Atomic coherence effects in a two-mode laser with coupled transitions

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An analytical nonperturbative theory of a two-mode laser with coupled transitions is presented. The gain medium is modeled as a homogeneously broadened Λ-type three-level system. Stable stationary solutions of such a two-mode laser are studied with particular attention paid to the two-photon-induced atomic coherence effects. Our analysis shows that the nonradiative collisional decay of atomic coherence affects substantially the extent of mode competition. In particular, the coexistence domain of the two modes increases with a faster decay of the quadrupole atomic coherence that also leads to an increase in the power extraction efficiency.

I. INTRODUCTION

Mode coupling in a multimode laser is a well-known phenomenon[1–3] and has been extensively studied in the context of bidirectional ring lasers,[4–6] Zeeman lasers,[7–10] and semiconductor lasers.[11,12] A feature common to this work is that different modes compete for gain from the same one-photon transition and the gain medium is often modeled in terms of a two-level atomic system.

In recent years the case wherein the gain medium is more appropriately modeled as a three-level system is getting considerable attention.[13–17] Here, even though the two modes experience gain from different atomic transitions, they are coupled since the two transitions share a common level. Furthermore, two-photon effects provide an additional coupling mechanism. One may expect that the generation of atomic coherence between the dipole-forbidden atomic levels should play an important role in such a two-mode laser. In most of the previous work,[13–17] however, these effects have not been fully incorporated. This may be partly so because, in contrast to a two-level system, a simple analytical expression for the three-level nonlinear susceptibility in the strong-signal regime is generally not available.

The objective of this paper is to present a simple but realistic model that provides an analytical nonperturbative approach to a two-mode laser with coupled transitions. To simplify the analysis, the gain medium is taken to be homogeneously broadened and dispersive effects are ignored by assuming exact coincidence between the atomic transition frequencies and the corresponding cavity-mode frequencies. On the other hand, the adopted relaxation-rate scheme is sufficiently general. In particular, it allows for the nonradiative collisional decay of the two-photon-induced atomic coherence. Stable stationary solutions of such a two-mode laser are analyzed using an analytical approach with specific attention paid towards the atomic coherence effects. The results are valid in the strong-signal regime and reduce to those obtained in previous work[13,16] in the appropriate limit. The analysis shows that mode coupling decreases with a faster decay of atomic coherence and suggests that collisions may be helpful in increasing the efficiency of a two-mode laser with coupled transitions.

II. COUPLED-MODE EQUATIONS

To obtain the coupled-mode equations, we follow the semiclassical formalism of Lamb[1,2] with the exception that the gain medium is now modeled as a Λ-type three-level system (see Fig. 1). The common upper level is assumed to be pumped and provides gain at two optical frequencies ω_1 and ω_2. For definiteness, we assume that the two modes have identical linear polarizations. The following analysis is, however, also applicable to the case of a Zeeman laser where the two modes have identical frequencies and are distinguished on the basis of their orthogonal circular polarizations.

The coupled-mode equations are obtained using Maxwell's equations with the plane wave and the slowly varying-envelope approximations. To simplify the analysis, standing-wave effects arising in a Fabry-Perot cavity are ignored.[18] When the medium response is governed by the steady-state complex susceptibility $\chi_n = \chi_n' + i\chi_n''$, where n equals 1 or 2 corresponding to the two laser modes, the coupled-mode equations for the amplitude E_n and the phase ϕ_n are[2]

$$\frac{dE_n}{dt} = -\tfrac{1}{2}\Gamma_n E_n + \tfrac{1}{2} i\omega_n \chi_n'' E_n , \tag{1}$$

$$\frac{d\phi_n}{dt} = (\Omega_n - \omega_n) - \tfrac{1}{2}\frac{\omega_n}{E_n}\chi_n' , \tag{2}$$

where Ω_n is the cavity-resonance frequency and Γ_n is a phenomenologically introduced cavity-decay rate.

