Propagation of nonlinear surface polaritons

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The propagation of nonlinear p-polarized electromagnetic surface waves in a model previously studied by Yu is studied here without restrictions on the field amplitudes. s-polarized waves are also studied. Results for the dispersion relations and field profiles are reported.

There have been theoretical[1-8] as well as experimental[9] interests in the past few years on the propagation of nonlinear surface polaritons (NLSP) along the interfaces of materials whose dielectric functions depend on the strength of the electric field.[10] Despite the complexity of the problem exact theoretical results have been obtained under a variety of circumstances.[1-8,10]

Some of the problems associated with the propagation of p- and s-polarized NLSP will be studied here. For p-polarized waves the model previously employed by Yu[8] will be used. His results will be extended to all strengths of the nonlinearties. For s-polarized waves general results will be derived and then applied to two specific examples. Information regarding the dispersion relations and wave profiles will be obtained.

We consider two media whose dielectric functions[11] have the form $\epsilon_{ij}=\delta_{ij}(\epsilon_i^{(0)}+\epsilon_i^{(2)}(|\vec{\mathscr{E}}|^2))$, where $\vec{\mathscr{E}}$ is the electric field in the medium. Medium I occupies the region $z<0$, and medium II occupies the region $z>0$. For a monochromatic plane wave propagating along $\hat{x}$ we write the fields in the form

$$\vec{\mathscr{E}}(\mathbf{r},t)=\mathbf{E}(z)e^{i(k_x x-\omega t)}\ ,\qquad \vec{\mathscr{B}}(\mathbf{r},t)=\mathbf{B}(z)e^{i(k_x x-\omega t)}\ . \tag{1}$$

Solutions to Maxwell's equation can be separated into a p-polarized and an s-polarized wave. The p-polarized wave has $E_y=B_x=B_z=0$ and the nonzero components obey the set of equations

$$B_y'=i\epsilon_x E_x\ ,\qquad \eta^{1/2}B_y=-\epsilon_z E_y\ ,\qquad E_x'=i\eta^{1/2}E_z+iB_y\ , \tag{2}$$

where the prime denotes differentiation with respect to the dimensionless variable $\xi\equiv z\omega/c$, and $\eta\equiv(k_x c/\omega)^2$. The s-polarized wave has $E_x=E_z=B_y=0$ and obeys the set of equations

$$E_y'=-iB_x\ ,\qquad \eta^{1/2}E_y=B_z\ ,\qquad B_x'=i\eta^{1/2}B_z-i\epsilon_y E_y\ . \tag{3}$$

p-polarized wave. Following Yu[8] we assume that ϵ_x and ϵ_z depend on E_x^2 only. A first integral can be obtained for E_x in the form[8]

$$\tfrac{1}{2}(E_x')^2+V(E_x)=0\ , \tag{4}$$

where

$$V(E_x)=-f(E_x^2)/2A^2(E_x^2)\ , \tag{5}$$

$$f(E_x^2)=\int_0^{E_x^2}du\,\epsilon_x(u)A(u)\ , \tag{6}$$

$$A(E_x^2)=\epsilon_z(E_x^2)/[\eta-\epsilon_z(E_x^2)]\ . \tag{7}$$

The integration constant has been determined by the requirement that $E_x=E_x'=0$ at $|\xi|=\infty$. As pointed out by Yu[8] the boundary condition[12] $[B_y]=0$ can be imposed immediately without having to solve (4) for the electric field profile. The result gives a kind of dispersion relation

$$[f(E_x^2(0))]=0 \tag{8}$$

connecting the frequency of the wave to the wave vector k_x for a given value of the electric field at the interface $E_x(0)$.

As in Ref. 8 we assume that $\epsilon_x^{(2)\mathrm{II}}=\alpha E_x^2$ and $\epsilon_z^{(2)\mathrm{II}}=\gamma E_z^2$; however, γE_z^2 needs not be small compared with $\epsilon_z^{(0)\mathrm{II}}$ here. The integral in (6) can in fact be evaluated analytically with the result

$$f(E_x^2)=-\left[\epsilon_x^{(0)\mathrm{I}}+\eta\frac{\alpha}{\gamma}\right]E_x^2-\frac{\alpha}{2}E_x^4-\frac{\eta}{\gamma}\left[\epsilon_x^{(0)\mathrm{I}}+(\eta-\epsilon_z^{(0)\mathrm{I}})\frac{\alpha}{\gamma}\right]\ln\left|1-\frac{\gamma E_x^2}{\eta-\epsilon_z^{(0)\mathrm{II}}}\right|\ . \tag{9}$$

