Semiclassical theory of bistable semiconductor lasers including radial mode variation

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The output of semiconductor laser resonators containing saturable absorbers is calculated with use of a semiclassical model that takes into account radial variation of the laser mode and the gain. The results show that more absorption is required for bistability to exist compared to the plane-wave case, and that transverse effects must be included to predict the switching intensities accurately.

I. INTRODUCTION

Bistable optical devices may play an important role in the fields of optical computing and optical signal processing. They have potential for use as optical transistors, memory elements, and logic elements. Most of the devices built to date have required both an optical resonator and a nonlinear medium for their operation. Passive devices have been demonstrated based on both nonlinear dispersive[1-3] and absorptive[4] media. The first active devices (i.e., laser resonators containing a saturable absorber) were demonstrated over 20 years ago,[5,6] and recent efforts have concentrated on semiconductor lasers.[7,8] Inhomogeneously excited semiconductor lasers are attractive for integrated optics applications, and single-chip[9] and multiple-chip[10] designs have been demonstrated.

With one exception,[11] previous theoretical studies of bistable laser resonators[12-15] have used the plane-wave approximation to predict the output. The results from analyses of passive resonators have shown that the radial structure of the modal intensity significantly affects the input-output characteristics in both the steady-state[16-23] and dynamic[24] regimes. In this paper we extend our previous results[11] by developing a semiclassical model for bistable laser resonators that includes the effects of standing waves, resonator detuning, and radial variation of the gain, absorption, and modal laser intensity. We treat the particular case of an inhomogeneously excited semiconductor laser, but our results can also be applied to other laser systems. We work in the mean-field envelope approximation, and although this model can be rather suspect for a high-gain laser such as a semiconductor laser, we expect our results to be at least qualitatively valid.

In Sec. II we develop the basic model and derive dynamical equations to describe the laser output. In Sec. III we obtain the steady-state solution to our set of equations and we derive a new state equation that takes into account the radial variation of the laser mode. In Sec. IV we compare plane-wave and Gaussian mode results foı the input-output curves, and we show that radial effects must be included if the hysteretic output curves are to be used to characterize the saturable absorber. In Sec. V we derive conditions for the onset of bistability. Section VI contains some concluding remarks.

II. THE SEMICLASSICAL MODEL

The physical system that we are considering is shown in Fig. 1. The semiconductor laser resonator contains two cells; one acts as a saturable amplifier, and the other is excited with a lower injection current and acts as a saturable absorber. The Hamiltonian of our system has three parts:[25,26] one for the free electrons in the amplifier and absorber, one for the free light field, and one for the interaction of the light field with the electron system:

$$H=H_{\mathrm{el}}+H_f+H_{\mathrm{el}\text{-}f}\ . \tag{2.1}$$

H_{el} is the electron Hamiltonian in the site representation, and is given by

$$H_{\mathrm{el}}=\sum_{m,n}\hbar\omega_m a^{\dagger}_{m,n}a_{m,n}+\sum_{\bar m,\bar n}\hbar\bar\omega_{\bar m}a^{\dagger}_{\bar m,\bar n}a_{\bar m,\bar n}\ . \tag{2.2}$$

$\hbar\omega_m$ is the energy of an amplifier electron in band m at lattice-site position $\mathbf{R}_n$, and $a_{m,n}$ is an annihilation operator for an electron in that energy state. Similar quantities for the absorber electrons are denoted by bars. The Hamiltonian of the free light field is given by

$$H_f=\sum_{\mathbf{q},\lambda}\hbar\omega_q b^{\dagger}_{\mathbf{q}\lambda}b_{\mathbf{q}\lambda}\ . \tag{2.3}$$

$\hbar\omega_q$ is the energy of a photon with wave vector $\mathbf{q}$ and polarization denoted by the polarization index λ ($\lambda=1,2$). $b_{\mathbf{q}\lambda}$ is an annihilation operator for photons in the state $(\mathbf{q},\lambda)$. Finally, the interaction Hamiltonian is

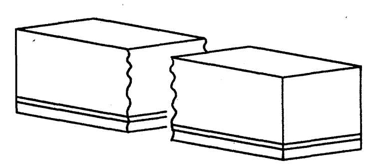

FIG. 1. Generalized geometry for the bistable semiconductor laser system.