The nonlinear susceptibility χ_n depends on the details

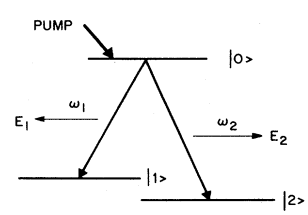

FIG. 1. Schematic illustration of the Λ-type transition scheme for a two-mode laser employing a three-level gain medium.

of the matter-radiation interaction. In the strong-signal regime it is obtained through the steady-state solution of the density-matrix equations obtained for a three-level system interacting with two arbitrarily intense optical fields.[19–23] In the most general case[22] that incorporates Doppler broadening, arbitrary level detunings, and arbitrary level-decay rates, a numerical approach is required. To present the main qualitative features as simply as possible, we assume homogeneous broadening and neglect dispersive effects by assuming that the laser frequency $\omega_n \equiv \Omega_n$ is coincident with the corresponding one-photon transition frequency. On the other hand, no severe restrictions are imposed on the population (longitudinal) decay rates γ_i and the coherence (transverse) decay rates γ_{ij} ($i,j=0$, 1, and 2 corresponding to the three levels in Fig. 1). In particular, the effect of phase-interrupting collisions on the coherence decay rates is allowed by assuming

$$\gamma_{ij}=(\gamma_i+\gamma_j)/2+\gamma_{ij}^{\text{ph}} , \tag{3}$$

where γ_{ij}^{ph} is the collisional decay rate. The contribution of inelastic collisions to the population decay rates γ_i can be easily included. It is worth emphasizing that, in contrast to a previous three-level model used extensively in high-resolution spectroscopy,[20] we consider an open three-level system that is applicable under more realistic experimental conditions. Recently it was used[23–25] to discuss polarization effects in saturated absorption and phase conjugation.

Using the density-matrix formalism, with the abovementioned simplifications, the following analytical expression for the susceptibility χ_n is obtained (see the Appendix of Ref. 23):

$$\chi_n=\frac{-ig_nc}{\omega_n}\left[\frac{(1+\tilde{p}I_{3-n})}{1+(1+q)(I_1+I_2)+\tilde{p}(1+2q)I_1I_2}\right] , \tag{4}$$

where n equals 1 or 2,

$$\tilde{p}=p[1+p+(I_1+I_2)/2]^{-1} \tag{5}$$

is itself intensity dependent,

$$g_n=|\mu_{n0}|^2\mathcal{N}\omega_n/(\epsilon_0\hbar c\gamma_{n0}) \tag{6}$$

is the small-signal gain, and $I_n=|E_n|^2/I_{sn}$ is the dimensionless mode intensity normalized to a three-level saturation intensity $I_{sn}=(\hbar^2\gamma_n\gamma_{n0})/|\mu_{n0}|^2$. Various decay rates enter through two dimensionless parameters

$$p=\gamma_{12}^{\text{ph}}/\gamma_1, \quad q=\gamma_1/\gamma_0 . \tag{7}$$

In Eq. (6), μ_{n0} is the transition dipole moment, ϵ_0 is the vacuum permittivity, and $\mathcal{N}$ is the population-inversion density assumed to be the same for both one-photon transitions. We further assumed that $\gamma_1=\gamma_2$ and $\gamma_{10}=\gamma_{20}$. These assumptions are justified if the two lower levels in Fig. 1 are relatively close compared to their separation from the upper level (e.g., hyperfine or Zeeman sublevels). Note that even when the population inversion density is the same for both modes, the small-signal gain is different depending on the corresponding transition dipole moment μ_{n0}. For the same reason the saturation intensities I_{sn} are also generally different.

Several features of the susceptibility expression (4) are noteworthy. It is purely imaginary owing to the assumption of exact one-photon resonance. The saturation denominator has contributions from self-saturation, cross saturation, and two-photon saturation. Optical pumping effects are governed by the parameter $p=\gamma_{12}^{\text{ph}}/\gamma_1$ that is related to the collisional relaxation of the two-photon-induced atomic coherence. It is shown later that the parameter p plays an important role and affects substantially the operating characteristics of such a two-mode laser.

