Early-stage domain formation and growth in one-dimensional systems

F. de Pasquale, P. Tartaglia, and P. Tombesi
Dipartimento di Fisica, Università di Roma "La Sapienza," Piazzale Aldo Moro,2 I-00185 Roma, Italy
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We study, theoretically and numerically, domain formation and growth in a one-dimensional system with nonconserved order parameter, which evolves from an initial unstable state, through the field model equation of Ginzburg and Landau. We are able to distinguish two time regimes and to give an estimate of the separation time, associated with early-stage domain formation and their final slow growth related to domain-wall interaction.

I. INTRODUCTION

The study of domain formation and growth in systems quenched below a critical point is one of the most challenging problems in nonequilibrium statistical mechanics. From the theoretical point of view systems of this kind have been extensively studied[1–3] since the pioneering works of Lifshitz[4] and Allen and Cahn.[5] More recently there have been many investigations using computer simulation especially for the late-stage development of domain growth.[1,6] Finally, several experiments have been done on various systems with particular preference for systems with a conserved order parameter, such as metal alloys and binary fluids.[1,7] What can be inferred from an analysis of the quoted literature is the existence of different time regimes, at least two, namely, the early stage of domain formation dominated by fluctuations followed by their growth and dominated by their interaction.[8] As a consequence of the existence of different stages of developments one can guess a strong dependence of the second stage, the growth, on the dimensionality of the system. These two regimes can be considered as being typical of the evolution of the system from an unstable configuration. The final approach to equilibrium dynamics[9] is a later regime which, we feel, needs further investigations. In particular it would be worth understanding the mechanisms responsible for the fact that the domain growth stops, since in one-dimensional systems with short-range interactions no phase transition is possible at finite temperature.

In this work we report the study of the kinetics of phase separation of a one-dimensional nonlinear dissipative stochastic model of the time-dependent Ginzburg-Landau (TDGL) type for a single-component nonconserved order parameter.[10] The theoretical technique we have adopted is the generalization of the quasideterministic theory (QDT) that was developed and successfully used in the case of systems without spatial diffusion and close to an unstable state.[11,12] This approach was applied to the study of the kinetics of systems such as the laser near threshold. We have also developed a scheme, based on the work of Rao, Borkwankar, and Ramkrishna,[13] to solve numerically the stochastic differential equation with space diffusion. The main result that comes from the numerical solution is the existence of at least two time regimes in the formation and growth of the domains. The early stage of formation is dominated essentially by diffusion, i.e., it has the characteristic $t^{1/2}$ time behavior, while the domain growth has the much slower $\ln(t)$ behavior. This latter result is in perfect agreement with the recent statistical mechanics theory of interacting kinks developed by Kawasaki and Nagai.[3] The overall behavior is in turn in good qualitative agreement with the recent experiment on layered dilute antiferromagnetic systems by Ikeda.[7] On the other hand, the QDT is able to reproduce the simulated data completely in the first time regime, i.e., the whole early stage of formation of domains. It is unable to explain the successive slow growth, although it is capable of giving a rather precise estimate of the time needed for the domains to form. In a rough sense this time can be considered to be the mean first passage time starting from the initial unstable state to the final stable one. It can be evaluated since the theoretical approximation can be controlled up to that time.

II. THE TDGL MODEL AND THE QDT

The field model equation one can use to describe in a simple way the complicated problem of ordering of a system with nonconserved order parameter is the TDGL model.[14] We consider the situation of a doubly degenerate ground state and an initially unstable configuration of a one-dimensional scalar field $\varphi(x,t)$. Strictly speaking, in one space dimension it is impossible to prepare such an unstable situation. However, it seems plausible that situations of this kind could arise in quasi-one-dimensional systems such as those investigated experimentally by Ikeda.[6,7]