$$H_{\text{el-}f}=\sum_{\mathbf{q},\lambda}\sum_{m',m}\sum_{n}(\hbar g_{m',m,\mathbf{q},\lambda,n}a^{\dagger}_{m',n}a_{m,n}b_{\mathbf{q}\lambda}+\text{H.c.})+\sum_{\mathbf{q},\lambda}\sum_{\bar m',\bar m}\sum_{\bar n}(\hbar\bar g_{\bar m',\bar m,\mathbf{q},\lambda,\bar n}\bar a^{\dagger}_{\bar m',\bar n}\bar a_{\bar m,\bar n}b_{\mathbf{q}\lambda}+\text{H.c.})\,, \tag{2.4}$$

where the coupling constant is

$$g_{m',m,\mathbf{q},\lambda,n}=-\frac{1}{\hbar}\frac{e}{mc}\frac{c}{n}\left[\frac{2\pi\hbar}{\omega_q}\right]^{1/2}\times\mathbf{u}_{\mathbf{q}\lambda}(\mathbf{R}_n)\cdot\langle m',n\,|\,\mathbf{p}\,|\,m,n\rangle\,. \tag{2.5}$$

e, m, and $\mathbf{p}$ are the electron charge, mass, and momentum, respectively. c is the speed of light, and n is the refractive index. $|\,m,n\rangle$ is the state vector for an electron in band m at lattice site $\mathbf{R}_n$. $\mathbf{u}_{\mathbf{q}\lambda}(\mathbf{r})$ is the spatial mode function for the light field, and it satisfies

$$\int d^3r\,\mathbf{u}^*_{\mathbf{q}'\lambda'}(\mathbf{r})\cdot\mathbf{u}_{\mathbf{q}\lambda}(\mathbf{r})=\delta_{\mathbf{q}\mathbf{q}'}\delta_{\lambda\lambda'}\,. \tag{2.6}$$

The operators for the electrons obey the Fermi commutation relations

$$\begin{aligned}&[a_{m',n'},a_{m,n}]_+=[a^{\dagger}_{m',n'},a^{\dagger}_{m,n}]_+=0\,,\\&[a_{m,n},a^{\dagger}_{m',n'}]_+=\delta_{mm'}\delta_{nn'}\,,\end{aligned} \tag{2.7}$$

where the plus sign denotes the anticommutator. The b's obey the boson commutation relations

$$\begin{aligned}&[b_{\mathbf{q}\lambda},b_{\mathbf{q}'\lambda'}]_-=[b^{\dagger}_{\mathbf{q}\lambda},b^{\dagger}_{\mathbf{q}'\lambda'}]=0\,,\\&[b_{\mathbf{q}\lambda},b^{\dagger}_{\mathbf{q}'\lambda'}]_-=\delta_{\mathbf{q}\mathbf{q}'}\delta_{\lambda\lambda'}\,.\end{aligned} \tag{2.8}$$

We now wish to derive equations of motion for the population, polarization, and light field operators. In the Heisenberg picture an operator O evolves according to

$$i\hbar\frac{dO}{dt}=[O,H]_-\,. \tag{2.9}$$

We find for the polarization operator $a^{\dagger}_{p,i}a_{s,i}$, using (2.2)—(2.4) in (2.9),

$$\frac{d}{dt}(a^{\dagger}_{p,i}a_{s,i})=-i\omega_{sp}a^{\dagger}_{p,i}a_{s,i}-i\sum_{\mathbf{q},\lambda}\left[\left[-\sum_{m'}g_{m',p,\mathbf{q},\lambda,i}a^{\dagger}_{m',i}a_{s,i}+\sum_{m}g_{s,m,\mathbf{q},\lambda,i}a^{\dagger}_{p,i}a_{m,i}\right]b_{\mathbf{q}\lambda}+b^{\dagger}_{\mathbf{q}\lambda}\left[-\sum_{m}g^*_{p,m,\mathbf{q},\lambda,i}a^{\dagger}_{m,i}a_{s,i}+\sum_{m'}g^*_{m',s,\mathbf{q},\lambda,i}a^{\dagger}_{p,i}a_{m',i}\right]\right]\,. \tag{2.10}$$

The difference frequency $\omega_{sp}=\omega_s-\omega_p$. The equation for the population operator $a^{\dagger}_{p,i}a_{p,i}$ can be obtained from (2.10) by setting $s=p$, giving

$$\frac{d}{dt}(a^{\dagger}_{p,i}a_{p,i})=-i\sum_{\mathbf{q},\lambda}\left[\left[-\sum_{m'}g_{m',p,\mathbf{q},\lambda,i}a^{\dagger}_{m',i}a_{p,i}+\sum_{m}g_{p,m,\mathbf{q},\lambda,i}a^{\dagger}_{p,i}a_{m,i}\right]b_{\mathbf{q}\lambda}+b^{\dagger}_{\mathbf{q}\lambda}\left[-\sum_{m}g^*_{p,m,\mathbf{q},\lambda,i}a^{\dagger}_{m,i}a_{p,i}+\sum_{m}g^*_{m',p,\mathbf{q},\lambda,i}a^{\dagger}_{p,i}a_{m',i}\right]\right]\,. \tag{2.11}$$

