Influence of damping on quantum interference: An exactly soluble model

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This paper reports the result of a calculation which shows the effect of damping on the quantum interference of two Gaussian wave packets in a harmonic potential. We use the influence-functional method, which seems to be the most appropriate one for this kind of calculation. It is shown that quantum interference effects are severely diminished by the presence of damping even when its influence on the system is only light. The corrections to the undamped formulas are always expressible in terms of the phenomenological damping constant γ, the temperature T (in the high-temperature limit), the cutoff frequency (Ω) of the reservoir oscillators, and the mean number (N) of quanta of energy initially present in the system. Both weakly and strongly damped systems are analyzed in the regime of low and high temperatures.

I. INTRODUCTION

During the past few years there has been an increasing interest in the study of the influence that damping in a physical system exerts on its quantum effects. Special attention has been paid to the phenomena of quantum tunneling[1,2] and quantum coherence,[3–5] when it is taken into account that the physical system under study is not isolated but is interacting with a heat bath instead. The main reason to deal with this type of problem arises from the very nature of the systems in question. In general, these systems are macroscopic ones such that quantum effects should involve a linear superposition of states which are macroscopically distinguishable.[6] On the other hand, it is well known that the macroscopic variable which describes the dynamics of the physical quantity to be measured in the system is always coupled to its internal (microscopic) degrees of freedom. Consequently, this variable can be thought of as being representative of a system coupled to a heat bath, and, when certain approximations are allowed,[7] its dynamics is governed in the classical limit by the standard Langevin equation [see Eq. (1.2) below]. Several examples can be given, but two of them are of special interest: SQUID (superconducting quantum interference device) rings and current-biased Josephson junctions. For the former, the flux trapped inside the ring is the variable which can display quantum effects, while for the latter this variable is the phase difference of the macroscopic wave function across the junction. In both cases normal electrons represent the heat bath.[8] These two systems have been investigated at very low temperatures on the theoretical[2,8] as well as on the experimental side,[9] and the results are in qualitative agreement with each other.

At this point the reader should be reminded that in both examples given above quantum tunneling plays an essential role. However, the influence of the coupling to the environment (or the influence of damping, to be more specific) [see Ref. (2) for the difference between the two cases] on the quantum properties of the system in question can be shown to exist even in the classically accessible region. To exemplify this point, suppose one wishes to investigate what happens to two undamped wave packets subject to the same harmonic potential when they are initially separated from each other with one of them at rest at the origin. This situation can be realized by preparing the initial state of the system as the sum of two Gaussian wave packets, one centered at the origin and the other one at a point far apart from the first. As one releases the finite-amplitude wave packet, the total probability density of the system starts to change in time. It is trivial to show that

$$\rho(x,t)\equiv\psi^*(x,t)\psi(x,t) = \rho_1(x,t)+\rho_2(x,t)+\rho_{\rm int}(x,t)\ , \tag{1.1}$$

where $\rho_1(x,t)=\psi_1^*(x,t)\psi_1(x,t)$ $[\rho_2(x,t)=\psi_2^*(x,t)\psi_2(x,t)]$ would be the probability density if the first (the second) packet were the only one representing the system and $\rho_{\rm int}(x,t)$ is the interference term. When the two packets are very far apart, $\rho_{\rm int}(x,t)$ is vanishingly small. However, as the second packet moves toward the one at rest, this term increases. When the two packets start to overlap, an interference pattern starts to develop due to the existence of $\rho_{\rm int}(x,t)$. This interference term achieves its maximum when the centers of the first and second packets coincide. So far, nothing is new. This is a standard problem of introductory quantum mechanics.

A very interesting question can now be raised if one studies the same problem in a viscous medium where the classical equation of motion for a particle reads

$$M\ddot{x}+\eta\dot{x}+M\omega_R^2 x=F(t) \tag{1.2}$$

with $\langle F(t)F(t')\rangle=2\eta kT\delta(t-t')$. It is a well-known result [see, e.g., Ref. (7)] that one of those packets moving

by itself in the harmonic potential would have its center performing the classical motion of a damped particle while its width would evolve in time in accordance with the asymptotic behavior given by the fluctuation dissipation theorem.[10] The time it takes for that packet to lose all its energy is of the order of γ^{-1} for very lightly damped systems and $\gamma\omega_R^{-2}$ for heavily damped ones ($\gamma \equiv \eta/2M$). The question is then what happens to the interference term of the preceding paragraph if one considers two packets instead of only one in the same potential. The centers of the two packets still perform classical motion as expected, but does the interference pattern still exist when the two packets go on coinciding cycle by cycle? Is it destroyed at the same rate as the finite-amplitude packet loses its energy? If not, can the formula expressing the destruction of the interference pattern be written in terms of the phenomenological quantities appearing in the classical equation of motion of the damped particle? What are the differences between underdamped and overdamped cases, and between low and high temperatures? These are the questions to be investigated from now on.

The principal aim of this paper is to give an explicit evaluation of the time-dependent density matrix of the system for an exactly soluble model of a quantum damped harmonic oscillator with the initial conditions specified above. The interpretation of these results raises some quite subtle points, not all of which we can claim to have completely explored in this paper.