The dispersion relation can now be expressed explicitly in the form

$$([\epsilon_x^{(0)}]+\eta[\alpha/\beta])E_x^2(0)+\frac{[\alpha]}{2}E_x^4(0)+\left[\frac{\eta}{\beta}\left[\epsilon_x^{(0)}+\frac{\alpha}{\beta}(\eta-\epsilon_z^{(0)})\right]\ln\left|1-\frac{\beta E_x^2(0)}{\eta-\epsilon_z^{(0)}}\right|\right]\ . \tag{10}$$

Our result is valid for all strength of $E_x^2(0)$. The result of Yu[8,13] can be recovered by taking the limit $\gamma E_x^2(0)/\epsilon_z^{(0)}<<1$. The dispersion relation obtained by Agranovich, Babichenko, and Chernyak[2] is also a special case of (10).

With E_x interpreted as the "coordinate" and ξ as the "time," Eq. (4) formally describes the "classical motion" of a unit mass moving in a one-dimensional "potential" V with zero total energy.[14] A qualitative picture of the motion (which describes the x component of the electric field profile as a function of ξ) can be obtained simply by analyzing the form of V without having to solve (4) explicitly.

For small E_x, V has the form

$$V = -\frac{\epsilon_x^{(0)}}{2\epsilon_z^{(0)}}(\eta - \epsilon_z^{(0)})E_x^2 \ . \tag{11}$$

Thus, in order for the existence of nonperturbative large-amplitude solutions we must require that $\epsilon_x^{(0)}(\eta-\epsilon_z^{(0)})/\epsilon_z^{(0)} > 0$. The asymptotic form of V also implies that[14]

$$E_x^{\mathrm{I}} \sim e^{\xi/\xi^{\mathrm{I}}} \text{ for } 0 < -\xi << \xi^{\mathrm{I}} \ ,$$
$$E_x^{\mathrm{II}} \sim e^{-\xi/\xi^{\mathrm{II}}} \text{ for } 0 \le \xi << \xi^{\mathrm{II}} \ , \tag{12}$$

where $\xi^{\lambda} = (\epsilon_z^{(0)\lambda}/\epsilon_x^{(0)\lambda})^{1/2}/(\eta-\epsilon_z^{(0)\lambda})^{1/2}$, with $\lambda = \mathrm{I}$ or II, are the decay widths of the p-polarized NLSP along z in the two media.

To know more about the form of the wave profile we need to find the specific form of V from Eqs. (5), (7), and (9). Owing to the large number of physical parameters involved there are a number of separate cases that require consideration. For simplicity we assume that medium I is linear with $\epsilon_{ij} = \delta_{ij}\epsilon^{\mathrm{I}}$, and $\epsilon_x^{(0)\mathrm{II}}$, $\epsilon_z^{(0)\mathrm{II}}$, α, γ, and $\eta - \epsilon_z^{(0)\mathrm{II}}$ all are positive quantities. If we further have

$$\eta > \frac{\epsilon^{\mathrm{I}}\epsilon_z^{(0)\mathrm{II}}(\epsilon^{\mathrm{I}} - \epsilon_x^{(0)\mathrm{II}})}{(\epsilon^{\mathrm{I}})^2 - \epsilon_x^{(0)\mathrm{II}}\epsilon_z^{(0)\mathrm{II}}} \ ,$$

then there can only be two types of wave forms. For $\epsilon^{\mathrm{I}}/(\eta-\epsilon^{\mathrm{I}}) > 0$ the p-polarized NLSP has a single peak located in region II. Otherwise it has a single peak in the form of a cusp at the interface.

s-polarized wave. In this case a first integral for E_y can be written as

$$\tfrac{1}{2}(E_y')^2 + V(E_y) = 0 \ , \tag{13}$$

where

$$V(E_y) = -\tfrac{1}{2}\kappa_y^2 E_y^2 - \int_0^{E_y^2} du\, \epsilon_y^{(2)}(u) \ , \tag{14}$$

$$\kappa_y = (\eta - \epsilon_y^{(0)})^{1/2} \ . \tag{15}$$

V has the form

$$V(E_y) \simeq -\tfrac{1}{2}\kappa_y^2 E_y^2 \tag{16}$$

near the origin; thus, we must require $\eta > \epsilon_y^{(0)}$ in both media in order to find large amplitude solutions to (13).[14] The asymptotic wave profiles are given by

$$E_y^{\mathrm{I}} \sim e^{K_y^{\mathrm{I}}\xi}, \quad E_y^{\mathrm{II}} \sim e^{-K_y^{\mathrm{II}}\xi} \ , \tag{17}$$

with K_y^{-1} the decay width of the NLSP.

