Bremsstrahlung and radiative-recombination emissivity coefficients in non-Maxwellian plasmas

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Emission for bremsstrahlung and direct radiative recombination is studied in typically non-Maxwellian plasmas, as found in the critical-density region of laser-produced plasmas. The strong departure of the spectrum from its Maxwellian counterpart is described by a general representation (which is rigorous for hydrogenic plasmas) by means of calculated reduced emissivity coefficients. This departure leads us to reconsider a current temperature diagnostic.

I. INTRODUCTION

The knowledge of the continuous emissivity coefficients is important in plasma physics because of their involvment in the rate equations and their predicting the continuous spectrum. Whereas considerable attention and efforts were spent in the past in calculating coefficients from atomic data of better and better quality,[1] little concern was shown about the fact that the distribution function followed by the free electrons may depart from the Maxwellian one, which is generally taken for granted. We will give an example where this second point is in fact of primary importance. The illustrative case corresponds to the critical-density region of a laser plasma, where the distribution function is of the type $\exp[(-v/v_0)^5]$. The emissivity coefficients values for bremsstrahlung and direct radiative recombination appear to differ much from their Maxwellian counterparts, and the modification of their evolution with photon energy has an important consequence on the validity of a current temperature diagnostic.

II. THEORETICAL DETERMINATION OF THE RADIATIVE EMISSIVITY COEFFICIENTS

We are interested in two atomic processes which contribute mostly to the continuum background spectrum, namely, direct radiative recombination and bremsstrahlung. Other processes such as dielectronic recombination, three-body recombination, etc., are not dealt with here. Emission is treated in the literal sense, i.e., without taking into account the stimulated emission and absorption aspects, which would become important at small photon energies for very dense plasmas. We furthermore take a refraction index equal to 1 all throughout.

The radiative power losses $P(\omega)$ and the emissivity coefficients $j(\omega)=dP(\omega)/d\omega$ are treated in the standard literature.[2,3] We remind their general expression without making any assumption on the form of the scalar electron distribution function $f(v)$, normalized here so that $\int_0^\infty f(v)v^2dv=1$. The $j(\omega)$'s are expressed in energy per volume unit, and ω is the angular frequency. The electron bremsstrahlung emissivity coefficients $j_{\rm br}(\omega)$ are given by the expression

$$j_{\rm br}(\omega)=N_eN\hbar Z^{*2}\sigma_{\rm Kramers}c^2\int_{v_{\min}}^{\infty}G(\hbar\omega,v)f(v)v\,dv\ , \qquad (1)$$

where the $G(\hbar\omega,v)$'s are the electron bremsstrahlung Gaunt factors, N_e and N are the electron and ion populations, and

$$\sigma_{\rm Kramers}=\frac{16\pi}{3\sqrt{3}}\alpha^3\left[\frac{\hbar}{m_ec}\right]^2$$

are the reduced Kramers cross section. The emission is due to all the incident electrons of velocity v greater than $v_{\min}=(2\hbar\omega/m_e)^{1/2}$. Concerning now the radiative capture into the n,l subshell of the ion considered, the emissivity coefficient j_R^n is equal to

$$j_R^n(\omega)=N_eN_i(\hbar^2\omega/m_e)\sigma_R^n(v)f(v)v^2\ . \qquad (2)$$

Here $\sigma_R^n(v)$ is the radiative-recombination cross section. The photon energy $\hbar\omega$, the ionization potential I_{nl}, and the velocity v are linked by the relation $\hbar\omega=\frac{1}{2}m_ev^2+I_{nl}$. The values of the calculated emissivity coefficients depend both on the atomic data chosen and on the type of distribution followed by the free electrons of the plasma.