We start then with the following stochastic differential equation (model A in the literature[15]):

$$\frac{\partial}{\partial t}\varphi(x,t)=-L\frac{\delta\mathscr{H}}{\delta\varphi(x,t)}+\sqrt{\epsilon}\xi(x,t)\ ,$$
$$\mathscr{H}[\varphi]=\int dx\left[-\frac{r}{2}\varphi^2(x,t)+\frac{u}{4}\varphi^4(x,t)-\frac{D}{2}\left[\frac{\partial\varphi(x,t)}{\partial x}\right]^2\right], \tag{1}$$

with L the kinetic coefficient, $r>0$, and u coupling constants. ϵ is related to L and the temperature T by the fluctuation-dissipation theorem

$$\epsilon=2Lk_BT\ , \tag{2}$$

where k_B is the Boltzmann constant and we assume natural boundary conditions at infinity. The properties of the Gaussian white noise $\xi(x,t)$ are

$$\langle \xi(x,t) \rangle = 0 \ ,$$
$$\langle \xi(x,t)\xi(x',t') \rangle = \delta(x-x')\delta(t-t') \ . \tag{3}$$

We are guided from our previous experience in homogeneous systems.[12] The system can be visualized as a chain of particles and the displacement of each point is given, at a given time, by $\varphi(x,t)$. On each location there is an on-site double-well potential, and linear interaction among nearest-neighbor sites is also present. In the unstable situation each particle is initially undisplaced but starts to move if the stochastic force acting on it is different from zero. When the displacement is sufficiently large the particle leaves the unstable position and its motion is essentially deterministic. When close to the equilibrium position, close to the bottom of either well, the stochastic force becomes effective again. This model is valid as long as the strength of the linear interaction between sites is small and each particle can be considered essentially independent of the adjacent particles. When the diffusive term is comparable with the other terms some particles very quickly tend to fall towards the stable position, carrying along the neighboring particles. Consequently some of them will be trapped in the unstable position since the closer particles have already fallen in different local equilibrium state positions. From this moment on the system evolves deterministically and domain walls form. At the end of this stage the dominant mechanism of evolution will be the interaction among kinks discussed by Kawasaki and Nagai.[3] This qualitative model is sketched in Fig. 1. The figure shows a particular realization of the time development of the stochastic field $\varphi(x,t)$ for successive instants labeled a–f. It shows how the local order parameter rapidly tends to saturation forming local domains and kinks.

The full dynamical solution of Eq. (1) has intrinsic difficulties which have inhibited so far any explicit solution. Also the perturbative approaches fail, in the sense that they only give an answer in very restricted regions of time. The reason is that essentially one performs a linearization close to the unstable state. Some more ingenious methods have been devised by Langer, Bar-on and Miller;[16] they are able to describe the early stage of binary-alloy spinodal decomposition although, as showed by Billotet and Binder,[17] there are some difficulties associated with this study.

In this work we want to contribute to the understanding of these phenomena by extending to this case the quasideterministic approach already introduced with some success in the spatially homogeneous case.[12] The central point of the QDT is the mapping between the original stochastic process and a new process which is associated with the initial condition. This mapping is induced by the deterministic solution of the evolution equation. From the analytical point of view this physical idea means a resummation of the nonuniform perturbative expansion, analogous to a singular perturbation expansion.[18]

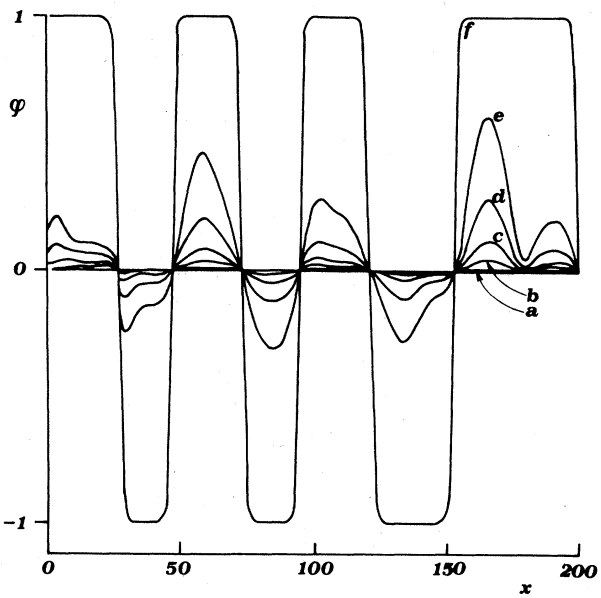

FIG. 1. A typical configuration of the field $\varphi(x,t)$ and its time evolution from an unstable configuration for successive instants labeled a,b,c,d,e,f.