Finally, we have for the light field operator $b_{\mathbf{q}\lambda}$

$$\frac{d}{dt}b_{\mathbf{q}\lambda}=-i\omega_q b_{\mathbf{q}\lambda}-i\sum_{m,m'}\sum_{n}g^*_{m',m,\mathbf{q},\lambda,n}a^{\dagger}_{m,n}a_{m',n}-i\sum_{\bar m,\bar m'}\sum_{\bar n}\bar g^*_{\bar m',\bar m,\mathbf{q},\lambda,\bar n}\bar a^{\dagger}_{\bar m,\bar n}\bar a_{\bar m',\bar n}\,. \tag{2.12}$$

We now proceed to a semiclassical description by taking expectation values of (2.10)—(2.12) and making the usual factorization assumption

$$\langle XY\rangle=\langle X\rangle\langle Y\rangle\,, \tag{2.13}$$

where X is an electron operator and Y is a light field operator. We choose for the initial system state

$$|\,\Psi\rangle=|\,\Psi_{\text{amplifier}}\rangle\,|\,\Psi_{\text{absorber}}\rangle\,|\,\Psi_{\text{light}}\rangle\,. \tag{2.14}$$

The state vector for the system of amplifier electrons is

$$|\,\Psi_{\text{amplifier}}\rangle=\sum_{\mu}\alpha_{\mu}\,|\,\{n_{\mu}\}\rangle\,, \tag{2.15}$$

where $|\,\{n_{\mu}\}\rangle$ is the μth occupation number state for electrons,[27] and α_{μ} is a coefficient in the linear expansion (2.15). Similarly for the absorber electrons we have

$$|\,\Psi_{\text{absorber}}\rangle=\sum_{\bar\mu}\bar\alpha_{\bar\mu}\,|\,\{\bar n_{\bar\mu}\}\rangle\,. \tag{2.16}$$

We assume that the light field is made up of a number of uncorrelated modes, and that it can be represented as[28]

$$|\,\Psi_{\text{light}}\rangle=\prod_{\mathbf{q},\lambda}|\,\beta_{\mathbf{q}\lambda}\rangle\,, \tag{2.17}$$

where $|\,\beta_{\mathbf{q}\lambda}\rangle$ is a coherent state with wave vector $\mathbf{q}$ and polarization index λ. Defining

$$\sigma_{ps,n}=\langle\Psi\,|\,a^{\dagger}_{p,n}a_{s,n}\,|\,\Psi\rangle\,, \tag{2.18}$$

$$\beta_{\mathbf{q}\lambda}=\langle\Psi\,|\,b_{\mathbf{q}\lambda}\,|\,\Psi\rangle\,, \tag{2.19}$$

we have

$$\frac{d}{dt}\beta_{q\lambda}=-i\omega_q\beta_{q\lambda}-i\sum_{m=v}\sum_{n} g^*_{c,m,q,\lambda,n}\sigma_{mc,n} -i\sum_{\bar m=\bar v}\sum_{\bar n}\bar g^*_{\bar c,\bar m,q,\lambda,\bar n}\bar\sigma_{\bar m\bar c,\bar n} , \tag{2.20}$$

$$\frac{d}{dt}\sigma_{cc,n}=P_{cc,n}-\gamma_{cc}(\sigma_{cc,n}-\sigma^0_{cc,n}) -i\sum_{q,\lambda}\sum_{l=v}(g_{c,l,q,\lambda,n}\sigma_{cl,n}\beta_{q\lambda} -g^*_{c,l,q,\lambda,n}\sigma_{lc,n}\beta^*_{q\lambda}) , \tag{2.21}$$

$$\frac{d}{dt}\sigma_{vv,n}=P_{vv,n}-\gamma_{vv}(\sigma_{vv,n}-\sigma^0_{vv,n}) -i\sum_{q,\lambda}(-g_{c,v,q,\lambda n}\sigma_{cv,n}\beta_{q\lambda} +g^*_{c,v,q,\lambda,n}\sigma_{vc,n}\beta^*_{q\lambda}) , \tag{2.22}$$

$$\frac{d}{dt}\sigma_{vc,n}=-(i\omega_{cv}+\gamma_{vc})\sigma_{vc,n} -i\sum_{q,\lambda}\left[-g_{c,v,q,\lambda,n}\sigma_{cc,n}+\sum_{m=v}g_{c,m,q,\lambda,n}\sigma_{vm,n}\right]\beta_{q\lambda} . \tag{2.23}$$