We now substitute $\chi_n=\chi_n'+i\chi_n''$ from Eq. (4) in Eqs. (1) and (2). Since we neglect dispersive effects by assuming exact resonance, $\chi_n'=0$ and $\omega_n=\Omega_n$, the normalized mode intensities $I_n=E_n^2/I_{sn}$ satisfy the following two coupled equations:

$$\dot{I}_1=\Gamma_1[A_1f_1(I_1,I_2)-1]I_1 , \tag{8}$$

$$\dot{I}_2=\Gamma_2[A_2f_2(I_1,I_2)-1]I_2 , \tag{9}$$

where

$$f_n=(1+\tilde{p}I_{3-n})/D \quad (n=1,2) \tag{10}$$

and the saturation denominator

$$D(I_1,I_2)=1+(1+q)(I_1+I_2)+\tilde{p}(1+2q)I_1I_2 \tag{11}$$

governs the extent of gain saturation. The pump parameter $A_n=g_nc/\Gamma_n$ represents the ratio of gain to loss for the nth mode. In the absence of mode coupling each mode reaches threshold when $A_n=1$.

Equations (8)—(11) are the main result of this section and will be analyzed for their stable stationary solutions. Compared to previous work,[13–17] the new feature is the introduction of the parameter p that governs the nonradiative collisional relaxation of the Raman-type atomic coherence generated between the two lower levels. Our analysis can be compared with previous work in two extreme limits of $p=0$ and $p=\infty$. The latter case was considered by Najmabadi *et al.*[13] who solved the density-matrix equations in the rate-equation approximation. In the appropriate limit our Eq. (8) and their Eq. (49) can be shown to yield identical results. In the other specific case of $p=0$, Eqs. (8)—(11) become

$$\dot{I}_n=\Gamma_n\left[\frac{A_n}{1+(1+q)(I_1+I_2)}-1\right]I_n . \tag{12}$$

This case was considered by Singh and Zubairy[16] who developed a quantum theory of the two-mode laser with coupled transitions.[17] When quantum correlations are neglected, their analysis yields a rate equation that is identical to our Eq. (12).

III. MODE COUPLING IN THE SMALL-SIGNAL REGIME

To obtain a qualitative understanding of the atomic coherence effects, it is instructive to consider the small-signal regime where mode intensities are much below the saturation level and a third-order perturbation theory is applicable. After expanding f_1 and f_2 in Eqs. (8) and (9) in powers of I_1 and I_2 and retaining only up to the linear terms, we obtain

$$\dot{I}_1=\Gamma_1 I_1(A_1-1-\beta_1 I_1-\theta_{12}I_2)\ , \tag{13a}$$

$$\dot{I}_2=\Gamma_2 I_2(A_2-1-\beta_2 I_2-\theta_{21}I_1)\ , \tag{13b}$$

where the self-saturation coefficients β_n and the cross-saturation coefficients θ_{nm} are given by

$$\beta_n=A_n(1+q),\ \ \theta_{nm}=A_n[(1+q)-p/(1+p)] \tag{14}$$

with $m,n=1$ or 2. Steady-state solutions of Eq. (13) and their stability criterion are well known.[2] The two-mode solution is found to be stable when the coupling constant

$$C=\theta_{12}\theta_{21}/\beta_1\beta_2\le 1\ . \tag{15}$$

Using Eqs. (14) and (15), the coupling constant in the present case is

$$C=\left[1-\frac{p}{(1+p)(1+q)}\right]^2 \tag{16}$$

and shows that the collisional relaxation $\gamma_{12}^{\mathrm{ph}}$ of the atomic coherence governed by the parameter p decreases the extent of mode competition. In the extreme limit of $p\to\infty$ (corresponding to the rate-equation approximation) $C=q^2/(1+q)^2$ and takes a value of $\frac{1}{4}$ when all population decay rates are equal[13] ($q=1$). The maximum coupling occurs for $p=0$. The important point to note is that since $C\le 1$ for all values of p, "strong coupling" does not occur and mode hopping leading to bistability is excluded within the framework of the present model.

IV. NONPERTURBATIVE ANALYSIS

To study mode competition in the strong-signal regime, we consider the steady-state solutions of Eqs. (8) and (9). As a specific case, A_2 is increased for a fixed value of $A_1>1$. Initially $A_2<1$, mode 2 is below threshold and $I_1=(A_1-1)/(1+q)$. When A_2 exceeds unity, mode 2 does not start lasing because gain saturation due to mode 1 inhibits its oscillation. The threshold A_2^L at which both modes starts to oscillate can be obtained using the condition $A_1 f_1=A_2 f_2$ that leads to an implicit relation