A procedure analogous to that just described when the system is inhomogeneous is impossible since no time-dependent general solution of the deterministic evolution is known. We have tried to overcome this difficulty using the simpler mapping which originates from a local deterministic solution, i.e., ignoring also the effect of the nearest-neighbor interaction ($D=0$)

$$\varphi(x,t) = \frac{h(x,t)e^{Lrt}}{[1+(u/r)h^2(x,t)(e^{2Lrt}-1)]^{1/2}} \ . \tag{4}$$

Since $h(x,t)$ is considered to be a stochastic process this equation gives the above-mentioned mapping between $\varphi(x,t)$ and $h(x,t)$. It is therefore apparent that the mapped process is the one associated with the initial configuration of the system. A further consequence of the mapping is the fact that the process $h(x,t)e^{Lrt}$ is the linearized process of Eq. (1) and consequently the behavior of $h(x,t)$ will be strongly influenced by the unstable initial condition $h(x,0)=0$.

It is worth mentioning that up to this point no approximation has been made; it is possible to derive from Eq. (4) a stochastic differential equation exactly equivalent to Eq. (1)

$$\frac{\partial h(x,t)}{\partial t} = LD\frac{\partial^2 h(x,t)}{\partial x^2} - 3LD\frac{\frac{u}{r}(e^{2Lrt}-1)h^2(x,t)}{1+\frac{u}{r}h^2(x,t)(e^{2Lrt}-1)}\frac{\left[\frac{\partial h(x,t)}{\partial x}\right]^2}{h(x,t)} + \sqrt{\epsilon}e^{-Lrt}\left[1+\frac{u}{r}h^2(x,t)(e^{2Lrt}-1)\right]^{3/2}\xi(x,t) \ . \tag{5}$$

As long as we can assume

$$\frac{u}{r}h^2(x,t)(e^{2Lrt}-1)\ll 1 \;, \tag{6}$$

Eq. (5) becomes

$$\frac{\partial h(x,t)}{\partial t}=LD\frac{\partial^2 h(x,t)}{\partial x^2}+\sqrt{\epsilon}e^{-Lrt}\xi(x,t) \;. \tag{7}$$

Equation (7) together with the mapping (4) represents the above-mentioned generalization of the QDT theory to the present system. From the assumption (6) we can obtain a rough estimate of the times region in which we expect the approximation should work, i.e.,

$$\sigma^2(t)(e^{2Lrt}-1)\ll\frac{r}{u} \;, \tag{8}$$

where

$$\sigma^2(t)=\langle h^2(x,t)\rangle \tag{9}$$

is the one-point correlation function obtained by solving Eq. (7) and taking the expectation value with respect to the assumed Gaussian distribution of the white noise $\xi(x,t)$.

III. PROPERTIES OF THE QDT

In the homogeneous case the QDT (Ref. 12) is very effective since the main features of the evolution from an initial unstable state are completely determined by the knowledge of the process $h(t)$, which is associated with the behavior of the system close to the unstable initial state. The probability distribution function of such a process becomes rapidly time independent; this amounts to saying that in each realization the process itself reaches a constant limit. Such a constant acts as an effective initial condition for the successive deterministic evolution.