A. Choice of the atomic data

Electron bremsstrahlung,[4] photoionization,[5,6] and radiative recombination are being studied with increased accuracy and insight by atomic physicists. The literature on photoionization is much more abundant than on radiative recombination but we can benefit from it as much since both processes are related by the detailed balance. Because of technical difficulties, scarcely any experiment could be carried out on ions, so that the very elaborate theoretical models developed in the 1970s were probed in reference to neutrals. They are expected to be as performing for ions, and were applied to them in a few cases.[7–9] However, it would be out of the question to use these models extensively for plasma physics needs because of the lengthy and costly calculations involved. Scaling laws linking the hydrogenic and the neutral limit cases is an alternative solution to treat bremsstrahlung of ions.[10] Let us mention at this point that it is sometimes not absolutely necessary to insist on having the best available data on isolated ions, because the influence of the plasma surrounding may be as important as the pure atomic-correlation effects. For example, for very dense plasmas the atomic process is somewhat modified as begins to be shown for photoionization[11] and bremsstrahlung[12] in the frame of averaged atom models, and as would be even more pronounced if averaging rates over the probable positions of the neighboring ions.[13] Therefore, in general, atomic data are at present of a sufficiently satisfying quality when obtained in independent-particle models, such as central-potential models or quantum-defect models. The dependence of the photoionization cross sections on the ionization degree was studied systematically.[14–18] A series of results, mostly for ions of astronomical interest, are expressed in a convenient analytical form.[8,19–21] Finally, we mention that the basic features about the Maxwellian emission were obtained for hydrogenlike plasmas. Therefore, we should start with completely stripped ions as well in studying the influence due to a departure from the Maxwellian distribution. For this purpose, hydrogenic cross sections[2,3] are suitable.

B. Free electron distributions considered

The emissivity coefficients are generally evaluated by assuming a Maxwellian free-electron distribution. Interesting situations with bi-Maxwellian distributions, either involving an additive drift[22] or not,[23] were investigated recently. Besides, there is a group of cases for which the $-v^2$ velocity exponential dependence is not even followed. For example, in laser plasma, the Maxwellian distribution is perturbed by various processes such as the resonant absorption, the electron-impact ionization, and the inverse bremsstrahlung. Theoretical approaches[24,25] lead to a general expression of $f(v)$ valid for a series of non-Maxwellian cases. The analytical distribution function writes

$$f(v)=\frac{m}{\Gamma(3/m)}\frac{1}{v_0^3}e^{-(v/v_0)^m}, \tag{3}$$

where m is an integer and Γ the γ function. The constant v_0 is related to the electron kinetic "temperature" T_e by the following expression:

$$kT_e=m_e\langle v^2/3\rangle=[\Gamma(5/m)/3\Gamma(3/m)]m_e v_0^2 . \tag{4}$$

For $m=2$, expression (3) reduces to the well-known Maxwellian function. The case $m=5$ corresponds to the distribution found in the critical-density region of a laser plasma, provided inverse bremsstrahlung was the dominant heating process[26,27] and the electron-electron collisions are negligible. Qualitatively speaking, the collision time is then too large in comparison with the heating time to restore the Maxwellian shape. We will be mostly interested in this distribution $m=5$, which is, furthermore, also reached in a plasma heated by ion-sound turbulence.[27] Though the probability itself of having electrons of velocity $v\to v+dv$ is $f(v)v^2dv$, we prefer to show curves $f(v)v$ in Fig. 1, since this last quantity is involved directly in relation (1). The maximum of the curves occurs at $\frac{1}{2}m_e m^{-2/m}v_0^2$. It is displaced towards higher energies with increasing m. The emission will be consequently modified when going from $m=2$ to $m=5$.

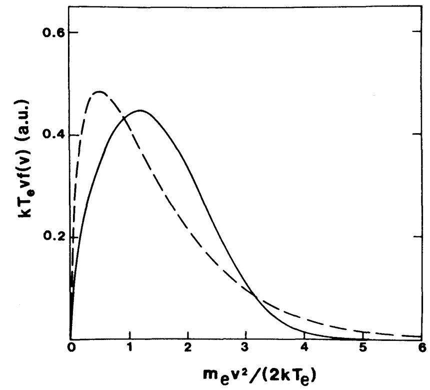

FIG. 1. Electron-distribution as a function of free-electron energy. The dashed curve corresponds to the Maxwellian ($m=2$) case, the solid curve to the non-Maxwellian $m=5$ case.

III. DISCUSSION OF THE CONTINUOUS EMISSION RESULTS

Our main purpose is to compare the emission produced in the critical-density region of a laser-produced plasma with the one which would originate in a Maxwellian plasma at the same temperature and density. The evolution of the emissivity with photon energy is modified in such a way that it leads to reconsider a current temperature diagnostic.