Section II is a brief review of the model to be employed and also contains the general results. In Secs. III and IV the high- and low-temperature cases are treated. Section V is devoted to the interpretation of the results and to further discussion of them. Finally, the conclusions are presented in Sec. VI.

II. MODEL

There are many ways to deal with damping in quantum mechanics but the system-plus-reservoir approach is the one that will be employed here. In particular it is the Feynman-Vernon[11] influence-functional technique, which has recently been applied to quantum Brownian motion,[7,12] which is the most suitable one to attack the above-mentioned questions. The reason for this is that in the Feynman-Vernon approach one directly finds the time evolution of the reduced density operator of the system in question as[7,11]

$$\rho(x,y,t)=\int J(x,y,t;x',y',0)\rho(x',y',0)dx'\,dy' , \quad (2.1)$$

where J, which is the propagator for ρ, can be written in the form[7]

$$J(x,y,t;x',y',0)$$
$$=\int\int Dx\,Dy \exp\left[\frac{i}{\hbar}\left[S_R[x]-S_R[y]-M\gamma\int_0^t[x(\tau)\dot{x}(\tau)-y(\tau)\dot{y}(\tau)+x(\tau)\dot{y}(\tau)-y(\tau)\dot{x}(\tau)]d\tau\right]\right]$$
$$\times\exp\left[-\frac{1}{\hbar}\frac{2M\gamma}{\pi}\int_0^\Omega \nu\coth\left[\frac{\hbar\nu}{2kT}\right]\int_0^t\int_0^\tau[x(\tau)-y(\tau)]\cos[\nu(\tau-s)][x(s)-y(s)]ds\,d\tau\,d\nu\right] . \quad (2.2)$$

Here T is the temperature, k is the Boltzmann constant, and

$$S_R[x]=\int_0^t(\tfrac{1}{2}M\dot{x}^2-\tfrac{1}{2}M\omega_R^2x^2)d\tau \quad (2.3)$$

is the action renormalized by the presence of the bath (see Ref. 7, Sec. III). Expression (2.2) is obtained by considering that the system of interest is linearly coupled to a bath of oscillators through a coordinate-coordinate coupling. Moreover, the set of oscillators is assumed to have a given spectral density and frequency cutoff Ω. For details of the influence-functional technique applied to quantum Brownian motion the reader is referred to Ref. 7.

It should be noted that the expression (2.2) is derived under the assumption that at $t=0$ the environment is in what would be its thermal-equilibrium state in the absence of coupling to the system. It is a delicate question how sensitive the various details of the results to be derived below are to this assumption.

In order to study the effect of damping on quantum interference the only thing one has to do is to follow the time evolution of $\rho(x,x,t)$ given by (2.1) when $y=x$ and $\rho(x',y',0)$ has the appropriate form; namely, it is a superposition of two well-localized Gaussian wave packets with centers far apart from each other. Suppose one initially prepares the system of interest in a state ψ given by

$$\psi(x)=\tilde{\mathcal{N}}\left[\exp\left[-\frac{x^2}{4\sigma^2}\right]+\exp\left[-\frac{(x-z)^2}{4\sigma^2}\right]\right] , \quad (2.4)$$

where $\tilde{\mathcal{N}}$ is the normalization constant, z is the initial distance between the centers of the packets, σ is the width of each packet (assumed to be equal only to simplify future calculations), and $z \gg \sigma$, which means that we can distinguish one packet from the other at $t=0$. (Note, however, that there is always an exponentially small overlap between the two packets.) Multiplying $\psi(y)$ by $\psi^*(x)$, one finds $\rho(x,y,0)$ which is given by (1.1) with

$$\rho_{\text{int}}(x,y,0)=\tilde{\mathcal{N}}^2\left[\exp\left[-\frac{x^2+(y-z)^2}{4\sigma^2}\right]+\exp\left[-\frac{y^2+(x-z)^2}{4\sigma^2}\right]\right] . \quad (2.5)$$

By linearity one can say that the time evolution of $\rho(x,x,t)$ is given by the sum of the time evolution of each of its initial components. The time evolution of $\rho_1(x,x,t)\equiv\rho_1(x,t)$ and $\rho_2(x,x,t)\equiv\rho_2(x,t)$ clearly describes the motion performed by each one of the packets if it

were alone in that potential (see Ref. 7). The interesting information is in $\rho_{\rm int}(x,x,t)\equiv\rho_{\rm int}(x,t)$ which is given by

$$\rho_{\rm int}(x,t)=\int J(x,x,t;x',y',0)\rho_{\rm int}(x',y',0)dx'\,dy' \tag{2.6}$$

with the initial reduced density operator as in (2.5). If we take $\gamma=0$ in (2.2), this interference term is the undamped one which has been previously discussed. The packet initially centered at z oscillates back and forth and each time it overlaps the one at the origin an interference pattern develops; at the time the centers coincide, we have

$$\rho_{\rm int}\left[x,x,t=\frac{n\pi}{\omega_R}+\frac{\pi}{2\omega_R}\right]=\text{const}\cos\left[\frac{z}{\sigma^2}x\right]\exp\left[-\frac{x^2}{\sigma^2}\right], \tag{2.7}$$

where the oscillatory term is due to the momentum acquired by the finite-energy wave packet as it moves under the influence of the harmonic force. However, this is not the limit in which one is interested. The term depending on γ must be kept finite in Eq. (2.2) if one wishes to study the influence damping has on quantum interference.