In the inhomogeneous fluctuations case we are now considering it is worthwhile to discuss the characteristic properties of the new space-dependent process $h(x,t)$. However, the space dependence makes it difficult to extract from the behavior of the process $h(x,t)$ its characteristic properties. A physical quantity which can give insight into the process itself is its two-point correlation function, or better, the averaged square correlation length defined as

$$l^2(t)=\frac{\int_{-\infty}^{+\infty}dx\,x^2\langle h(x,t)h(x,0)\rangle}{\int_{-\infty}^{+\infty}dx\,\langle h(x,t)h(x,0)\rangle} \;, \tag{10}$$

where the expectation is with respect the probability distribution functional of $h(x,t)$. The calculation of this quantity is straightforward. In fact, the solution of the space Fourier transform of Eq. (7)

$$\frac{d}{dt}\tilde{h}(q,t)=-LDq^2\tilde{h}(q,t)+\sqrt{\epsilon}e^{-Lrt}\tilde{\xi}(q,t) \tag{11}$$

is

$$\tilde{h}(q,t)=\sqrt{\epsilon}e^{-Lrt}\int_0^t dt'\tilde{\xi}(q,t')e^{L(r-Dq^2)(t-t')} \;, \tag{12}$$

where $\tilde{\xi}(q,t)$ is the Fourier transform of the Gaussian white noise whose correlation is

$$\langle\xi(q,t)\xi(q',t')\rangle=\delta(q+q')\delta(t-t') \tag{13}$$

and we have chosen $\tilde{h}(q,0)=0$ as initial condition. The two-point correlation function is

$$\langle h(x,t)h(0,t)\rangle=\frac{\epsilon}{\sqrt{2\pi}}\int_0^t dt' e^{-2Lrt'}\frac{e^{-x^2/8LD(t-t')}}{\sqrt{4LD(t-t')}} \;, \tag{14}$$

which when substituted into Eq. (10) gives

$$l^2(t)=4LD\left[\frac{t}{1-e^{-2Lrt}}-\frac{1}{2Lr}\right] . \tag{15}$$

We see that for long times $l^2(t)$ is linearly divergent. The effect of the mapping on a single realization of the process $h(x,t)$ is to give rise to a configuration of $\varphi(x,t)$ that, eventually, leads to a saturation in amplitude. No analogous phenomenon concerning the width is expected because of the local character of the mapping. As a consequence, the extension of our approximation in the time region beyond its validity, as given by Eq. (6), predicts a time growth of the length of the correlation which is quite different from the true one as we shall see in the following sections.

IV. GROWTH OF THE SPATIAL CORRELATION

We now describe how the QDT allows us to predict the growth of spatial correlation in the early stage of the evolution of the system. The time-dependent correlation length has been defined in Eq. (10); in the present case taking into account the spatial homogeneity of the system we have

$$\xi^2(t)=\frac{\int_{-\infty}^{+\infty}dx\,x^2\langle\varphi(x,t)\varphi(0,t)\rangle}{\int_{-\infty}^{+\infty}dx\,\langle\varphi(x,t)\varphi(0,t)\rangle} \;. \tag{16}$$

The process $\varphi(x,t)$ is given by the mapping of Eq. (4) in terms of the effective initial configuration process $h(x,t)$ which is approximated according to Eq. (7).

The statistical properties of the process $\varphi(x,t)$, because of the mapping, can be derived in terms of the probability distribution functional associated with the configurations of the process $h(x,t)$. This functional is known because $h(x,t)$ is a linear functional of the Gaussian white noise $\xi(x,t)$. In order to calculate the correlation length (16) we only need the reduced two-point joint probability distribution function.[19] Defining h_i for $i=1,2$ as $h_i=h(x,t)$ and $h_2=h(0,t)$ we have

$$P_2(h_1,h_2;t)=\frac{|W^{-1}|^{1/2}}{2\pi}\exp\left[-\tfrac{1}{2}\sum_{i,j=1}^{2}h_iW_{ij}^{-1}h_j\right] , \tag{17}$$

where $\underline{W}$ is the covariance matrix associated with the process h, i.e.,

$$W_{ij}=\langle h_ih_j\rangle \tag{18}$$

and $|\underline{W}^{-1}|$ is the determinant of the inverse of the covariance matrix. An explicit calculation gives

$$| \underline{W} | = (1-\rho^2)\sigma^4 , \tag{19}$$

where the one- and two-point correlation function

$$\sigma^2(t) = \langle h_j^2 \rangle = \frac{\epsilon}{4L} \frac{1}{\sqrt{Dr}} w_2(\sqrt{2Lrt}) \tag{20}$$