In the above v stands for the valence band(s) and c for the conduction band. The rotating-wave approximation has been made in the above equations, and phenomenological decay and pumping terms have been added. γ_{vc} is the dipole dephasing rate, $P_{cc,n}$ is the pumping rate of electrons into the conduction band at lattice site $\mathbf{R}_n$, $P_{vv,n}$ is the pumping rate for electrons in the valence band, and $\sigma^0_{cc,n}$ and $\sigma^0_{vv,n}$ are equilibrium populations. γ_{cc} and γ_{vv} are the decay rates for electrons in the conduction and valence bands, respectively. The assumption of a constant decay rate is reasonable only for heavily doped active layers.[29]

We note the presence of a $\sigma_{v_1v_2,n}$ term in (2.23) (an intervalence band polarization term); we assume that this term can be neglected since it is nonresonant with the light field at the frequencies of interest. A closed set of equations can now be obtained by multiplying (2.23) by the electron-light coupling constant:

$$\frac{d}{dt}g^*_{c,v,p,\eta,n}\sigma_{vc,n} =-(i\omega_{cv}+\gamma_{vc})g^*_{c,v,p,\eta,n}\sigma_{vc,n} +i\sum_{q,\lambda}g^*_{c,v,p,\eta,n}g_{c,v,q,\lambda,n}(\sigma_{cc,n}-\sigma_{vv,n})\beta_{q\lambda} . \tag{2.24}$$

We now make the quasimonochromatic, single-mode approximation and write[13]

$$\beta_{q\lambda}(t)=\tilde\beta_{q\lambda}(t)\exp(-i\Omega t) , \tag{2.25}$$

$$g^*_{c,v,q,\lambda,n}\sigma_{vc,n}(t)=[C_{v,c,q,\lambda,n}(t)+iS_{v,c,q,\lambda,n}(t)]\exp(-i\Omega t) . \tag{2.26}$$

The sum over $\mathbf{q}$ in all equations will now be taken to mean a sum over propagation direction. Our basic set of equations, finally, becomes

$$\frac{d}{dt}\tilde\beta_{q\lambda}(t)=\sum_{v}\sum_{n}S_{v,c,q,\lambda,n}(t)+\sum_{\bar v}\sum_{\bar n}\bar S_{\bar v,\bar c,q,\lambda,\bar n}(t) , \tag{2.27}$$

$$(\omega-\Omega)\tilde\beta_{q}(t)=\sum_{v}\sum_{n}C_{v,c,q,\lambda,n}(t)+\sum_{\bar v}\sum_{\bar n}\bar C_{\bar v,\bar c,q,\lambda,\bar n}(t) , \tag{2.28}$$

$$\frac{d}{dt}\sigma_{cc,n}(t)=P_{cc,n}(t)-\gamma_{cc}[\sigma_{cc,n}(t)-\sigma^0_{cc,n}]-2\sum_{q,\lambda}\sum_{v}S_{v,c,q,\lambda,n}(t)\tilde\beta_{q\lambda}(t) , \tag{2.29}$$

$$\frac{d}{dt}\sigma_{vv,n}(t)=P_{vv,n}(t)-\gamma_{vv}[\sigma_{vv,n}(t)-\sigma^0_{vv,n}]+2\sum_{q,\lambda}S_{v,c,q,\lambda,n}(t)\tilde\beta_{q\lambda}(t) , \tag{2.30}$$

$$\frac{d}{dt}C_{v,c,p,\eta,n}(t)=-\gamma_{vc}C_{v,c,p,\eta,n}(t)-(\Omega-\omega_{cv})S_{v,c,p,\eta,n}(t)-[\sigma_{cc,n}(t)-\sigma_{vv,n}(t)]\,\mathrm{Im}\left[\sum_{q,\lambda}g^*_{c,v,p,\eta,n}g_{c,v,q,\lambda,n}\tilde\beta_{q\lambda}(t)\right] , \tag{2.31}$$

$$\frac{d}{dt}S_{v,c,p,\eta,n}(t)=-\gamma_{vc}S_{v,c,p,\eta,n}(t)+(\Omega-\omega_{cv})C_{v,c,p,\eta,n}(t)+[\sigma_{cc,n}(t)-\sigma_{vv,n}(t)]\,\mathrm{Re}\left[\sum_{q,\lambda}g^*_{c,v,p,\eta,n}g_{c,v,q,\lambda,n}\tilde\beta_{q\lambda}(t)\right] . \tag{2.32}$$

Equations for the absorber population and polarization can be obtained from (2.29)—(2.32) by adding bars to the appropriate quantities.