So as to give an idea of the type of plasma found in the critical-density region, we indicate its characteristics concerning an aluminium ($Z=13$) plasma produced by a neodymium laser. The critical-density amounts to $10^{21}e^-/\text{cm}^3$ or $4\times10^{21}e^-/\text{cm}^3$, depending on whether the laser is working at its simple or double frequency. Typical temperatures found in the experiments are 0.1–5 keV. For such sets of parameters, the average ionization degree of aluminium is larger than 11, according to evaluations made in average atom models.[28,29] These estimations are consistent with alternative studies dealing more closely with the various ionic populations.[30,31]

In order to determine the main features of the non-Maxwellian emissivity coefficients, hydrogenic data are used, as appropriate in the present highly ionized plasma. We evaluate the bremsstrahlung emission due to the various ions, as a whole, by means of the average ionization degree Z^*, and we use Kramers Gaunt factors, i.e., $G(\hbar\omega,v)=1$ whichever $\hbar\omega$ and v. For direct radiative recombination, the cross section relative to the capture into the n,l subshell has the general expression[2]

$$\sigma_{nl}=\frac{32\pi}{3\sqrt{3}}\alpha^3\frac{I_{1s}^2}{(\hbar\omega)(\hbar\omega-I_{nl})}\frac{1}{n^3}a_0^2 ,$$

a_0 being the Bohr radius. In the common Maxwellian case, the emissivity coefficients j_{br} and j_R have the well-known $\exp(-\hbar\omega/kT_e)$ dependence on $\hbar\omega$. More exactly, their expressions given per steradian and per mode are

$$j_{\rm br}=N_eN\frac{8\pi}{3\sqrt{3}}\left[\frac{m_e}{2\pi}\right]^{3/2}\frac{\alpha^3}{(kT_e)^{1/2}}\left[\frac{\hbar}{m_e}\right]^3 Z^{*2}e^{-\hbar\omega/kT_e}\ , \tag{5}$$

$$j_R^n=N_eN_i\frac{8\pi}{3\sqrt{3}}\left[\frac{m_e}{2\pi}\right]^{3/2}\frac{\alpha^3}{(kT_e)^{1/2}}\left[\frac{\hbar}{m_e}\right]^3\frac{I_{1s}}{kT_e}\frac{2Z^2}{n^3}$$

$$\times e^{-\hbar\omega/kT_e}e^{+I_{nl}/kT_e}\ . \tag{6}$$

As a consequence, the calculated curve $\ln y_{\rm br}(\hbar\omega)$ with $y_{\rm br}(\hbar\omega)=j_{\rm br}(\hbar\omega)(kT_e)^{1/2}/(N_eNZ^{*2})$ drawn versus $x_{\rm br}=\hbar\omega/kT_e$ is rigorously a straight line independent of the parameter set T_e,N_e,N,Z^*. The same can be said for $\ln y_R^n(\hbar\omega)$ with $y_R^n(\hbar\omega)=j_R^n(\hbar\omega)(kT_e)^{3/2}n^3/(N_eN_iZ^4)$ when drawn versus $x_R=(\hbar\omega-I_{nl})/kT_e$, whichever T_e, N_e, N_i, Z, and n. The reduced emissivity results $y_{\rm br}(\hbar\omega)$ and $y_R^n(\hbar\omega)$ are plotted in a semilogarithmic representation in Figs. 2 and 3. The dashed straight lines correspond to the well-known Maxwellian case. Curves corresponding to $m=5$ are drawn in solid lines in the same figures, so as to illustrate clearly how the emission is affected by a departure from the Maxwellian situation. These curves meander around the referential Maxwellian straight lines, reflecting—directly for radiative recombination, according to relation (2)—the comparative evolution of the two types of distribution functions. Notice that the universality of the $y(\hbar\omega)$ curves, i.e., their independence of the physical parameters, which is obvious for $m=2$, is still verified and can be demonstrated easily for any other value of m.