(independent of j) and

$$\rho(x,t) = \frac{\langle h_1 h_2 \rangle}{\sigma^2} = e^{-x^2/8LDt} \frac{w_2 \left[\sqrt{2Lrt} + i \dfrac{x}{\sqrt{8LDt}} \right]}{w_2(\sqrt{2Lrt})} \tag{21}$$

can be obtained from the explicit evaluation of Eq. (14). $w_2(z)$ is the imaginary part of the error function for complex arguments.[20] Thus Eq. (17) becomes

$$P_2(h_1,h_2;t) = \frac{1}{2\pi\sigma^2(1-\rho^2)^{1/2}} \times \exp\left[-\frac{h_1^2+h_2^2-2\rho h_1 h_2}{2\sigma^2(1-\rho^2)} \right] , \tag{22}$$

a bivariate normal probability function. The correlation function of the field $\varphi(x,t)$ is then given by

$$\langle \varphi(x,t)\varphi(0,t) \rangle = \frac{e^{2Lrt}}{2\pi\sigma^2(1-\rho^2)^{1/2}} \int_{-\infty}^{+\infty} dh_1 \int_{-\infty}^{+\infty} dh_2 \exp\left[-\frac{h_1^2+h_2^2-2\rho h_1 h_2}{2\sigma^2(1-\rho^2)} \right] \frac{h_1 h_2}{[(1+f^2h_1^2)(1+f^2h_2^2)]^{1/2}} , \tag{23}$$

where

$$f^2(t) = \frac{u}{Lr}(e^{2Lrt}-1) . \tag{24}$$

The dependence on the space variable x appears only through the normalized correlation function $\rho(x,t)$ of the linearized process given by (21). Given the values of the functions f^2, σ^2, and ρ the double integral (23) can be partially performed and some limiting cases evaluated as in Kawasaki, Yalabik, and Gunton.[21] However, we want to stress here that the functions f^2, σ^2, and ρ have different behaviors with respect to the similar expressions of Ref. 21. The reason for that is the fact that they use an early version of Suzuki's[11] mapping which fails in that region.

Before closing this section we want to add some remarks on the space-independent moments of the field. In particular we want to check whether a phenomenon such as anomalous fluctuations typical of the homogeneous system is still present. For this purpose we introduce the one-point reduced probability distribution function

$$P_1(h,t) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-h^2/2\sigma^2} . \tag{25}$$

By using (25) we can calculate the expectation $\langle \varphi^2(x,t) \rangle$ and its variance $\langle (\varphi^2 - \langle \varphi^2 \rangle)^2 \rangle$. The theoretical results obtained in this section will be discussed in the following section in connection with the numerical solution of the full problem.

In closing this section we want to comment on the above-mentioned work of Ref. 21. The application of their global relaxation theory to one-dimensional systems has two difficulties. The first one is its failure for very short times; the second is its inadequacy to describe the long-time behavior. As a consequence, the overall range of validity of the theory is restricted to a time regime between the early stage of formation of domains and the interacting kink regime. Naturally it is worth stressing that the above authors did not intend to treat the one-dimensional case which is rather peculiar. In fact the $t^{1/2}$ growth is valid for late times for higher space dimensionality and the theory of Ref. 21 should be valid then. The $\ln t$ growth for late times is typical only for one-dimensional systems.

V. NUMERICAL SOLUTION AND COMPARISON WITH THE QDT

We have solved the TDGL equation (1) numerically in order to check the range of validity of the QDT. At the same time we have studied the slow evolution of the domain size, a process dominated by kink interaction.