The path integral (2.2) can be evaluated exactly and this has been done in Eq. (6.26) of Ref. 7 [in this equation $\xi\equiv(x-y)$ and $X\equiv(x+y)$]. Although there is no conceptual difficulty in doing this, its evaluation is too lengthy to be reproduced in this paper. Therefore, what is quoted here is only the final form of $\rho_{\rm int}(x,t)$ given by (2.6). In order to do this in a convenient way a new set of parameters and functions should be introduced. For the underdamped motion ($\omega_R>\gamma$) they are

$$\sigma^2=\frac{\hbar}{2M\omega_R},\quad \omega^2\equiv\omega_R^2-\gamma^2,\quad R\equiv\frac{\gamma}{\omega_R},\quad S\equiv\frac{\omega}{\omega_R},\quad \theta\equiv\omega_R t,\quad K\equiv\frac{\hbar\omega_R}{2kT},\quad \lambda_c\equiv\frac{\Omega}{\omega_R},$$

$$z(\theta)\equiv z\left[\frac{R\sin(S\theta)}{S}+\cos(S\theta)\right]\exp(-R\theta),$$

$$C_R(\theta,\lambda)\equiv\frac{1}{\sin^2(S\theta)}\int_0^\theta\int_0^\theta \sin[S(\theta-\theta_1)]\cos[\lambda(\theta_1-\theta_2)]\sin[S(\theta-\theta_2)]\exp[R(\theta_1+\theta_2)]d\theta_1\,d\theta_2, \tag{2.8}$$

$$I_R(\theta)\equiv\frac{4}{\pi}\int_0^{\lambda_c}d\lambda\,\lambda C_R(\theta,\lambda)\coth(K\lambda),\quad Q(\theta)\equiv 1+RI_R(\theta)+(R+S\cot\theta)^2,$$

$$\sigma^2(\theta)\equiv\frac{\sigma^2 Q(\theta)\sin^2(S\theta)\exp(-2R\theta)}{S^2},$$

while for the overdamped motion one should make the replacement $\omega^2\to\tilde{\omega}^2$ ($\tilde{\omega}^2\equiv\gamma^2-\omega_R^2$), $S\to i\tilde{S}$ ($\tilde{S}\equiv\tilde{\omega}/\omega_R$). Notice that σ was chosen to be the minimum uncertainty width only to simplify the expressions defined above. This will not change the physical significance of the final results.

With the help of all these newly defined quantities the expression for $\rho_{\rm int}(x,t)$ can be finally written down as

$$\rho_{\rm int}(x,t)\equiv\rho_{\rm int}(x,x,\theta)=A(\theta)\cos\left[\frac{1}{2\sigma^2(\theta)[R+S\cot(S\theta)]}\{[x-z(\theta)]^2-x^2\}\right]\times\exp\left[-\frac{x^2}{4\sigma^2(\theta)}\right]\exp\left[-\frac{[x-z(\theta)]^2}{4\sigma^2(\theta)}\right]\exp\left[-\frac{z^2RI_R(\theta)}{8\sigma^2Q(\theta)}\right], \tag{2.9}$$

where $A(\theta)$ is a time-dependent amplitude which is given by $4\sigma^{-1}(\theta)\sigma\tilde{\mathcal{N}}^2$. It is easy to identify $z(\theta)$ and $\sigma(\theta)$ as describing the time evolution of the center and width of the wave packets since the total reduced density operator can always be cast into the form

$$\rho(x,\theta)=\rho_1(x,\theta)+\rho_2(x,\theta)+2[\rho_1(x,\theta)]^{1/2}[\rho_2(x,\theta)]^{1/2}\cos\phi(x,\theta)\times\exp\left[-\frac{z^2RI_R(\theta)}{8\sigma^2Q(\theta)}\right], \tag{2.10}$$

where the third term on the right-hand side (rhs) clearly corresponds to $\rho_{\rm int}$. This expression can be easily verified if one assumes that $\tilde{\mathcal{N}}^2\simeq(2\sqrt{2\pi\sigma^2})^{-1}$ which means that the initial overlap between $\psi_1(x)$ and $\psi_2(x)$ has been neglected.

A glance at (2.10) is enough to convince oneself that the only term which will modulate the intensity of the interference fringes is the last exponential on the rhs of that expression. Thus one has to analyze this term very carefully in order to obtain any nontrivial information about the destruction of quantum interference.

