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Atomic bremsstrahlung produced by heavy-ion bombardments

Keizo Ishii
Cyclotron and Radioisotope Center, Tohoku University, Sendai 980, Japan

Susumu Morita
Research Center of Ion Beam Technology, Hosei University,
Koganei, Tokyo 184, Japan

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In accordance with the plane-wave Born approximation, contribution of atomic bremsstrahlung (AB) in heavy-ion–atom collisions has been estimated and compared with experimental results. It is found that AB is negligibly small in symmetric or nearly symmetric collisions, whereas it becomes predominant in asymmetric collisions. Thus continuum x rays produced in asymmetric heavy-ion–atom collisions, which have not fully been interpreted yet, can be understood by taking account of the atomic bremsstrahlung.

Continuum or broad spectral x rays produced in heavy-ion–atom collisions have been interpreted in terms of molecular-orbital (MO) process[1] and radiative electron capture.[2] Especially, the intensity of MO x rays is much enhanced when an electron orbital of the projectile well overlaps with that of the target atom.[3] Up to the present, several ideas and experimental results on the production of these x rays have been reported and recently summarized by Anholt.[4] Stott and Waddington[5] have observed the x rays in cases of asymmetric collision and reported that the intensity of the continuum x rays from a Au target bombarded with C, O, Cl, and Br ions, cannot be explained in terms of Mo x rays. Anholt and Saylor[6] have calculated the cross sections of radiative ionization (RI) for O + Zr and O + Au collisions. Further, Anholt and Salin[7] have estimated the contribution of quasimolecular bremsstrahlung (QMB) in Kr + Ti collisions. The contribution of RI and QMB, however, was found to be negligibly small in comparison with the experimental x-ray yields observed in these collisions, and the continuum x rays produced in asymmetric collision still remain to be solved.

Recently,[8] we have calculated the cross section of atomic bremsstrahlung (AB) in the case of light-ion bombardments and have shown that AB is predominant in the region of high x-ray energy. By using the method previously developed, we estimate here the contribution of AB in the heavy-ion–atom collision, which corresponds to QMB. The T-matrix element T_{fi} of AB for heavy-ion bombardments, following the same way as was done in our recent report,[8] is given by

$$\begin{aligned}T_{fi}=&-\frac{Z_T e^3}{m_e}\frac{4\pi}{q^2}\left(\frac{2\pi}{\omega}\right)^{1/2}\left(\sum_m \frac{\left\langle \chi_i\left|\sum_{i=1}^{N_p}(-i\hbar\mathbf{e}\cdot\nabla_i)\right|\chi_m\right\rangle\left\langle\chi_m\left|\sum_{i=1}^{N_p}e^{-\mathbf{q}\cdot\mathbf{r}_i}\right|\chi_i\right\rangle}{\epsilon_{p,i}+\hbar\omega-\epsilon_{p,m}+i\epsilon}+\frac{\left\langle \chi_i\left|\sum_{i=1}^{N_p}e^{-i\mathbf{q}\cdot\mathbf{r}_i}\right|\chi_m\right\rangle\left\langle\chi_m\left|\sum_{i=1}^{N_p}(-i\hbar\mathbf{e}\cdot\nabla_i)\right|\chi_i\right\rangle}{\epsilon_{p,i}-\hbar\omega-\epsilon_{p,m}+i\epsilon}\right)\\&-\frac{Z_p e^3}{m_e}\frac{4\pi}{q^2}\left(\frac{2\pi}{\omega}\right)^{1/2}\left(\sum_m \frac{\left\langle \psi_i\left|\sum_{k=1}^{N_T}(-i\hbar\mathbf{e}\cdot\nabla_k)\right|\psi_m\right\rangle\left\langle\psi_m\left|\sum_{k=1}^{N_T}e^{i\mathbf{q}\cdot\mathbf{r}_k}\right|\psi_i\right\rangle}{\epsilon_{T,i}+\hbar\omega-\epsilon_{T,m}+i\epsilon}+\frac{\left\langle \psi_i\left|\sum_{k=1}^{N_T}e^{i\mathbf{q}\cdot\mathbf{r}_k}\right|\psi_m\right\rangle\left\langle\psi_m\left|\sum_{k=1}^{N_T}(-i\hbar\mathbf{e}\cdot\nabla_k)\right|\psi_i\right\rangle}{\epsilon_{T,i}-\hbar\omega-\epsilon_{T,m}+i\epsilon}\right),\end{aligned}\qquad(1)$$