III. STATE EQUATION FOR THE INTENSITY

The steady-state solution is found by setting all time derivatives equal to zero in (2.27)—(2.32). We find for the out-of-phase component of the polarization and for the inversion

$$S_{v,c,\mathbf{p},\eta,n}=\frac{1}{\delta_{vc}}(\sigma_{cc,n}-\sigma_{vv,n})\sum_{\mathbf{q},\lambda}\tilde{\beta}_{\mathbf{q}\lambda}\,[\mathrm{Re}(g^*_{c,v,\mathbf{p},\eta,n}g_{c,v,\mathbf{q},\lambda,n})-\Delta\,\mathrm{Im}(g^*_{c,v,\mathbf{p},\eta,n}g_{c,v,\mathbf{q},\lambda,n})]\;, \tag{3.1}$$

$$\sigma_{cc,n}-\sigma_{vv,n}=[(\sigma^0_{cc,n}-\sigma^0_{vv,n})+(P_{cc,n}-P_{vv,n})/\gamma_{cc}]\times\left[1+4\sum_{\mathbf{p},\eta}\sum_{\mathbf{q},\lambda}\tilde{\beta}_{\mathbf{q}\lambda}\tilde{\beta}_{\mathbf{p}\eta}(1/\gamma_{cc}\delta_{vc})[\mathrm{Re}(g^*_{c,v,\mathbf{p},\eta,n}g_{c,v,\mathbf{q},\lambda,n})-\Delta\,\mathrm{Im}(g^*_{c,v,\mathbf{p},\eta,n}g_{c,v,\mathbf{q},\lambda,n})]\right]^{-1}, \tag{3.2}$$

where

$$\Delta=(\Omega-\omega_{cv})/\gamma_{vc}\;, \tag{3.3}$$

$$\delta_{vc}=\gamma_{vc}(1+\Delta^2)\;. \tag{3.4}$$

($\gamma_{vv}=\gamma_{cc}$ has been assumed.) We have neglected terms involving all but one valence band in order to keep the equations tractable. For GaAs, e.g., one should include the effect of the light-hole valence band. We now insert the above into (2.27) and introduce the standard notion

$$I_{\mathbf{q}\lambda}=S\tilde{\beta}^2_{\mathbf{q}\lambda}\;, \tag{3.5}$$

$$S=4\,|\,g_{cv}\,|^2/(\gamma_{cc}\delta_{vc})\;, \tag{3.6}$$

where $I_{\mathbf{q}\lambda}$ is the optical intensity normalized to the saturation intensity, S is a saturation parameter for the amplifier medium, and the spatially invariant coupling constant g_{cv} is defined through

$$g_{c,v,\mathbf{q},\lambda,\mathbf{R}_n}=g_{cv}f_{\mathbf{q}\lambda}(\mathbf{R}_n)e^{i\mathbf{q}\cdot\mathbf{R}_n}\;. \tag{3.7}$$

$f_{\mathbf{q}\lambda}(\mathbf{r})$ is the slowly varying envelope of the field mode function $\mathbf{u}_{\mathbf{q}\lambda}(\mathbf{r})$. We obtain as the equation for the intensity $I_{\mathbf{Q}\lambda}$

$$\begin{aligned}0=\mathscr{n}\int_V d^3r\frac{|\,g_{cv}\,|^2}{\delta_{vc}}&\{[\sigma^0_{cc}(\mathbf{r})-\sigma^0_{vv}(\mathbf{r})]+[P_{cc}(\mathbf{r})-P_{vv}(\mathbf{r})]/\gamma_{cc}\}\\&\times\{2I_{\mathbf{Q}\lambda}f^2_{\mathbf{Q}\lambda}(\mathbf{r})+2\sqrt{I_{\mathbf{Q}\lambda}I_{-\mathbf{Q}\lambda}}f_{\mathbf{Q}\lambda}(\mathbf{r})f_{-\mathbf{Q}\lambda}(\mathbf{r})[\cos(2\mathbf{Q}\cdot\mathbf{r})+\Delta\sin(2\mathbf{Q}\cdot\mathbf{r})]\}\\&\times[1+I_{\mathbf{Q}\lambda}f^2_{\mathbf{Q}\lambda}(\mathbf{r})+I_{-\mathbf{Q}_\lambda}f^2_{-\mathbf{Q}\lambda}(\mathbf{r})+2\sqrt{I_{\mathbf{Q}\lambda}I_{-\mathbf{Q}\lambda}}f_{\mathbf{Q}\lambda}(\mathbf{r})f_{-\mathbf{Q}\lambda}(\mathbf{r})\cos(2\mathbf{Q}\cdot\mathbf{r})]^{-1}+\cdots\;,\end{aligned} \tag{3.8}$$

where the ellipsis represents similar absorber terms; in the above $\mathscr{n}$ is the density of amplifier lattice sites, V is the amplifier volume, and $\mathbf{Q}$ is the wave vector of the forward-propagating wave. Equation (3.8) is the standing-wave result; the traveling-wave result can be obtained by setting the $I_{\mathbf{Q}\lambda}I_{-\mathbf{Q}\lambda}$ product terms equal to zero.