This careful analysis will be carried on in what follows. However, there are some very general features about that attenuation term which are worth mentioning before any detailed procedure. To start with, the expression for $RI_R(\theta)$ [see definitions (2.8) above] can be evaluated in the underdamped limit ($R<1$) as

$$RI_R(\theta)\sin^2(S\theta)=\left[\frac{S^2\exp(2R\theta)}{\pi}+\frac{[R\sin(S\theta)+S\cos(S\theta)]^2}{\pi}\right]I(0)$$
$$-\frac{2S^2\cos(S\theta)+2RS\sin(S\theta)}{\pi}\exp[(R\theta)I(\theta)]+\frac{2S}{\pi}\sin(S\theta)\exp(R\theta)\frac{dI(\theta)}{d\theta}$$
$$-\frac{\sin^2(S\theta)}{\pi}\frac{d^2I(\theta)}{d\theta^2}\bigg|_{\theta=0}, \tag{2.11}$$

where

$$I(\theta)\equiv\int_0^{\lambda_c}d\lambda\frac{4R\lambda}{(\lambda^2-1)^2+4R^2\lambda^2}\coth(K\lambda)\cos(\lambda\theta) \tag{2.12}$$

and the exponent of the attenuation factor becomes [see (2.8)]

$$\frac{z^2}{8\sigma^2}\frac{RI_R(\theta)}{Q(\theta)}=\frac{z^2}{8\sigma^2}\frac{R\sin^2(S\theta)I_R(\theta)}{\sin^2(S\theta)+R\sin^2(S\theta)I_R(\theta)+[R\sin(S\theta)+S\cos(S\theta)]^2}. \tag{2.13}$$

Now, it is easy to see from (2.11)–(2.13) that $RI_R(\theta)/Q(\theta)$ is equal to zero when $\theta=0$ and tends to one when $\theta\to\infty$. Consequently, the attenuation factor tends to $\exp(-N/2)$ at long times, where $N\equiv z^2/4\sigma^2$ (notice that N is a measure of the initial mean energy of the system in units of $\hbar\omega_R$). As one is interested in the situation where the two packets are quite far apart at $t=0$ ($z\gg\sigma$), the mean number of quanta present at this moment is certainly much greater than one. Therefore, for long times, one can say that the attenuation factor is practically zero [for example, if $z/\sigma\sim10$, $\exp(-N/2)<10^{-5}$]. This approximation is particularly important when the two packets are macroscopically distinguishable at $t=0$ because in this case N turns out to be a huge number. The main question of this work is then, how long does it take for the attenuation factor to become negligible?

Actually the existence of this residual interference term is quite artificial. Since ρ_1 and ρ_2 must be correctly normalized to $\frac{1}{2}-\epsilon$, where ϵ is a measure of the overlap between $\psi_1(x)$ and $\psi_2(x)$ at $t=0$ [consequently $O(\exp(-N))$], one expects that corrections proportional to ϵ will cancel that residual term when $t\to\infty$.

In order to simplify the final results, four specially interesting limits are taken. Firstly, the high-temperature limit ($\hbar\omega_R\ll kT$) is considered for two extreme situations, namely, weakly damped ($\omega_R\gg\gamma$) and strongly damped systems ($\omega_R\ll\gamma$). Secondly, the low-temperature limit ($\hbar\omega_R\gg kT$) is taken where, once again, weakly as well as strongly damped systems are considered. Before proceeding any further one should keep in mind that all conclusions to be reached below are valid only when $z\gg\sigma$.

III. HIGH-TEMPERATURE CASE

When $\hbar\omega_R\ll\hbar\Omega<2kT$ the expression for $I_R(\theta)$ can be simplified because (2.12) can be evaluated by residues as

$$I(\theta)\simeq\frac{\pi}{KS}[S\cos(S\theta)+R\sin(S\theta)]\exp(-R\theta). \tag{3.1}$$

With this result it is trivial to show that (for $R<1$)

$$RI_R(\theta)=\frac{RS\exp(R\theta)}{K\sin^2(S\theta)}\times\left[\frac{2S}{R}\sinh(R\theta)-\exp(-R\theta)\times\left[\frac{2R}{S}\sin^2(S\theta)+2\sin(S\theta)\cos(S\theta)\right]\right]. \tag{3.2}$$

On the other hand, if $\hbar\omega_R\ll2kT\ll\hbar\Omega$, the last term on the rhs of (2.11) gives (in dimensional units)

$$\frac{1}{\pi}\frac{d^2I(\theta)}{d\theta^2}\bigg|_{\theta=0}\simeq\frac{2kT}{\hbar\omega_R}+\frac{4\gamma}{\pi\omega_R}\ln\left[\frac{\hbar\Omega}{2kT}\right] \tag{3.2'}$$

and because $2kT\gg\hbar\omega_R$, the first term on the rhs of (3.2') might still dominate even for moderately overdamped systems. As all the other integrals in (2.11) do not change much in this new range of temperatures, one can still say that (3.2) is a justifiable approximation. However, for heavily damped systems it can only be used if $2kT>\hbar\Omega$. Now, expression (3.2) will be studied when $R\to0$ ($S\to1$) and $R\to\infty$ ($S\to iR$).