From the boundary conditions[12]

$$[E_y] = [E_y'] = 0 \tag{18}$$

we obtain a general dispersion relation of the form

$$[E_y^{(0)}]E_y^2(0) - \int_0^{E_y^2(0)} du\,[E_y^{(2)}(u)] = 0 \ . \tag{19}$$

Note that regardless of the form of $\epsilon_y^{(2)}(E_y^2)$, Eq. (19) when expressed in terms of the field at the interface, $E_y(0)$, is always independent of η as long as $\eta > \epsilon_y^{(0)}$.

To illustrate our method further we first consider the case where medium I is linear with $\epsilon_{ij}^{\mathrm{I}} = \delta_{ij}\epsilon_i^{(0)\mathrm{I}}$, and medium II has $\epsilon_i^{(2)\mathrm{II}}(|\vec{\mathscr{E}}|^2) = \alpha_i^{\mathrm{II}}E_y^2$. Then Eq. (14) gives

$$V^{\mathrm{I}}(E_y) = -\tfrac{1}{2}(\kappa_y^{\mathrm{I}})^2E_y^2 \ ,$$
$$V^{\mathrm{II}}(E_y) = -\tfrac{1}{2}[(\kappa_y^{\mathrm{II}})^2E_y^2 - \tfrac{1}{2}\alpha_y^{\mathrm{II}}E_y^4] \ . \tag{20}$$

The situation for $\kappa_y^{\mathrm{II}} > \kappa_y^{\mathrm{I}}$ (or equivalently $\epsilon_y^{(0)\mathrm{I}} > \epsilon_y^{(0)\mathrm{II}}$) and $\alpha_y^{\mathrm{II}} > 0$ (self-focusing) is depicted in Fig. 1(a). Since $E_y = E_y' = 0$ at $\xi = \pm\infty$, we imagine a particle starting at $\xi = -\infty$ from rest at the origin, rolling down the potential well V^{I}, and eventually rolling back up V^{II} to reach the origin again at $\xi = +\infty$. The boundary conditions force the particle to cross from V^{I} to V^{II} at the point where V^{I} and V^{II} intersect. The position where this happens is just $E_y(0)$. The "velocity" of the particle entering and leaving this point must remain unchanged. It is easy to see that these conditions give precisely the dispersion relation as in Eq. (19), which in the present case takes the form

$$\epsilon_y^{(0)\mathrm{I}} - \epsilon_y^{(0)\mathrm{II}} = \tfrac{1}{2}\alpha_y^{\mathrm{II}}E_y^2(0) \ . \tag{21}$$

This relation has been obtained before by Maradudin.[3,15] The only classically allowed motion is that shown by the arrows in Fig. 1(b). The NLSP must have a single peak located in medium II of height $E_{y,m} = (2/\alpha_y^{\mathrm{II}})^{1/2}\kappa_y^{\mathrm{II}}$, which is determined by the condition $V^{\mathrm{II}}(E_{y,m}) = 0$, or $E_{y,m}' = 0$. The energy flux per unit length in the y direction can be obtained by integrating the x component of the Poynting vector over z:

$$S_x = \frac{c}{8\pi}\,\mathrm{Re}\int_{-\infty}^{+\infty} dz\, \mathscr{E}_y \mathscr{B}_z^* = \frac{c}{8\pi}\eta^{1/2}\int_{-\infty}^{+\infty} dz E_y^2 \ . \tag{22}$$