$$I_2=aI_1-b\ , \tag{17}$$

where

$$a=\frac{A_2 p-(A_1-A_2)/2}{A_1 p+(A_1-A_2)/2}\ , \tag{18a}$$

$$b=\frac{(A_1-A_2)(1+p)}{A_1 p+(A_1-A_2)/2}\ . \tag{18b}$$

At the second-mode threshold, $I_2=0$ and $I_1=b/a$. Using Eq. (18) we thus obtain

$$A_2^L=\frac{A_1(1+p+I_1/2)}{1+p+(p+\frac{1}{2})I_1}\ . \tag{19}$$

For $A_2>A_2^L$ both modes have nonzero intensities until I_2 increases up to a point that mode 2 inhibits oscillations of mode 1. This critical value A_2^U can be obtained from Eq. (17) by setting $I_1=0$ so that $I_2=-b$ or

$$I_2=\frac{A_2-1}{(1+q)}=\frac{(A_2-A_1)(1+p)}{A_1 p+(A_1-A_2)/2}\ . \tag{20}$$

This is a quadratic equation in A_2 and the only physical solution is

$$A_2^U=A_1+[h^2+2pA_1(A_1-1)]^{1/2}-h \tag{21}$$

with

$$h=(1+p)(1+q)+(A_1-1)/2-A_1 p\ . \tag{22}$$

Figure 2 shows the mode intensities I_1 and I_2 as a function of A_2 after choosing $A_1=2$, $p=1$, and $q=0.1$. Dashed lines show the corresponding intensities in the absence of mode competition. Clearly the total intensity I_1+I_2 is always reduced in the presence of mode couplings because both modes compete for the same gain. Because of an inherent symmetry with respect to the two modes, an identical behavior is observed when the pump parameter A_1 varies with fixed A_2. In particular, the bifurcation thresholds A_1^L and A_1^U can be obtained from Eqs. (19) and (21) by interchanging subscripts 1 and 2.

Each steady-state solution of the coupled-mode Eqs. (8) and (9) should be examined for its stability under weak fluctuations. This is achieved by carrying out a linear stability analysis.[26] The mode intensities $I_n=I_n^s+\delta_n$ are expanded around their steady-state value I_n^s in powers of δ_n and quadratic and higher powers of δ_n are neglected. This leads to a set of two linear equations

$$\dot{\delta}_m=\sum_{n=1}^{2} S_{mn}\delta_n\ \ (m=1,2) \tag{23}$$

where the elements of the stability matrix S are expressed

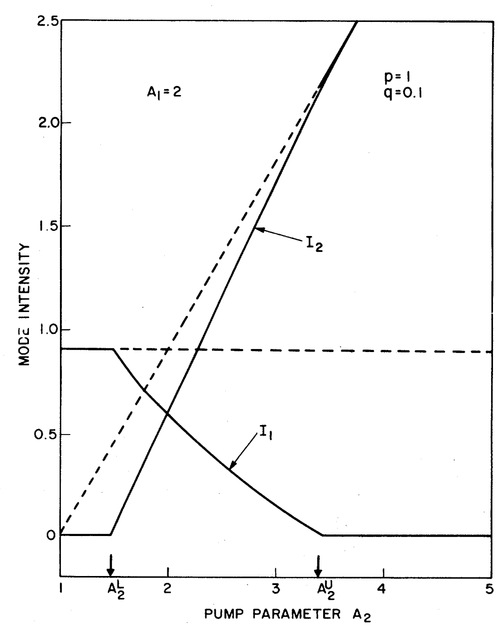

FIG. 2. Variation of the normalized mode intensities as a function of the pump-parameter A_2 while $A_1=2$ is kept fixed. Dashed lines correspond to the behavior expected in the absence of mode competition.

in terms of f_n and their derivatives [see Eq. (10)] as follows:

$$S_{nn}=\Gamma_n(A_n f_n-1+A_n I_n \partial f_n/\partial I_n)\,, \tag{24a}$$

$$S_{mn}=\Gamma_m A_m I_m \partial f_m/\partial I_n\,. \tag{24b}$$

A stationary solution is stable if the real part of both eigenvalues of the stability matrix S is negative indicating an exponential decay of fluctuations. In Fig. 2 the dashed-line solutions were found to be unstable as a direct consequence of mode coupling. The critical thresholds A_2^L and A_2^U given by Eqs. (19) and (21) represent bifurcation points and both modes coexist only in the range $A_2^L<A_2<A_2^U$.