A. Weakly damped limit

Taking the limit $R\to0$ in (3.2) and in $Q(\theta)$ [see (2.13)], one has

$$\frac{RI_R(\theta)}{Q(\theta)}\sim\frac{2\theta-\sin(2\theta)}{\frac{K}{R}+2\theta-\sin(2\theta)}\quad\text{as } R\to0. \tag{3.3}$$

As both K and R are very small ($K,R\ll1$) one has to study three new situations separately. These are the limits $K/R\gg1$, $K/R\ll1$, and $K/R\simeq1$. In the first case ($K/R\gg1$) only long times ($\theta\gg1$) will make $RI_R(\theta)/Q(\theta)\simeq1$. Then for times such that $2\theta\gg\sin\theta$, one can write

$$\exp\left[-\frac{z^2RI_R(\theta)}{8\sigma^2Q(\theta)}\right]\simeq\exp(-\Gamma_1t), \tag{3.4}$$

where $\Gamma_1 \equiv (NR/K)\omega_R$ $[=(2NkT/\hbar\omega_R)\gamma]$. As $R/K \ll 1$, the time it takes for the interference pattern to be washed out (Γ_1^{-1}) depends on the ratio between N and KR^{-1}. For example, take $N \gg 1$ but $N/R^{-1}K \ll 1$. Then, $\Gamma_1^{-1} \gg \omega_R^{-1}$ which means that it will take many cycles before the interference pattern disappears. On the other hand, if $N \gg 1$ is such that $N/KR^{-1} > 1$, one has $\Gamma_1^{-1} < \omega_R^{-1}$ and there would be no interference even at the time the two packets would first overlap. Nevertheless, since NK^{-1} is always much greater than one, one can safely say that $\Gamma_1 \gg \gamma$ no matter whether $\Gamma_1 \gg \omega_R$ or $\Gamma_1 \ll \omega_R$. For later convenience it is important to define the regime $NRK^{-1} \ll 1$ as the extremely weakly damped limit. The foregoing discussion is clearly in agreement with the fact that when R is strictly zero, it takes an infinite time for the interference pattern to disappear.

The other two cases ($K/R \simeq 1$ and $K/R \ll 1$) actually represent a single situation, namely, $K/R < 1$. This is due to the fact that when it happens, times that are $O(\omega_R^{-1})$ or greater make $RI_R(\theta)/Q(\theta) \sim 1$. Then for $K/R < 1$ only short times are important to the study of the time evolution of $RI_R(\theta)/Q(\theta)$. When $\theta \ll 1$, (3.3) can be written as

$$\exp\left[-\frac{z^2 RI_R(\theta)}{8Q(\theta)\sigma^2}\right] \simeq \exp[-(\Gamma_2 t)^3] , \tag{3.5}$$

where $\Gamma_2 \equiv (\frac{2}{3}NR/K)^{1/3}\omega_R$, and one always has $\Gamma_2^{-1} < \omega_R^{-1}$. However, in order to be consistent with the short time approximation ($\Gamma_2^{-1} \ll \omega_R^{-1}$), one needs either RK^{-1} or $N \gg 1$.

Therefore one can finally conclude that the interference pattern is destroyed in a typical time which is shorter than γ^{-1}, possibly being of the order of (or shorter than) ω_R^{-1} if the system is not extremely weakly damped.

B. Strongly damped limit

In the limit $R \to \infty$, the ratio $RI_R(\theta)/Q(\theta)$ becomes

$$\frac{RI_R(\theta)}{Q(\theta)} \sim \frac{\theta}{\theta + KR} \tag{3.6}$$

which allows one to write

$$\exp\left[-\frac{z^2 RI_R(\theta)}{8Q(\theta)\sigma^2}\right] \approx \exp[-(\Gamma_3 t)] , \tag{3.7}$$

where $\Gamma_3 = N\omega_R/2KR = (2NkT/\hbar\omega_R)\omega_R^2/2\gamma$. For the overdamped motion the finite-amplitude wave packet comes to equilibrium at the origin without oscillating at all and the time it takes to achieve its equilibrium position is $O(\gamma\omega_R^{-2})$. On the other hand, the expression for Γ_3 indicates that $\Gamma_3^{-1} \ll \gamma\omega_R^{-2}$, which means that the interference pattern is destroyed much faster than the relaxation time of the system. Actually, overdamped systems are not the most appropriate ones for the present discussion since there is no interference pattern to be washed out. In the beginning of the motion the packets do not overlap each other, and in the end (when they overlap) the system is in thermal equilibrium with the heat bath and once again no interference is present. However, a very similar calculation can be applied to free damped packets which are allowed to overlap during their motion (see Sec. V).

IV. LOW-TEMPERATURE CASE

When $\hbar\omega_R \gg 2kT$ ($K \gg 1$), the approximation used in (3.2) is no longer valid. One has to evaluate (2.12) as

$$I(\theta) = \lim_{K\to\infty}\left[\tfrac{1}{2}\int_{-\lambda_c}^{\lambda_c} d\lambda \frac{4R\lambda}{(\lambda^2-1)^2+4R^2\lambda^2}\coth(K\lambda) \times\cos(\lambda\theta)\right], \quad \lambda_c \gg 1 , \tag{4.1}$$

which can be done by residues as in the limit $K \to 0$. The attenuation factor can be rewritten (for $R < 1$)

$$\exp\left[-\frac{z^2}{8\sigma^2}\frac{RI_R(\theta)}{Q(\theta)}\right] = \exp\left[-\frac{N}{2}\frac{R\sin^2(S\theta)I_R(\theta)}{\sin^2(S\theta)+R\sin^2(S\theta)I_R(\theta)+[R\sin(S\theta)+S\cos(S\theta)]^2}\right] \tag{4.2}$$

which with the help of (2.11) and (2.12) turns out to be an extremely cumbersome expression. However, its time dependence can be studied in certain limits.