with

$$H_p\chi_m(\mathbf{r}_1,\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_{N_p})=\epsilon_{p,m}\chi_m(\mathbf{r}_1,\ldots,\mathbf{r}_i,\ldots,\mathbf{r}_{N_p})$$

and

$$H_T\psi_m(\mathbf{r}_1,\ldots,\mathbf{r}_k,\ldots,\mathbf{r}_{N_T})=\epsilon_{T,m}\psi_m(\mathbf{r}_1,\ldots,\mathbf{r}_k,\ldots,\mathbf{r}_{N_T})$$

where Z_p (Z_T), N_p (N_T), $\epsilon_{p,i}$ ($\epsilon_{T,i}$), χ_i (ψ_i), and H_p (H_T) are, respectively, the atomic number, the number of electrons, energy of the initial electron state, the wave function of the initial electron state, and the Hamiltonian of electrons of the projectile ion (the target atom); m_e is the electron rest mass, $\hbar q$ the transfer momentum of the incident ion, $\hbar\omega$ energy of the emitted photon, $\mathbf{r}_i$ ($\mathbf{r}_k$) the position vector with respect to the projectile nucleus (the target nucleus), ∇_i the differential operator of $\mathbf{r}_i$, and $\mathbf{e}$ the polarization vector of the photon.

In the derivation of Eq. (1), the condition $qa \ll m_T/m_e$ and m_p/m_e is assumed, where m_p (m_T) is mass of the projectile nucleus (the target nucleus) and a is the radius of the target atom or the incident ion; this means that the nuclear bremsstrahlung can be neglected. Then, the dipole approximation can be applied to the radiative interaction. Electrons of the target atom and the projectile are classified according to the nucleus with which they belong, and the Coulomb interaction between a target electron and a projectile electron might be neglected.

By taking the wave functions χ and ψ as a single product of hydrogenlike functions, the production cross section of atomic bremsstrahlung for heavy-ion bombardments can be expressed by

$$\frac{d\sigma^{\mathrm{AB}}}{d(\hbar\omega)\,d\Omega_\omega}=\frac{8a_0^2\alpha^5}{\pi\hbar\omega}\left(\frac{c}{v_p}\right)^2\int_{\omega/v_p}^{\infty}\frac{dq}{q}\left\{1-\left(\frac{\omega}{qv_p}\right)^2+\left[\frac{3}{2}\left(\frac{\omega}{qv_p}\right)^2-\frac{1}{2}\right]\sin^2\theta_\omega\right\}|Z_pS(Z_T,N_T,q)-Z_TS(Z_p,N_p,q)|^2\ , \qquad (2)$$

with

$$S(Z_T,N_T,q)=S_1(Z_T,N_T,q)-S_2(Z_T,N_T,q)\ ,$$

where, a_0, α, v_p, and θ_ω are the Bohr radius, the fine-structure constant, velocity of the projectile, and the emission angle of photon with respect to the direction of incident particles, respectively, and the expressions of S_1 and S_2 have been given in our recent article.[8]

It is seen from Eq. (2) that the cross section of AB vanishes in the case of symmetric collision ($Z_p=Z_T$ and $N_p=N_T$). This fact corresponds to the case of $Z_p/m_p=Z_T/m_T$ in the nuclear-bremsstrahlung production, and just reflects the characteristic of the dipole radiation.

The calculation is now compared with experimental results on heavy-ion bombardments. Figure 1 shows comparison of the present calculation with the experimental result on Ti bombarded with 1.4-MeV/amu Kr ions. The ordinate represents the continuum x-ray yields for the thick target. The experimental results were taken from Figs. 4 and 7 in Ref. 9 and are shown by the dot and dashed line. The thick-target yields of AB were estimated from

$$\frac{dY^{\mathrm{AB}}}{d(\hbar\omega)}=\int_0^{E_p}dE_p'\frac{1}{S(E_p')}\int d\Omega_\omega\frac{d^2\sigma^{\mathrm{AB}}}{d(\hbar\omega)\,d\Omega_\omega}(Z_T,N_T,Z_p,N_p(E_p'))\ , \qquad (3)$$

where E_p is incident energy of the projectile, $S(E_p')$ the stopping power[10] of the target for the projectile with energy E_p', and $N_p(E_p')$ the number of electrons of the incident ion with energy E_p' and was estimated from the equilibrium charge state[11] of the projectile in the target material. The solid line in Fig. 1 represents the prediction of AB calculated from Eq. (3). It is seen in Fig. 1 that the present theory of AB can well reproduce the experimental result. The target-atom dependence of the continuum x-ray yield for 1.4-MeV/amu Kr, Ni, and Ti bombardments is shown, respectively, in Figs. 2, 3, and 4, where the data points[9] represent the thick-target yields at the reduced x-ray energy $\nu(\equiv\hbar\omega/E^U_{K_{\alpha_1}})=0.7$, $E^U_{K_{\alpha_1}}$ being the K_{α_1} x-ray energy of the united atom; the dashed curves show the prediction of MO x rays based on the scaling-law method by Anholt[8] multiplied by factors of 1.1, 2.1, and 1.5, respectively, for Kr, Ni, and Ti bombardments. The solid curves are the thick-target