We now make a mean-field approximation (for the direction along the resonator) by adding to (3.8) a $-2\Gamma I_{\mathbf{Q}\lambda}$ term [cf. (2.27)] and by assuming that $f_{\pm\mathbf{Q}\lambda}(\mathbf{r})$ depends only on the direction transverse to $\pm\mathbf{Q}$. We also define a spatially dependent pumping parameter for the amplifier cell (cf. Ref. 12)

$$A(\mathbf{r})=\frac{|\,g_{cv}\,|^2\mathscr{n}V}{\delta_{vc}\Gamma}\{[\sigma^0_{cc}(\mathbf{r})-\sigma^0_{vv}(\mathbf{r})]+[P_{cc}(\mathbf{r})-P_{vv}(\mathbf{r})]/\gamma_{cc}\}\;, \tag{3.9}$$

and an absorbing cell parameter

$$C(\mathbf{r})=1-\frac{|\,\bar{g}_{\bar{c}\bar{v}}\,|^2\bar{\mathscr{n}}\bar{V}}{\bar{\delta}_{\bar{v}\bar{c}}\Gamma}\{[\bar{\sigma}^0_{\bar{c}\bar{c}}(\mathbf{r})-\bar{\sigma}^0_{\bar{v}\bar{v}}(\mathbf{r})]+[\bar{P}_{\bar{c}\bar{c}}(\mathbf{r})-\bar{P}_{\bar{v}\bar{v}}(\mathbf{r})]/\bar{\gamma}_{\bar{c}\bar{c}}\}\;. \tag{3.10}$$

$A(\mathbf{r})$ and $C(\mathbf{r})$ are related to the actual gain $g(\mathbf{r})$, and absorption $\alpha(\mathbf{r})$, by

$$A(\mathbf{r})=\frac{V}{V_{\mathbf{Q}\lambda}}\frac{g(\mathbf{r})}{\alpha_r}\;, \tag{3.11}$$

$$C(\mathbf{r})=1+\frac{\bar{V}}{V_{\mathbf{Q}\lambda}}\frac{\alpha(\mathbf{r})}{\alpha_r}\;, \tag{3.12}$$

where $V_{\mathbf{Q}\lambda}$ is the laser mode volume, and α_r is a distributed loss coefficient for the resonator, given by

$$\alpha_r=2\frac{n}{c}\Gamma\;. \tag{3.13}$$

Using the above in (3.8) gives

$$0=-2\Gamma I\left[1-V^{-1}\int_V d^3r\, f^2(r_\perp)A(\mathbf{r})[2\cos^2(Qz)+\Delta\sin(2Qz)][1+4f^2(r_\perp)I\cos^2(Qz)]^{-1}\right.$$
$$\left.-\bar{V}^{-1}\int_{\bar{V}} d^3\bar{r}\, f^2(\bar{r}_\perp)[1-C(\bar{\mathbf{r}})][2\cos^2(Q\bar{z})+\bar{\Delta}\sin(2Q\bar{z})][1+4f^2(\bar{r}_\perp)aI\cos^2(Q\bar{z})]^{-1}\right], \quad (3.14)$$

where

$$a=\bar{S}/S \quad (3.15)$$

is the ratio of the amplifier saturation intensity to the absorber saturation intensity, and $I=I_{Q\lambda}=I_{-Q\lambda}$. The traveling-wave result is

$$0=-2\Gamma I\left[1-V^{-1}\int_V d^3r\frac{f^2(r_\perp)A(\mathbf{r})}{1+2f^2(r_\perp)I}-\bar{V}^{-1}\int_{\bar{V}} d^3\bar{r}\,\frac{f^2(\bar{r}_\perp)[1-C(\bar{\mathbf{r}})]}{1+2f^2(\bar{r}_\perp)aI}\right]. \quad (3.16)$$

Equations (3.14) and (3.16) are the central results of this paper. They show how the plane-wave result must be modified to determine the effects of radial variation in the mode intensity, gain, and absorption. The integrals over z and $\bar{z}$ can readily be performed in both equations to give

$$0=-2\Gamma I\left[1-(2\mathscr{A})^{-1}\int_{\mathscr{A}} dr_\perp A(r_\perp)\frac{1}{I}\{1-[1+4f^2(r_\perp)I]^{-1/2}\}\right.$$
$$\left.-(2\bar{\mathscr{A}})^{-1}\int_{\bar{\mathscr{A}}} d\bar{r}_\perp[1-C(\bar{r}_\perp)]\frac{1}{aI}\{1-[1+4f^2(\bar{r}_\perp)aI]^{-1/2}\}\right] \quad (3.17)$$