Derivatives of the emissivity coefficients over photon energy are more affected by the type of distribution

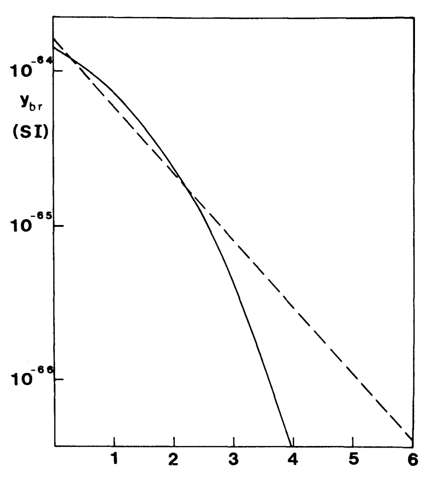

FIG. 2. Bremsstrahlung emissivity as a function of photon energy. Scaled ordinate is $y_{\rm br}(\hbar\omega)=j_{\rm br}(\hbar\omega)(kT_e)^{1/2}/(N_eNZ^{*2})$, $j_{\rm br}(\hbar\omega)$ being the bremsstrahlung emissivity coefficient. Scaled abscissa is $x_{\rm br}=\hbar\omega/kT_e$, photon energy over electron kinetic temperature. The dashed line corresponds to the Maxwellian case, the solid curve to the $m=5$ case.

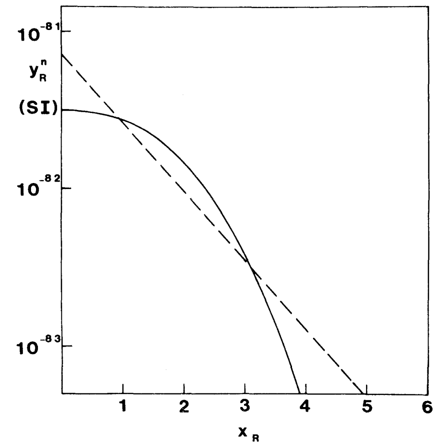

FIG. 3. Radiative-recombination emissivity relative to the n,l subshell as a function of the photon energy. Scaled ordinate is $y_R^n(\hbar\omega)=j_R^n(\hbar\omega)(kT_e)^{3/2}n^3/(N_eN_iZ^4)$. Scaled abscissa is $x_R^n=(\hbar\omega-I_{nl})/kT_e$. The dashed line corresponds to the Maxwellian case, the solid curve to the $m=5$ case.

reached than the emissivity coefficients themselves, as seen from the slopes of the curves in Figs. 2 and 3. This remark is of importance for the validity of a current temperature diagnostic.[32] Let us define the quantity t_e by

$$-\frac{1}{kt_e}=\frac{d\ln j(\hbar\omega)}{d(\hbar\omega)}=\frac{d\ln y(\hbar\omega)}{d(\hbar\omega)}\ . \tag{7}$$

The diagnostic under consideration is based on the identity $t_e=T_e$, which is verified (rigorously with Kramers atomic data) in the Maxwellian case, according to relations (5) and (6). The diagnostic is applicable both to the bremsstrahlung and to the radiative-recombination processes, and therefore—and even more interestingly—to any mixture of them both. For this reason, the drawing of experimental curves $\ln j(\omega)$ versus $\hbar\omega$ is an immediate means to determine the temperature of Maxwellian plasmas. This simple diagnostic is evidently invalidated in a non-Maxwellian plasma, where the slopes $d\ln j(\hbar\omega)/d(\hbar\omega)$ are far from being constant along the spectrum, as seen in Figs. 2 and 3. For bremsstrahlung, the ratio of the quantity t_e defined by (7) over the actual kinetic temperature T_e is given by the expression

$$\left[\frac{t_e}{T_e}\right]_{\rm br}=e^{0.134x_{\rm br}^{5/2}}\int_{x_{\rm br}}^{\infty}e^{-0.134t^{5/2}}dt\ .$$

For the recombination spectrum into the n,l subshell, the equivalent ratio amounts to

$$\left[\frac{t_e}{T_e}\right]_R=2.98x_R^{-3/2}\ .$$

The t_e/T_e ratios are plotted in Figs. 4 and 5. For bremsstrahlung, they are given until $\hbar\omega=0$ for completeness, but become meaningless at very low photon energies because the absorption effect is not included. According to Figs. 4 and 5 the traditional temperature diagnostic would be very misleading in the non-Maxwellian case, except if it were made by chance around $\hbar\omega=1.3kT_e$ for