A. Weakly damped limit

When $R < 1$, $I(\theta)$ [see (4.1)] can be approximated by

$$\frac{I(\theta)}{\pi} \simeq \frac{\exp(-R\theta)}{S}\cos(S\theta) + \frac{\exp(R\theta)}{\pi}\int_R^\infty dx \frac{\exp(-\theta x)}{x^2+S^2} - \frac{\exp(-R\theta)}{\pi}\int_{-R}^\infty dx \frac{\exp(-\theta x)}{x^2+S^2} \tag{4.3}$$

which when substituted into (2.11) still leaves the attenuation factor with too complicated a structure to be analyzed in a simple way. However, when $R \ll 1$ and $\theta \gg 1$ such that $R\theta \ll 1$, one can expand the attenuation factor in a power series of R and write those integrals in (4.3) in their asymptotic forms. The final result is (to first order in R)

$$\exp\left[-\frac{z^2}{8\sigma^2}\frac{RI_R(\theta)}{Q(\theta)}\right] \simeq \exp(-NR\theta + \cdots) , \tag{4.4}$$

where the ellipsis also includes unspecified oscillatory terms. Therefore one can say that the time for the destruction of the interference pattern is approximately $\tilde{\Gamma}_1^{-1} \equiv (N\gamma)^{-1}$ which is clearly much shorter than γ^{-1}. Notice that depending on the product NR, this time can be $O(\omega_R^{-1})$ and once again it is possible that one observes

no interference at all even when two packets overlap for the first time.

B. Strongly damped limit

In the limit $R \to \infty$ one initially needs to rewrite expressions (2.11), (4.1), and (4.2) with the appropriate modifications, namely, the replacements $\omega^2 \to \tilde{\omega}^2$ and $S \to i\tilde{S}$ [see definitions below (2.8)]. After that, it is easy to show that the overdamped analog of (4.3) can be written as

$$\frac{I(\theta)}{\pi} \simeq \exp\left[-\frac{\theta}{2R}\right] + \frac{1}{\pi}\exp(R\theta)\int_R^\infty dx \frac{\exp(-\theta x)}{x^2 - R^2 + 1} - \frac{1}{\pi}\exp(-R\theta)\int_{-R}^\infty dx \frac{\exp(-\theta x)}{x^2 - R^2 + 1}, \quad (4.5)$$

where the two integrals must be evaluated in principal value. Substituting this expression in the overdamped version of the attenuation factor which reads

$$\exp\left[-\frac{z^2}{8\sigma^2}\frac{RI_R(\theta)}{Q(\theta)}\right] \simeq \exp - \frac{N}{2}\frac{R\sinh^2(R\theta)I_R(\theta)}{\sinh^2(R\theta) + R\sinh^2(R\theta)I_R(\theta) + R^2[\sinh(R\theta)+\cosh(R\theta)]^2}, \quad (4.6)$$

one can finally write (for $R\theta \gg 1$)

$$\exp\left[-\frac{z^2}{8\sigma^2}\frac{RI_R(\theta)}{Q(\theta)}\right] \simeq \exp(-\tilde{\Gamma}_2 t), \quad (4.7)$$

where $\tilde{\Gamma}_2 \equiv N\omega_R^2/2\gamma$ and as in the previous approximation, the time for the destruction of interference is shorter than the relaxation time of the finite energy wave packet by a factor N^{-1}.

V. DISCUSSION

The central result of the previous sections is that the time it takes for the interference pattern to be blurred out by the coupling to the environment is much shorter than the relaxation time of the system when the two wave packets in question are initially prepared far apart from one another. Although this has been proved to be valid for some specific limits, there is no apparent reason why those results should be drastically modified in more general situations.

At this point the reader should certainly be worried about two basic questions upon which no comment has been made so far: (a) What is the physical interpretation of the results found in this work? (b) Is there any real situation where this theory could be tested?

In order to answer the first of these two questions, take for simplicity the zero-temperature case. At $t=0$ the system of interest is prepared in a state $|\psi\rangle = |\psi_z\rangle + |\psi_0\rangle$, where $|\psi_z\rangle$ is a Gaussian centered at $x=z$ and $|\psi_0\rangle$ is the ground state of the oscillator. This state initially contains a mean number N of energy quanta $\hbar\omega_R$ as mentioned in Sec. II. As one is dealing with the zero-temperature case, the environment is clearly in its ground state which is here denoted by $|0\rangle$. Therefore one can say that the state of the universe (system plus environment) is initially given by

$$|\Phi_i\rangle = (|\psi_z\rangle + |\psi_0\rangle) \otimes |0\rangle . \quad (5.1)$$

After a time τ (the relaxation time of the system of interest), the universe is in its final state which can be written as

$$|\Phi_f\rangle = |\psi_0\rangle \otimes |N\rangle , \quad (5.2)$$

where $|N\rangle$ is a state of the environment containing N quanta of energy. Notice that (5.2) is valid only when one further assumes that the coupling between system and environment is vanishingly small, otherwise $|\Phi_f\rangle$ would not have that simple form.