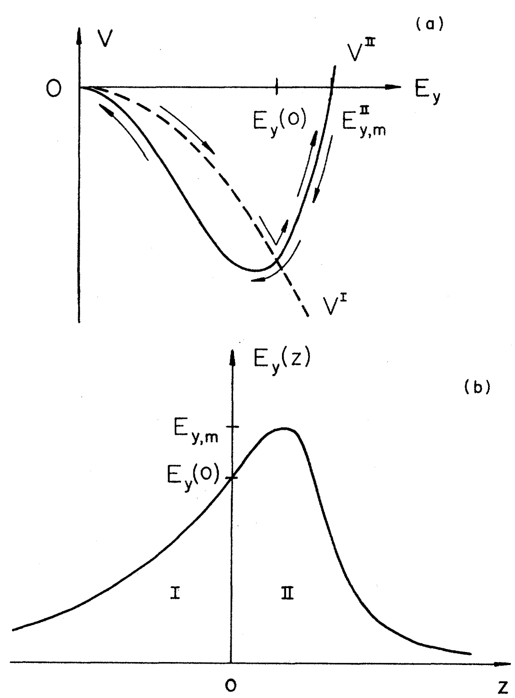

FIG. 1. (a) The form of V in regions I and II for the case where $\kappa_y^{\mathrm{II}} > \kappa_y^{\mathrm{I}}$ and $\alpha_y^{\mathrm{II}} > 0$. The classically allowed "motion" is shown by the arrows. (b) The corresponding wave profile for the s-polarized NLSP.

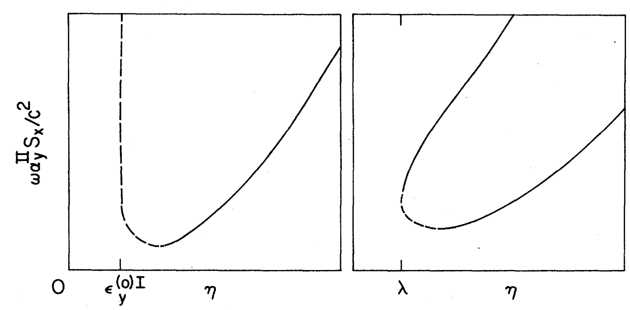

FIG. 2. (a) Schematic plot of the dispersion relation in terms of the averaged energy flux along $\hat{x}$ as a function of $\eta=(k_xc/\omega)^2$. The y axis has been expressed in a dimensionless form. (b) Schematic plot of the dispersion relations for mode (1) and (2) in terms of the averaged energy flux along $\hat{x}$ as a function of $\eta=(k_xc/\omega)^2$. λ is defined as

$$[\epsilon_y^{(0)II}-\epsilon_y^{(0)I}(\alpha_y^{II}/\alpha_y^{I})]/(1-\alpha_y^{II}/\alpha_y^{I}) \ .$$

Parameter values correspond to $\epsilon_y^{(0)II}>\epsilon_y^{(0)I}>0$ and $\alpha_y^{I}>\alpha_y^{II}>0$.

The result is shown in Fig. 2(a). The dotted line denotes part of the curve where $\alpha_y^{II}\omega S_x/c^2$ decreases with increasing η.

No s-polarized NLSP solution can be allowed for $\kappa_y^{II}<\kappa_y^{I}$ with $\alpha_y^{II}>0$, or for $\kappa_y^{II}>\kappa_y^{I}$ with $\alpha_y^{I}<0$, since V^{I} and V^{II} do not intersect at all except at $E_y=0$. For $\kappa_y^{II}<\kappa_y^{I}$ and $\alpha_y^{II}<0$ the available solutions represent propagating modes with an infinite flux of electromagnetic energy, and must be ignored.[16] Thus the only allowed modes are of the type shown in Fig. 1.

Next consider the case where both media have a dielectric function of the form $\epsilon_{ij}=\delta_{ij}(\epsilon_i^{(0)}+\alpha_i|\vec{\mathscr{E}}|^2)$. In both regions V is given by

$$V(E_y)=-\tfrac{1}{2}(\kappa_y^2E_y^2-\tfrac{1}{2}\alpha_yE_y^4) \ . \tag{23}$$

There are two subcases of interest here. Subcase (i) has $\kappa_y^{I}>\kappa_y^{II}$, $\alpha_y^{I}>\alpha_y^{II}>0$, and $E_{y,m}^{II}>E_{y,m}^{I}$. V is plotted in Fig. 3(a). It is clear that two different NLSP modes can exist. Mode (1) has a peak located in medium II. It corresponds to the motion of a particle rolling down V^{I} from the origin, crossing over to V^{II} on the first passage at $E_y(0)$, bouncing off the wall at $E_{y,m}^{II}$ and then staying on V^{II} until finally rolling back to the origin. Mode (2) has a peak which is relatively narrower and lower compared with mode (1), and is located in medium I. It corresponds to the motion where the particle rolls down from the origin on V^{I}, continues on V^{I} after passing through $E_y(0)$, bounces off from the wall at $E_{y,m}^{I}$ until it reaches $E_y(0)$ again, where it now crosses over to V^{II} and rolls back to the origin [see Fig. 3(b)].

