\frac{I(\eta_0)}{I(0)}=\eta_0\sqrt{\pi/2}\,\sigma_t\frac{4\pi}{k}\mathrm{Im}f(0)$$
$$-\frac{\eta_0\sqrt{2\pi}(\pi a^2)\sigma_t|f(0)|^2}{[1+(\sigma_p/\sigma_t)^2]^{1/2}(1-e^{-a^2/2\sigma_p^2})r_0^2} \ , \tag{11}$$

where σ_t and σ_p are the standard deviations of the target and projectile beams, respectively. For a Gaussian-electron-beam standard deviation of 200 μm, a flow rate of 5×10^{18} molecules/sec, an average molecular velocity of 5×10^4 cm/sec (N_2 at 298 K), and a gas beam standard deviation of 1 mm, the beam attenuation by the first term on the right-hand side of the equal sign in Eq. (11) should be of the order of 1% for a total crosss section of 10^{-17} cm^2, with the second term on the right-hand side making a negligible contribution with a 1-mm beam trap diameter ($2a$). This result is in qualitative agreement with the experimental observations of Wellenstein.[4]

It is also of interest to consider the ratio R of the interference term to the scattering term. Assuming parallel Gaussian beams and constancy of the scattered amplitude in the

TABLE I. The ratio of interference scattering to regular scattering for 25- and 35-keV electrons scattered from atomic helium with a scattering center to detector distance of 50 cm for various Gaussian projectile, σ_p, and target, σ_t, standard deviations. The term in parentheses denotes the distance of the detector from the center of the projectile beam in projectile-beam standard deviations.

		25 keV		35 keV
θ_s (mrad)	$\sigma_p=100$ μm, $\sigma_t=1$ mm, t_y	$\sigma_p=500$ μm, $\sigma_t=1$ mm, t_x	$\sigma_p=500$ μm, $\sigma_t=1$ mm, t_y	$\sigma_p=200$ μm, $\sigma_t=1$ mm, t_y
1	-1.08 $(5\sigma_p)$	-7.9×10^4 $(1\sigma_p)$	-7.0×10^4 $(1\sigma_p)$	1.9×10^3 $(2.5\sigma_p)$
2	-6×10^{-17} $(10\sigma_p)$	-1.8×10^4 $(2\sigma_p)$	-1.1×10^4 $(2\sigma_p)$	1.1×10^{-1} $(5\sigma_p)$
3		-1.4×10^3 $(3\sigma_p)$	-4.7×10^2 $(3\sigma_p)$	1.0×10^{-8} $(7.5\sigma_p)$
4		-4.4×10^1 $(4\sigma_p)$	-5.9 $(4\sigma_p)$	
5		-4.8×10^{-1} $(5\sigma_p)$	-2.1×10^{-2} $(5\sigma_p)$	
6		-2×10^{-3} $(6\sigma_p)$	-2.2×10^{-5} $(6\sigma_p)$	

small-angle region the ratio is given by

$$R=-\frac{\frac{4\pi}{k}\,\mathrm{Im}f(0)r_0^2\,[1+(\sigma_p^2/\sigma_t^2)]^{1/2}}{|f(0)|^2(2\pi\sigma_p^2)}\times\exp\left[-\frac{r_0^2\theta_s^2}{2}\left(\frac{1}{\sigma_p^2}+\frac{1}{\sigma_t^2}\right)\right]. \tag{12}$$

For He with $\sigma_{tot}\sim1.4\times10^{-18}$ cm^2 (Ref. 5) and $|f(0)|\sim0.439\times10^{-8}$ cm (Ref. 6) one obtains the results given in Table I for various choices of the parameters. The values for the experiment reported by Geiger and Morón-León[7] of $r_0\sim50$ cm, $\sigma_p\sim7$ μm, and $\sigma_t\sim3$ mm do not give a measurable effect in the milliradian angular range. The values used by Fink, Wellenstein, and Coffman[8] are given in the fourth column in Table I and suggest that the effect would also not be a factor in their experiment. Note that σ_p and σ_t refer to beam dimensions in the scattering region. In addition, if Lorentzian distributions are used the interference effect should be broadened to even larger scattering angles. Table I illustrates the total dominance of the interference term inside the scattered beam.

In summary, it has been shown that the interference scattering (1) averaged over a finite scattering volume leads to true shadow scattering (diminution of the scattered intensity in the forward direction), (2) should be measurable with current experimental capabilities,[4,9] and (3) can in no way explain the enhancement of elastic scattering in the forward direction observed by Geiger and Morón-León.[7]

Observation of the beam attenuation in the forward direction under single scattering conditions appears feasible. Absolute determination of the cross section requires careful characterization of the target beam which is normally a difficult task. On the other hand it may be possible to obtain the projectile energy dependence on the relative total cross section without characterization of the target-beam distribution. This could provide valuable information on certain parameters characterizing the elastic and inelastic scattering[5] at high projectile energies.

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[1]L. I. Schiff, Prog. Theor. Phys. **11**, 288 (1954).

[2]K. Rubin and I. Efremov have considered this same interference term using momentum wave-packet descriptions for both projectile and target particles. Their premise was that non-plane-wave effects could extend the influence of the interference term to larger scattering angles. Their experiments, however, differ from the ones discussed here in that the target atom beam was detected rather than the projectile electron beam. The nature of their electron beam was also significantly different. They concluded that the interference term would reduce the forward scattering. As shown in this Brief Report the effect of the interference term may be extended to larger angles, but not outside the boundaries of the unscattered beam, if a plane-wave description is employed. K. Rubin and I. Efremov, in *Coherence and Correlation in Atomic Collisions*, edited by H. Kleinpoppen and J. F. Williams (Plenum, London, 1980), p. 651.

[3]B. Friedman, *Lectures on Applications-Oriented Mathematics* (Holden-Day, San Francisco, 1969), pp. 93–97.

[4]H. F. Wellenstein (private communication). He has reported observation of $\sim1\%$ beam attenuation in single scattering experiments using keV electrons and conditions similar to those used here.

[5]See M. Inokuti, Y. K. Kim, and R. L. Platzman, Phys. Rev. **164**, 55 (1967), for the total inelastic scattering; M. Inokuti and M. R. C. McDowell, J. Phys. B **7**, 2382 (1974), for the elastic scattering cross sections.

[6]Interpolated from the partial-wave values given in *International Tables for X-Ray Crystallography,* edited by J. A. Ibers and W. C. Hamilton (Kynoch, Birmingham, 1974), p. 182.

[7]J. Geiger and D. Morón-León, Phys. Rev. Lett. **42**, 1136 (1979).

[8]M. Fink, H. F. Wellenstein, and D. Coffman (private communication).

[9]It may be advantageous to carry out such experiments using a pulsed target beam with a position-sensitive electron detector. See Ref. 2 above.