We have used an array of 200 particles evolving according to the space-discretized equation

$$\frac{d}{dt}\varphi_i(t) = Lr\varphi_i - Lu\varphi_i^3 + \frac{DL}{a^2}(\varphi_{i-1} - 2\varphi_i + \varphi_{i+1}) + \sqrt{\epsilon/a}\,\xi_i \quad (i=1,\ldots,200) , \tag{26}$$

where a is the lattice spacing and ξ_i independent random Gaussian white noises with correlation function

$$\langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\delta(t-t'), \quad \langle \xi_i(t) \rangle = 0 . \tag{27}$$

Equation (26) is solved with periodic boundary conditions, and a totally unstable initial configuration $\varphi_i(0)=0$ ($i=1,\ldots,200$). In order to solve the set of coupled equations (26) we have used the algorithm developed in Ref. 13, suitably modified to take into account the space diffusion. The detailed approximate equations are correct up to the order $\Delta t^{3/2}$ in the elementary time interval Δt.

The averaged quantities have been calculated using both averages over many configurations (typically 1000) and over the sites. Since we start from an homogeneous initial state we expect this procedure to work; in order to check its validity in the situation we have considered, we checked the equality of the results with long chains ($\sim$2000 sites) and only space averages, or relatively short ones ($\sim$200) and many configurational averages ($\sim$1000). To improve the efficiency of the averaging we have used both methods, i.e., averaging over 200 sites and 1000 configurations. Finally, we have checked whether there is any effect due to the finite size of the sample. This has been done in the only case analytically accessible, that is,

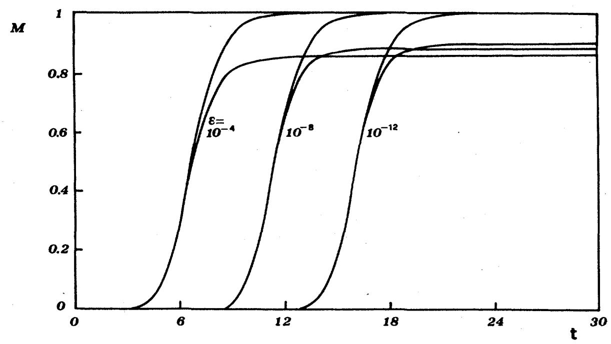

FIG. 2. The averaged squared field $M(t)$ vs time for the parameters given in the text and $\epsilon=10^{-4}$, 10^{-8}, 10^{-12}. The QDT goes asymptotically to one, the numerical results to lower values.

the linearized form of the TDGL model which is valid only for short times. The continuum model, the discretized model, and the numerical solution all give the same results in this linear regime.

The quantities we have computed are

$$M(t)=\frac{1}{NN_c}\sum_{i=1}^{N}\sum_{\alpha=1}^{N_c}\varphi_i^2(t\mid\alpha)\ , \tag{28}$$

$$V(t)=\frac{1}{NN_c}\sum_{i=1}^{N}\sum_{\alpha=1}^{N_c}[\varphi_i^2(t\mid\alpha)-M(t)]^2\ , \tag{29}$$

$$C_j(t)=\frac{1}{NN_c}\sum_{i=1}^{N}\sum_{\alpha=1}^{N_c}\varphi_i(t\mid\alpha)\varphi_{i+j}(t\mid\alpha)\ , \tag{30}$$

i.e., the average of the field squared, its fluctuations, and the field space correlation function, respectively. N is the number of sites and N_c the number of the stochastic configurations $\varphi_i(t\mid\alpha)$ of the field. The correlation length $\xi(t)$ given by Eq. (16) has been evaluated at the width at half amplitude of the normalized correlation function $C_j(t)/M(t)$, a quantity which is numerically much easier to evaluate than the integral defining $\xi(t)$, and contains the same information.

Figures 2—5 show our numerical and theoretical results for the values $Lr=1$, $Lu=1$, $DL/a^2=1$, and $\epsilon/a=10^{-4}$, 10^{-8}, and 10^{-12}.