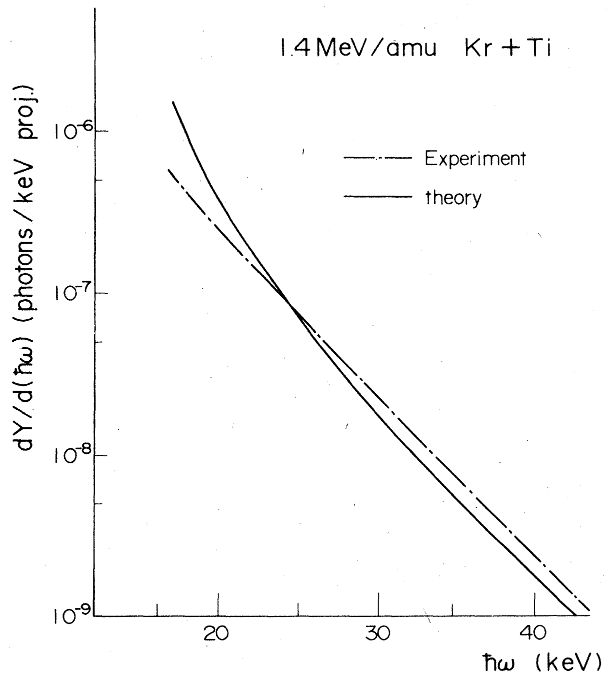

FIG. 1. Continuum x-ray yields for a thick Ti target bombarded with 1.4-MeV/amu Kr ions. The dot and dashed line represents the experimental results taken from Figs. 4 ands 7 in Ref. 9. The solid line is the prediction of atomic bremsstrahlung calculated from Eq. (3).

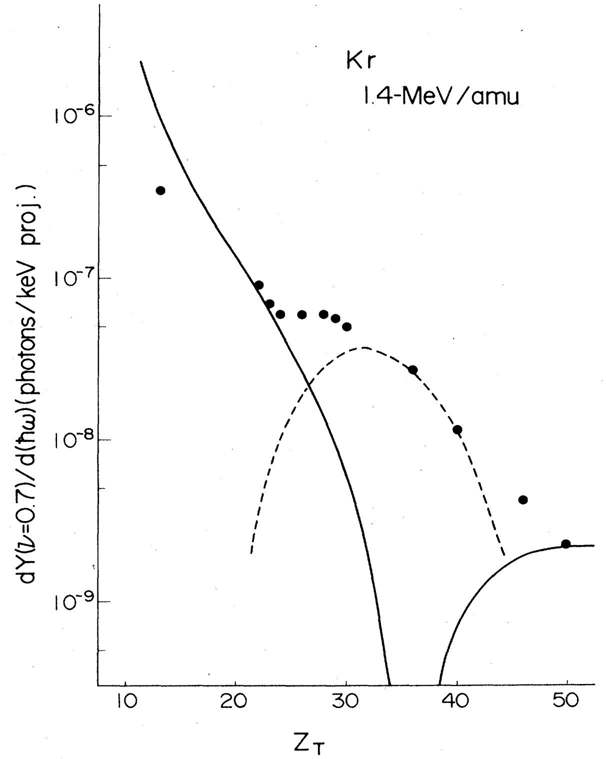

FIG. 2. Target-atom dependence of the continuum x ray produced by 1.4 MeV/amu Kr ion bombardments. The data points (Ref. 9) show thick-target yields of continuum x rays at the photon energy $\nu=0.7$. The dashed line represents the scaling-law calculation of two-collision Mo x rays by Anholt (Ref. 9), multiplied by 1.1, and the solid line is the prediction of atomic bremsstrahlung from Eq. (3).