V. ATOMIC COHERENCE EFFECTS

A physical understanding of the atomic coherence effects on mode competition can be achieved using phase diagrams in the pump-parameter space. Figure 3 shows the behavior for four values of p. In each case A_2^L and A_2^U are plotted as a function of A_1. This leads to two lines of bifurcation points and the region bounded by these two curves corresponds to the domain where both modes coexist. Outside this region the mode competition is so strong that one mode inhibits oscillation of the other. Figure 3 clearly shows that the domain of coexistence depends strongly on the parameter p that governs the collisional relaxation of atomic coherence. The mode competition is strongest for $p=0$. In this case both modes coexist only for $A_1=A_2$. Any departure from the exact equality extinguishes one of the modes. Clearly, this case corresponds to the limiting case of neutral coupling ($C=1$ in the third-order perturbation theory, see Sec. III). The mode competition decreases when p increases as is evident by the increase in the size of the coexistence domain. This is a consequence of the faster decay of atomic coherence induced by phase-interrupting collisions.

From a practical viewpoint it is of interest to consider the variation of the mode intensity when both pump parameters are varied simultaneously such that their ratio A_2/A_1 is kept fixed. In the specific case of $A_1=A_2\equiv A$ the symmetry considerations dictate that $I_1=I_2\equiv I$. The coupled-mode equations (8) and (9) are now readily solved and the mode intensity is given by

$$I=\{[u^2+(A-1)(1+p)v]^{1/2}-u\}/v\,, \tag{25}$$

where

$$u=(1+p)(1+q-A/2)+\tfrac{1}{2}\,, \tag{26a}$$

$$v=p(1+q)+2(1+q)\,. \tag{26b}$$

Figure 4 shows the variation of the mode intensity I with the pump parameter A for several values of p after choosing $q=0.1$. The variation is almost linear for all values of p. In the extreme cases of $p=0$ and $p=\infty$, the mode intensity is given by

$$I(p=0)=(A-1)/(2+2q)\,, \tag{27}$$

$$I(p=\infty)=(A-1)/(1+2q)\,. \tag{28}$$

The interesting point to note is that the power extraction efficiency related to the slope dI/dA increases by a factor of $2(1+q)/(1+2q)$ as p increases from 0 to ∞. For $q=\gamma_1/\gamma_0\ll 1$, this implies an increase by a factor of 2. Since the parameter $p=\gamma_{12}^{\text{ph}}/\gamma_1$ is related to the collisional decay of the atomic coherence, the result implies that phase-interrupting collisions lead to higher efficiencies in a two-mode laser with coupled transitions.

In a typical experimental situation it is more likely that $A_1\neq A_2$ (owing to different cavity losses or different dipole moments) but increases simultaneously with fixed ratio $r=A_2/A_1$. The variation of the mode intensities with the pump-parameter A_1 is illustrated in Fig. 5 for several values of r with $p=1$ and $q=0.1$. The dashed line corresponds to $r=1$ and in that case $I_1=I_2$, as discussed earlier. When $r\neq 1$, the mode intensities are different, as

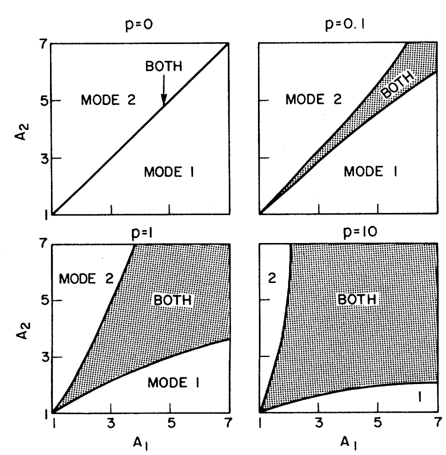

FIG. 3. Phase diagrams in the pump-parameter space. Shaded areas show the coexistence domain of the two modes. In other regions one mode inhibits lasing of the other mode. Increasing values of p indicate a faster collisional decay of the atomic coherence.