The dispersion relations in terms of the energy flux can also be calculated for these two modes. The results are shown in Fig. 2(b). For mode (2) a portion of the curve (denoted by the dots) has $\alpha_y^{II}\omega S_x/c^2$ decreasing with increasing η. Note that by virtue of the nonlinearity these two modes cannot be excited simultaneously.

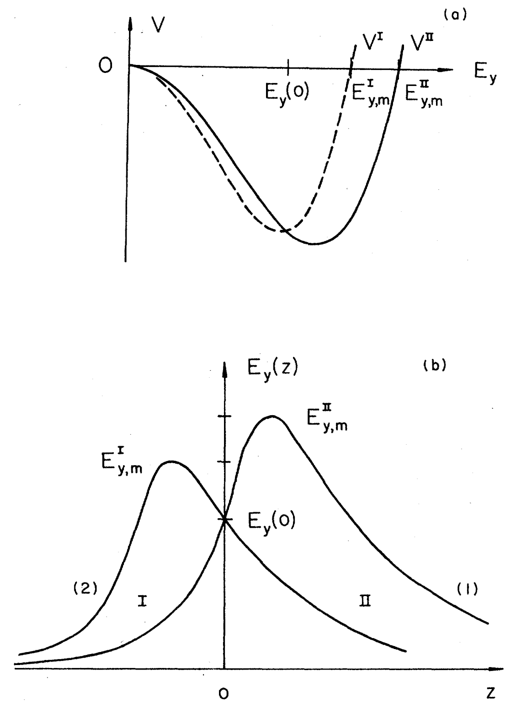

FIG. 3. (a) The form of V in regions I and II for the case where $\kappa_y^{I}>\kappa_y^{II}$, $\alpha_y^{I}>\alpha_y^{II}>0$, and $(\kappa_y^{II})^2/\alpha_y^{II}>(\kappa_y^{I})^2/\alpha_y^{I}$. (b) Schematic picture of the wave profiles for mode (1) and (2).

From Figs. 2(a) and 2(b) one can see that S_x has a threshold characteristic, which means that these modes can have no counterpart whatsoever in the corresponding linear regime.

Subcase (ii) has $\kappa_y^{I}<\kappa_y^{II}$, $\alpha_y^{I}<0$, and $\alpha_y^{II}>0$. The situation here is very similar to that discussed before where medium I is linear and medium II is self-focusing. There is only one allowed s-polarized NLSP mode. E_y has a single peak in medium II similar to that shown in Fig. 1(b).

Clearly there are two more subcases which can be obtained from subcases (i) and (ii) by interchanging κ_y^{I} with κ_y^{II}, α_y^{I} with α_y^{II}, and ξ with $-\xi$. Besides those discussed above there are no other s-polarized NLSP that can exist in this model.

A natural question to ask now is whether these NLSP are stable. In order to answer this question one needs to find the modes propagating along z in addition to the surface localized modes discussed here. These interesting and important problems will be left for future studies.

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[10]For a review on the subject see A. A. Maradudin, in *Proceedings of the Second International School on Condensed Matter Physics*, Varna, Bulgaria (Scientific, New York, 1982).

[11]We assume that the dielectric functions are all real.

[12]The square brackets will be used to denote the discontinuity of the value of its contents across the boundary at $z=0$.

[13]There is a misprint in Ref. 8. The definition of b above Eq. (12) should have been $b=\beta k^2 \kappa_z^{-4}$, in his notation.

[14]At this stage ideas from soliton studies can be utilized. See K.M. Leung, Phys. Rev. B **26**, 226 (1982).

[15]There is a misprint in Eq. (19) of Ref. 3. The right-hand side of Eq. (19) should have an overall minus sign.

[16]Although it is possible to find solutions with the correct boundary conditions but with part of the motion consists of the particle rolling off to $V=-\infty$ with E_y going to $+\infty$ and then returning from $V=-\infty$ with E_y increasing from $-\infty$. Solutions of this type carry an *infinite energy flux* and *must be ignored.* To see this, let ξ_0 be the "time" when the particle reaches $V=-\infty$. Then in this vicinity we have $V \sim -E_y^4$ and so $E_y^1 \sim E_y^2$. Thus, $E_y \sim (\xi-\xi_0)^{-1}$ diverges at z_0. The energy flux which goes as $\int d\xi (\xi-\xi_0)^{-2}$ therefore also diverges.