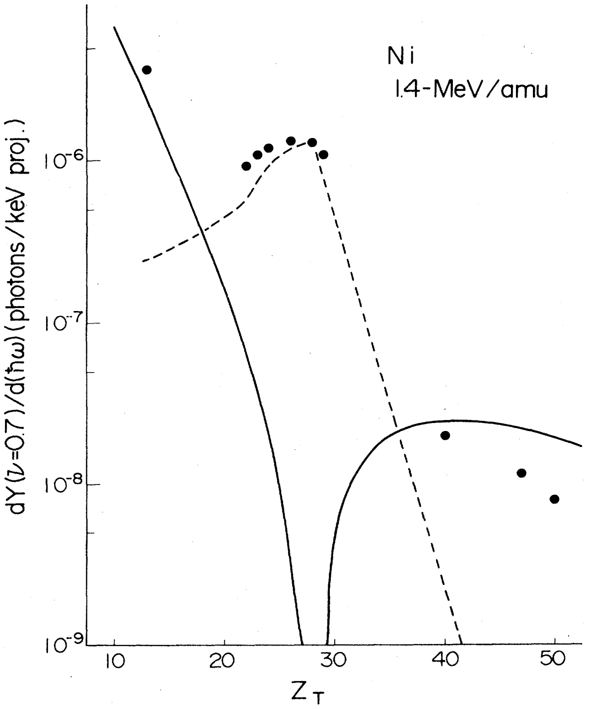

FIG. 3. Same as Fig. 2, except for Ni-ion bombardments and the multiplication factor of 2.1 for the scaling-law calculation.

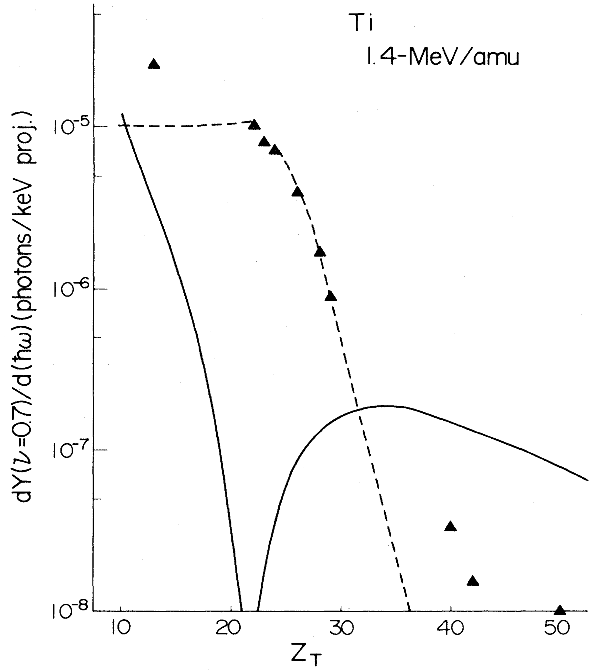

FIG. 4. Same as Fig. 2, except for Ti-ion bombardments and the multiplication factor of 1.5 for the scaling-law calculation.

yields of AB calculated from Eq. (3). As expected from Eq. (2), the contribution of AB is negligible in the vicinity of symmetric collisions and becomes predominant in the region of asymmetric collisions. Generally, the present theory reproduces well the experimental results, while the prediction overestimates the experiment in the region of the target-atomic number $Z_T = 40$–50 for Ti bombardment. Since the present theory is simply based on the plane-wave Born approximation, the Coulomb deflection effect[12] of the projectile by the target nucleus, the screening effect for the electric charge of the projectile, and the binding-energy[13] and polarization[14] effects, i.e., the molecular-orbital effect, should be taken into account. The relativistic effect[15] on the electronic wave function should also be taken into account. In the present case, however, the Coulomb deflection effect has been estimated to be small on the analogy of K-shell ionization,[12] and the screening effect has also been found to be small, since, if we consider a screened potential of Yukawa type $e^{-r/a_K}/r$, where a_K is the K-shell radius and, if we estimate the screening effect from the Fourier transformation of this potential, the transfer momentum q is found to be much larger than $a_K{}^{-1}$.

Anholt and Salin[6] have calculated the production cross section of AB for heavy-ion bombardments, that is, QMB, on the basis of a semiclassical method, and have shown that QMB can be neglected in comparison with MO x rays and rapidly decreases with increase in the x-ray energy. This discrepancy between our calculation and that of Anholt and Salin for the high-energy photon emission might be solved by calculating QMB process on the basis of the quantum method.[16]

Although our present theory for the atomic bremsstrahlung production may be improved by taking account of the effects mentioned above, especially the MO process, it must be noticed that the present theory can well represent the characteristics of AB and the predominant part of the continuum x rays produced by asymmetric collisions should be interpreted in terms of the atomic bremsstrahlung.

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