for the standing-wave result and

$$0=-2\Gamma I\left[1-\mathscr{A}^{-1}\int_{\mathscr{A}} dr_\perp\frac{f^2(r_\perp)A(r_\perp)}{1+2f^2(r_\perp)I}\right.$$
$$\left.-\bar{\mathscr{A}}^{-1}\int_{\bar{\mathscr{A}}} d\bar{r}_\perp\frac{f^2(\bar{r}_\perp)[1-C(\bar{r}_\perp)]}{1+2f^2(r_\perp)aI}\right] \quad (3.18)$$

for the traveling-wave result. $\mathscr{A}$ and $\bar{\mathscr{A}}$ are the amplifier and absorber cross-sectional areas, and $A(\mathbf{r})$ and $C(\mathbf{r})$ have been assumed to depend on the transverse coordinate only. Equation (3.18) was derived in our earlier work[15] using generalized laser parameters from the outset. A result similar to (3.17) has been derived by Sandle *et al.*[18] in a treatment of passive resonators.

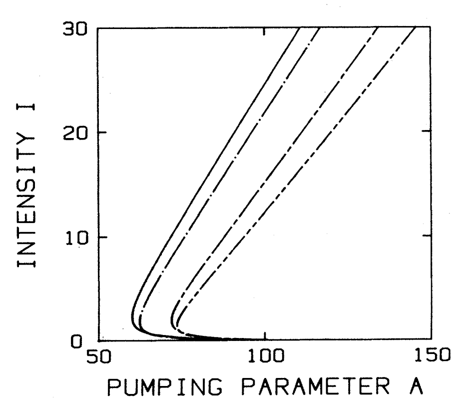

FIG. 2. Output intensity I vs amplifier cell pumping parameter A, for $C=100$ and $a=2$. The curves are the traveling-wave plane-wave case (solid), standing-wave plane-wave case (single dot), traveling-wave Gaussian mode case (single dash), and the standing-wave Gaussian mode case (double dash).

IV. INPUT-OUTPUT CURVES

In order to illustrate the effects of transverse mode and gain variation we integrated (3.17) and (3.18) numerically for the case of a semiconductor laser with an active layer thickness of 0.15 μm and a stripe width of 3 μm. The gain and absorption were assumed to be uniform inside and zero outside this region. The waist parameters of the elliptic Gaussian beam were taken to be $w_x=0.81$ μm (parallel to the active layer) and $w_y=0.48$ μm (perpendicular to the active layer). We treat here only the pure absorptive case ($\Delta=0$). Shown in Fig. 2 are the resulting curves of intensity I versus amplifier cell pumping parameter A for the particular case of $C=100$ and $a=2$. The effect of standing waves is to reduce the output intensity and increase the value of A for switch-off; the value of A for switch-on, however, is unaffected, contrary to the results for passive resonators.[19] The effect of radial mode

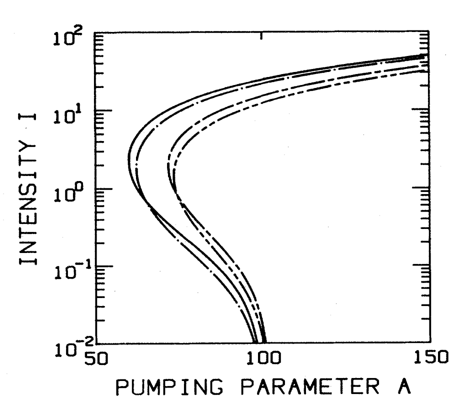

FIG. 3. Results of Fig. 2 replotted on a semilogarithmic scale. The labeling is as in Fig. 2.

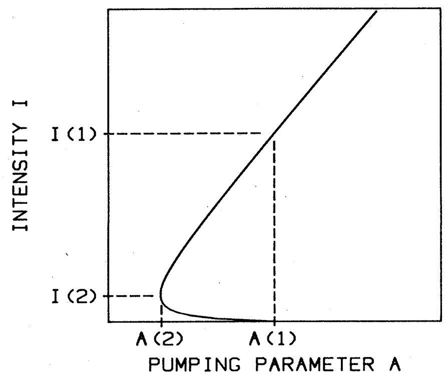

FIG. 4. Input-output curve showing the labeling of the switching points $I(1)$, $I(2)$, $A(1)$, and $A(2)$.