Now, suppose one wishes to investigate the state of the universe after the oscillator has emitted one quantum of energy $\hbar\omega_R$ to the environment. Since the emission of N quanta takes place in a time τ, the emission of only one quantum must occur in a time which is of the order of τ/N. At this time the state of the universe is given by

$$|\Phi_1\rangle = |\tilde{\psi}_z\rangle \otimes |1\rangle + |\psi_0\rangle \otimes |0\rangle , \quad (5.3)$$

where $|\tilde{\psi}_z\rangle$ is the initial $|\psi_z\rangle$ after having emitted $\hbar\omega_R$ and $|1\rangle$ is the state of the environment containing one quantum of energy. What allows one to write the time evolution of (5.1) after τ/N as (5.3) is the fact that the interaction Hamiltonian between system and environment (being a coordinate-coordinate coupling) correlates $|\psi_z\rangle$ and $|\psi_0\rangle$ to different states of the environment. It was also assumed that the ground state $|\psi_0\rangle$ of the system does not change much during the emission of one quantum of energy. However, the state which has been investigated throughout this work is the reduced density matrix of the system only, or

$$\hat{\rho} \equiv \mathrm{tr}_\epsilon |\Phi\rangle\langle\Phi| , \quad (5.4)$$

where tr_ϵ indicates that one must trace over the states of the environment. Thus, the reduced density operator at $t=\tau/N$ is given by

$$\hat{\rho} = |\tilde{\psi}_z\rangle\langle\tilde{\psi}_z| + |\psi_0\rangle\langle\psi_0| , \quad (5.5)$$

where orthogonality between $|0\rangle$ and $|1\rangle$ has been used.

It is now a trivial matter to argue that after a time τ/N has elapsed no interference between $\tilde{\psi}_z(x)$ and $\tilde{\psi}_0(x)$ can be observed since $\hat{\rho}$ is a mixture of these two states. The emission of more energy quanta by $\tilde{\psi}_z(x)$ will clearly reinforce this conclusion because the new states of the environment will always be orthogonal to the previous ones.

Although this simple argument has been used only for $T=0$, the reader will not find it much harder to generalize it to the finite-temperature case. The only difference now is that one needs to take into account that the reservoir induces transitions between the harmonic-oscillator

states. In order to make life simpler, consider only two neighboring energy eigenstates of the system of interest labeled by 1 and 2. The equation of motion for their occupation number can be written as (see, for example, Sec. II of Ref. 13)

$$\dot{n}_1 = -An_1 - BU(\omega_R)(n_1 - n_2) ,$$
$$\dot{n}_2 = An_1 + BU(\omega_R)(n_1 - n_2) , \tag{5.6}$$

where A is obviously $O((\tau/N)^{-1})$ (the spontaneous emission rate for one quantum of energy) because at $T=0$ the emission of one quantum is enough to dephase $|\psi_0\rangle$ and $|\psi_z\rangle$, and

$$U(\omega_R) = \frac{A}{B}\left[\frac{1}{\exp(\hbar\omega_R/kT)-1}\right] \equiv \frac{A}{B}n(\omega_R) . \tag{5.7}$$

The solution of (5.6) is very simple and the difference between the occupation number of the two states considered above reads

$$n_1 - n_2 \propto \exp\{-[2n(\omega_R)+1]At\} , \tag{5.8}$$

allowing one to write the new dephasing time as

$$\tau_d \propto \frac{1}{2n(\omega_R)+1}\frac{\tau}{N} \tag{5.9}$$

which agrees with the high-temperature limit where $\tau_d \simeq \hbar\omega_R\tau/2NkT$. Notice that (5.9) is also in agreement with the zero-temperature limit, namely, $\tau_d \simeq \tau/N$. Of course all these arguments could be made more rigorous but it would be an unnecessary complication since the simpler approach is enough to explain the basic results of this work.

The occurrence of the last term in (2.11) which is logarithmically dependent on the environmental upper cutoff frequency Ω, raises some interesting questions which we hope to discuss in the future, together with the (possibly related) question of the influence of the precise initial conditions on the environment.