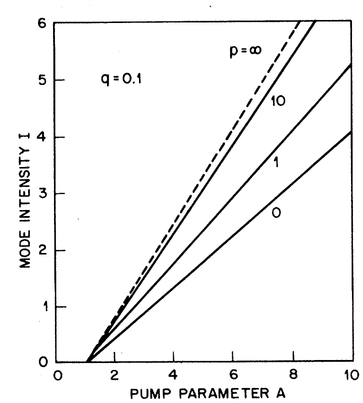

FIG. 4. Mode intensity $I_1=I_2=I$ as a function of the pump parameter for the symmetric case of $A_1=A_2=A$. Note that the power extraction efficiency goverened by the slope dI/dA increases with the parameter p.

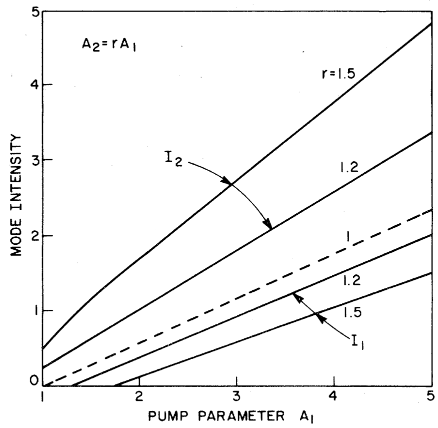

FIG. 5. Variation of the mode intensities when A_1 and A_2 are increased simultaneously with a fixed ratio $r = A_2/A_1$. Other parameters are $p=1$ and $q=0.1$.

expected. The qualitative behavior, however, remains unchanged. Note also that for $A_2 > A_1$, I_1 remains zero until A_1 reaches a critical threshold. This is due to the phenomenon of mode inhibition as discussed before [see Eq. (19)].

VI. DISCUSSION AND CONCLUSION

This paper has presented a nonperturbative analytical theory of a two-mode laser when the gain medium is modeled as a Λ-type homogeneously broadened three-level system. The emphasis has been on the atomic coherence effects that arise because of a two-photon coupling of the two lower states (see Fig. 1). It is found that the lasing characteristics are strongly influenced by the nonradiative collisional decay of atomic coherence. In general mode competition decreases with a faster decay of atomic coherence. An interesting result is that the power extraction efficiency increases in the presence of phase-interrupting collisions. Although not discussed explicitly, the results presented here also apply for a Zeeman laser[7–9] with $J=1 \rightarrow J=0$ transition. In this case the two lower states are degenerate Zeeman sublevels of the ground state. The two lasing modes are distinguished from their orthogonal circular polarizations and mode-competition effects manifest through the variation of beam polarization. Our analysis indicates that the beam polarization is strongly affected by Zeeman coherence and its collisional decay.

We have considered only the competitive case by assuming a Λ-type three-level system. This case is often of practical importance.[14] A V-type system, where the two transitions share a common lower level and the upper two levels are pumped simultaneously, can be treated along similar lines. In the cascade case[13,15] the two transitions help each other to lase rather than compete. Our analysis can be extended for this case if the assumption of equal population inversions is removed. This leads to additional terms in the susceptibility expression and the analytical simplicity of the present approach is likely to be lost. The cascade case has been discussed in detail by Najmabadi *et al.*[13]

In our work a major simplification is achieved by assuming homogeneous broadening of the gain medium. This implies that, strictly speaking, the results apply to a solid-state or a dye laser. For a gas laser the inclusion of Doppler broadening in the strong-signal regime will require an extensive numerical analysis. It is, however, expected that the qualitative features of the atomic coherence effects discussed here should remain largely unaffected.

An experimental verification of the atomic coherence effects presented in this paper would be of considerable interest. The two lower levels may correspond to different hyperfine sublevels or Zeeman sublevels of the same electronic state. In the case of a gas laser the pressure of the buffer gas controls the contribution of phase-interrupting collisions to the decay of the sublevel atomic coherence. In a solid-state laser phonons provide the nonradiative decay mechanism and temperature controls the extent of phase-interrupting collisions. Simultaneous oscillation of two (or more) modes in the Λ configuration has been reported in several solid-state lasers[14,27] and discussed within the framework of a rate-equation model. Our analysis has shown that the two-photon-induced atomic coherence affects substantially the coexistence domain of the two modes and should be included.

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