In Fig. 2 we show $M(t)$ vs t; the curves obtained numerically tend asymptotically to a value less than 1, the value roughly obtained by the theoretical results. It is clear that a large part of the temporal development is well described by the QDT. The approach to the equilibrium value is instead much faster in the QDT, reflecting the fact that the mapping (4) tends to the local equilibrium value r/u for large times. In the homogeneous case[12] the QDT leads to the correct equilibrium value; the discrepancy in the present case is obviously due to the presence of space diffusion.

In Fig. 3 we report the fluctuation $V(t)$ of φ^2 which shows the phenomenon of anomalous fluctuations peculiar to systems close to an instability.[11,12,22] Again, the QDT gives a final state without fluctuations for the same reason outlined above. The numerical result clearly shows the fluctuations in the neighborhood of the local equilibrium positions.

It is worth noting that the characteristic time corresponding to the maximum fluctuations can be estimated from the time up to which the QDT is valid. In fact Eq. (8) furnishes the following values for the limit times t_0 of validity of the theory: $t_0\approx 15.65$, 10.95, and 6.19 for $\epsilon=10^{-12}$, 10^{-8}, and 10^{-4}, respectively. The significance of t_0 is related to its meaning of characteristic time of domain formation. This time can also be roughly estimated from the mean first passage time that can be effectively evaluated as $\bar{t}\approx\frac{1}{2}\ln 2/\epsilon$ (Ref. 23) and gives values only slightly lower than the previous ones.

Figure 4 summarizes our results for the correlation length (width at half-height), both numerically and theoretically. In the first case ξ has an initial purely diffusive behavior $t^{1/2}$, then a saturation occurs, and finally becomes almost constant or grows very slowly.

Theoretically, the QDT is capable of reproducing the entire initial stage up to the mean first passage time which corresponds to a formed interface among domains. At that moment a saturation and a small shrinking of the average width of the domains occur, which is qualitatively well reproduced by the theory. The shrinking involves only one or two adjacent sites and tends to disappear for larger values of the noise. The QDT is controllable up to this point; in fact for later times it gives a growth of the

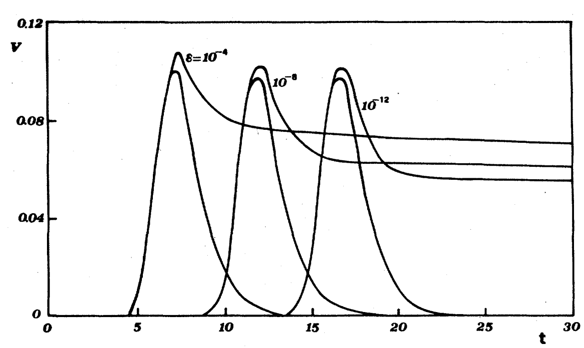

FIG. 3. The same as Fig. 2 for the variance $V(t)$. The QDT tends to zero.

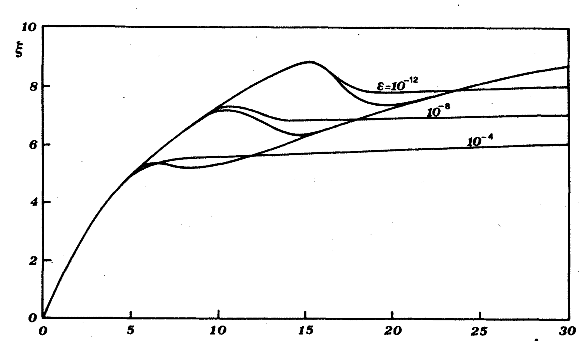

FIG. 4. The correlation length $\xi(t)$ vs t for the same values of the parameters as in Fig. 2. The numerical solution tends to grow very slowly compared to the QDT.