Another very interesting example that could be studied with the same techniques which have been used so far is that of an undamped oscillator which is subject to an external stochastic force satisfying

$$\langle F(t)\rangle = 0, \quad \langle F(t)F(t')\rangle = F_0^2\bar{\tau}\delta(t-t') , \tag{5.10}$$

where F_0 is the intensity of the force and $\bar{\tau}$ is the mean time elapsed between successive actions of $F(t)$ on the oscillator. Taking the limits $T\to\infty$ and $\gamma\to 0$ (but keeping $4kM\gamma T = F_0^2\bar{\tau}$) in (2.2), one easily sees that it now corresponds to the above-mentioned case. Notice that those limits are only formal and the new problem involves no finite-temperature effects. It is a simple harmonic particle which is not coupled to any environment ($\gamma=0$) but is acted on by an external force instead. Since previous finite-temperature results depend on γ and T only through the product γT, one can replace

$$\tau_d = \frac{\hbar\omega_R}{2N\gamma kT} \to \tau_d = \frac{2M\hbar\omega_R}{NF_0^2\bar{\tau}} \tag{5.11}$$

which on its turn can be rewritten as

$$\tau_d = 4\frac{\langle p^2\rangle}{\langle I^2\rangle}\frac{\bar{\tau}}{N} , \tag{5.12}$$

where $\langle I^2\rangle \equiv F_0^2\bar{\tau}^2$ is the mean-square impulse transmitted through the stochastic force to the oscillator and $\langle p^2\rangle$ is the mean-square momentum of a particle in the ground state of the oscillator. Since one assumes that the external force is only weakly disturbing the system ($\langle p^2\rangle \gg \langle I^2\rangle$) and that $\omega_R \ll \tau^{-1}$, the dephasing time can be $O(\bar{\tau})$ which is obviously much shorter than the natural period of oscillations ω_R^{-1}.

Before leaving the subject of the first of the two questions raised in this discussion it should be very instructive to realize that the results presented in this work are also in agreement with the quantum theory of measurement.[14] The environment acts as if it were measuring the position of the oscillator through the coordinate-coordinate coupling. As the initial state of the oscillator is a superposition of two approximate eigenstates of position (two Gaussians centered at $x=0$ and $x=z$) they become a mixture in a very short time scale. Notice, however, that the concept of pointer basis[14] should be carefully applied to the position operator since the very act of measuring clearly modifies the value to be measured. In other words, the position operator of the harmonic oscillator does not commute with its own Hamiltonian. This concept is only valid if the measurement is performed in a time interval Δt such that $\Delta t \ll \omega_R^{-1}$ in the underdamped case and $\Delta t \ll \gamma\omega_R^{-2}$ in the overdamped case.

The second question which was raised in the beginning of this section deals with the possibility of testing the results investigated so far in real life systems. Although no real experiment is proposed in this work, the following "gedanken" experiment can be inspiring to some experimentalists.

Consider the double-slit experiment designed in Fig. (1) below. In region I there is a source of charged particles which are directed toward the double slit. Around each slit one has identical magnetic fields oppositely directed which are used to bend the charged particle's beam. In region II there is only a screen. Now suppose that the width of each slit is given by σ and they are separated by a distance z. When $z \gg \sigma$ and the magnetic field is zero it is obvious that one can see no interference on the screen

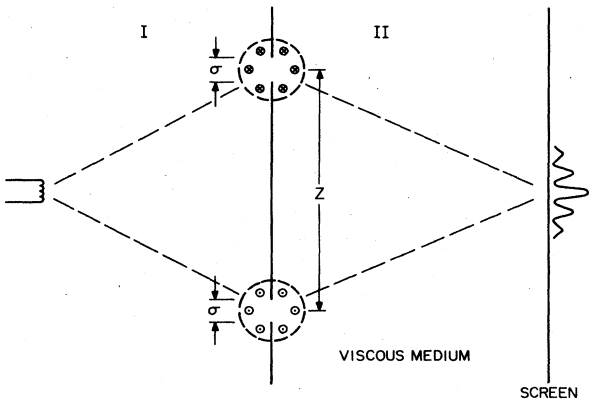

FIG. 1. Schematic diagram of the double-slit experiment.

in region II. However as one switches on the magnetic field the semiclassical paths followed by the two distinct beams start to be bent and it is possible to detect a quite intense interference pattern on the region the two beams coincide. Now, if one introduces a "viscous" medium in region II, the effect of damping on quantum interference can be investigated by varying the intensity of the magnetic field and the position of the screen.

Notice that this example is not entirely equivalent to the one that has previously been investigated. In this new situation there is no potential to which the wave packets are subject. Now, one initially has the superposition of two free wave packets of width σ which are separated by a distance z and directed against each other with momenta $\vec{p}$ and $-\vec{p}$ (here, one obviously means the components of momenta parallel to the screen). By symmetry the maximum intensity of the interference fringes occurs when the two packets coincide halfway between their initial positions. The magnetic fields can be clearly used to vary the initial momenta of the packets.

In the viscous medium one expects the intensity of the interference pattern to be a function of the initial component of the momentum of each packet parallel to the screen and of the ratio z/σ. It is hoped to give a more thorough treatment of this problem elsewhere.

VI. CONCLUSIONS

The main result presented in this paper is that damping tends to destroy interference effects on a time scale shorter than the relaxation time of the system. In the specific case of the two Gaussian wave packets subject to a harmonic potential one easily sees that they behave as two classical Brownian particles exhibiting no quantum interference when they overlap except in the extremely weakly damped limit. Perhaps this is a justification for the use of nonlinear wave equations (for example, the Schrödinger-Langevin equation[15]) to describe quantum Brownian motion; however this claim deserves a much deeper investigation.

Finally, it should be mentioned that the results obtained in the extremely weakly damped limit can be partially deduced with the use of quantum master equations.[16]

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