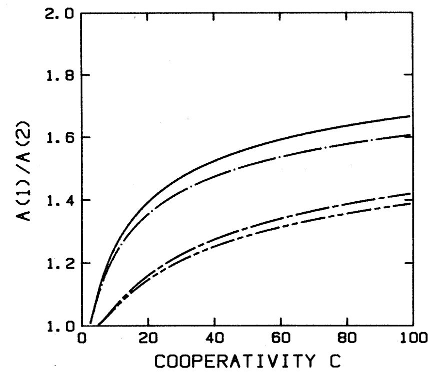

FIG. 6. Ratio of switching points $A(1)/A(2)$ vs C, for the case $a=2$. Labeling is as in Fig. 2.

and gain variation is to reduce the output intensity also, but both A-switching points are increased. The results are replotted in Fig. 3 on a semilogarithmic scale, where it is easier to see the increase in threshold due to radial effects. It is clear that the effects of radial variation are more significant than the effects of standing waves.

An interesting point to consider is whether the hysteretic output curves can be used to make spectroscopic measurements on the absorber cell. Using the plane-wave, ring resonator theories of bistability one can find analytic expressions for the switching points (cf. Fig. 4) $I(1)$, $I(2)$, $A(1)$, and $A(2)$ as functions of C and a.[13] In principle, then, C and a could be determined by an experimental measurement of $I(1)/I(2)$ and $A(1)/A(2)$. Unfortunately, for the general case of Gaussian laser mode and nonuniform gain and absorption, analytic expressions cannot be obtained. We have therefore calculated the ratios of these switching points numerically for the case mentioned in the beginning of this section; the results are plotted in Figs. 5 and 6 versus C, for the particular choice $a=2$. The effect of radial variation is again much greater than that of standing waves; however, the percentage error of the traveling-wave theory relative to the standing-wave theory increases with the amount of hysteresis (i.e., with C). One rather curious result of the theory is that the relative height of the hysteresis loop undergoes a stretching in going from traveling waves to standing waves; $I(1)/I(2)$ is greater for standing waves for all values of C, whereas $A(1)/A(2)$ is greater for traveling waves. Thus we see that in order to use the output curves to determine C and a the radial variation in the system must be accounted for.

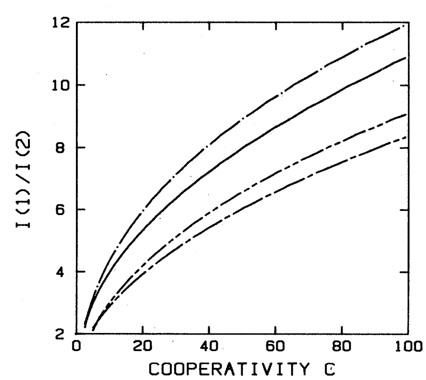

FIG. 5. Ratio of switching points $I(1)/I(2)$ vs C, for the case $a=2$. Labeling is as in Fig. 2.

V. REQUIREMENT FOR BISTABILITY

Finally, we would like to calculate the requirements on the absorber, amplifier, and resonator for bistability. This can be done rather easily by noting that when the laser goes from a system displaying bistability to one displaying no nonlinearities the slope of the curve of A as a function of I, evaluated at $I=0$, changes sign. Thus at the onset of bistability $(dA/dI)|_{I=0}=0$. We therefore expand (3.17) and (3.18) in a Taylor series about $I=0$, take the derivative dA/dI and set it equal to zero. Only the result is quoted here (for our laser diode system), namely

$$C > 1+\frac{1}{(a-1)G}\ , \tag{5.1}$$

$$G=[\pi w_x w_y/(2Sd)]\mathrm{erf}(S/\sqrt{2}w_x)\mathrm{erf}(d/\sqrt{2}w_y)\ . \tag{5.2}$$

Here S is the stripe width and d is the active layer thickness. The requirement (5.1) for bistability is the same for the traveling-wave and standing-wave cases. The plane-wave result is recovered by setting $G=1$ in (5.1); G, therefore, accounts for the effects of radial variation. Using the numbers quoted in Sec. IV we have $C \gtrsim 4.004$; therefore in this case radial effects cause the required amount of cooperativity to increase by about a factor of 2.

VI. CONCLUDING REMARKS

We have developed a semiclassical theory of bistable lasers that takes into account radial variation in the gain, absorption, and intensity, and we have applied the results to semiconductor lasers. A new state equation was found which included four quantities: the ratio of the saturation intensities of the amplifier and absorber (a), and the gain $[A(\mathbf{r})]$, absorption $[C(\mathbf{r})]$, and intensity $[f^2(\mathbf{r})]$ as functions of position. We have found that standing waves decrease the output intensity and increase the switch-off value of A. Radial effects modify the output curves to a greater degree by shrinking the hysteresis loop in both directions. We have also found, at least within the present approximation, that only radial variation increases the amount of cooperativity required for bistability; this minimum requirement is unaffected by the presence of standing waves, contrary to the results for passive resonators. Our results are general enough to provide insight into other bistable laser systems.

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