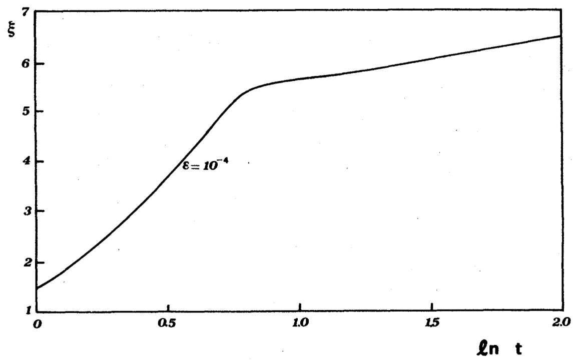

FIG. 5. $\xi(t)$ vs $\ln t$ for $\epsilon=10^{-4}$ which shows the two time regimes discussed in the text.

domain size which seems to follow a $t^{1/2}$ law. This fact is a consequence of the already discussed property of the process $h(x,t)$ whose correlation function tends to grow too rapidly in time.

The results are those of an early-stage theory which describes how the local order parameter tends to saturation. It would be very useful to compare this with a theory of the type of Ref. 16, which is also an early-time theory. This approach was extended to the case of a nonconserved order parameter in Ref. 17. Unfortunately the theory only allows a numerical solution which was not developed for the one-dimensional case. The two theories start from different points of view, the QDT makes assumptions on the stochastic process while the Langer—Bar-on—Miller decoupling is made on the correlation functions. The two schemes coincide in the linear instability region but are expected to disagree in the time region around the mean first passage time. In other words, no phenomenon related to anomalous fluctuations would appear in a theory in which the homogeneous limit reduces to a mean-field approximation.

In order to check the later stage of growth we have performed a numerical calculation for longer times. It is reported on a semilogarithmic scale in Fig. 5 for $\epsilon=10^{-4}$ and shows a $\ln t$ law of growth. This behavior allows us to make contact, as we mentioned earlier, with the results of Kawasaki and Nagai.[3,6] In particular, with our choice of the parameters, the exponent ν of the growth law ($\ln t^{\nu}$) given in Refs. 3 and 6 would be $\nu \approx \sqrt{12} = 3.46$ in agreement with the value reported by the quoted authors.

VI. DISCUSSION AND CONCLUSION

To summarize and discuss our study of the TDGL model close to an instability let us start from the analogies between our results and the experimental data on layered dilute antiferromagnetic systems reported by Ikeda.[7] The most striking similarity is the existence of two distinct temporal regimes. According to our model theory, the first can be interpreted as being related to the formation of the domains, while the second reflects the much slower domain growth. Our interpretation is supported by the following fact. Since we assume to start from an initial unstable configuration the fluctuations are extremely important in starting the entire process. The QDT gives a detailed explanation of the behavior of the system for very short times and in the subsequent regime in which the system itself is still dominated by the initial fluctuations. The important point here is the fact that our numerical solution indicates then a transition from the previous behavior to a much slower one when the domains are fully formed. This is in agreement with the conclusions of Kawasaki and Nagai who developed a theory based on interacting domain walls.[3] Their independent numerical simulation[6] further confirms the given explanation.

Two further consequences of the QDT are worth mentioning. The first is the good estimate of the time needed by the systems to fully separate in well-formed domains, essentially the mean first passage time. The second is the existence of some structure in the behavior of the correlation length in the region of transition between the initial diffusive and the final logarithmic growth. The latter fact might be worth investigating experimentally. It would also be of interest to check the predicted logarithmic dependence on the effective noise ϵ of the transition time, in order to verify indirectly that a quasi-one-dimensional system of the type studied in Ref. 7 can be considered as starting from an initial unstable configuration.

As a final point we want to mention the fact that all the observed results remain valid even if we do not start from a totally unstable configuration, but also if we use an initial quasiunstable one, i.e., a configuration with a narrow probability distribution for the initial state on each site.

ACKNOWLEDGMENTS

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[10]The choice of this model can be questionable in the case in which the system evolves from an initial state far from equilibrium. However, our final results are very reasonable and

support this choice.
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