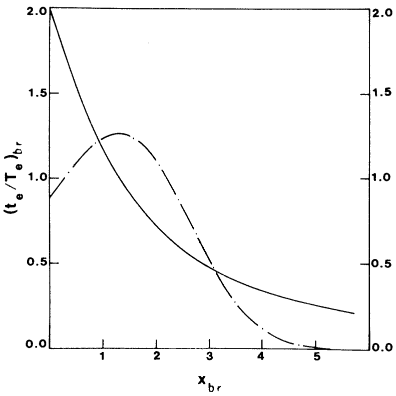

FIG. 4. Comparative results of temperature diagnostic for bremsstrahlung according to the type of distribution function. Solid curve: ratio of t_e, defined for $m=5$ by $-1/kt_e = d\,\ln j_{\rm br}(\hbar\omega)/d(\hbar\omega)$, to the electron temperature kT_e. Dotted-dashed curve: ratio of the emissivity coefficient $j_{\rm br}(\hbar\omega)$ obtained with $m=5$, to that obtained for the Maxwellian plasma.

bremsstrahlung and $\hbar\omega=2kT_e+I_{nl}$ for radiative recombination. Figures 4 and 5 show also the ratios of the emissivity coefficients obtained with $m=5$ over the corresponding values with $m=2$. The ratio found for bremstrahlung at the neodymium laser frequency amounts to 0.884, as already indicated by Jones and Lee.[33] Note that the curves drawn in Figs. 4 and 5 are universal again; by this we mean independent of temperature, density, and atomic number.

At this point, it is interesting to mention that the general conclusions of the preceding paragraphs are likely to be still valid, at least qualitatively, if other atomic data than Kramers data have to be employed. In a first step, we will comment shortly on the appropriateness of Kramers data even for hydrogenic ions. For bremsstrahlung, the exact hydrogenic Gaunt factors[10,34] depend on the only variable $\mu=Z^*\hbar\omega/(m_e v^3)$, and the exact values are close to unity as long as μ is larger than around 0.5, which is reasonably verified in the temperature range involved in laser plasmas. As for radiative recombination,

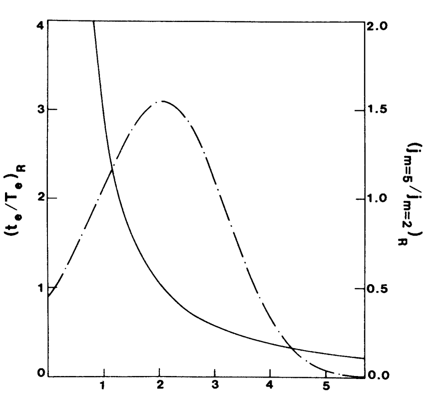

FIG. 5. Comparative results of temperature diagnostic for radiative recombination according to the type of distribution function (see Fig. 4).

Kramers formula is known to be valid until $\hbar\omega=10I_{1s}$. Indeed, for $n=1$ use of the exact quantum-mechanics expression[35] leads to similar $j(\omega)$'s, and more similar or even the same ratios t_e/T_e, as was recently shown for $\hbar\omega<5$ keV.[36] We indicate in passing that the above comments on the appropriateness of Kramers data are valid over the whole spectrum only for moderately dense plasmas. For densities much higher than the solid density, the radiative-recombination cross sections would not be infinite at threshold anymore due to the short-range character of the potential. As a consequence, the emissivity coefficients would become much smaller at lower photon energies in comparison with the isolated case. This effect on the emissivity is similar to that caused by an electron distribution favoring higher velocity electrons compared to the Maxwellian electrons. Let us raise the question in a second step of the behavior of the nonhydrogenic ions. A few examples are treated elsewhere[36] concerning the radiative capture on Al^{11+} ions at the temperatures of 1 keV, and for the densities of $4.5\times10^{21}e^-/\text{cm}^3$. Emissivity coefficients are of course different from the hydrogenic case, but the qualitative evolution from $m=2$ to $m=5$ remains unchanged except at threshold. Generally speaking, we would expect that this evolution still be obeyed as long as the recombination cross sections have a monotonic behavior. This should therefore be verified, at least qualitatively, for atoms ionized more than a couple of times.

We add now a few comments on the total radiative-recombination spectrum. The total coefficient $J_R(\hbar\omega)$ is obtained by adding up the $j_R^n(\hbar\omega)$ over n. However, the emission at photon energies higher than I_{1s} is particular to that respect: this threshold lies relatively well above those relative to the captures into $n>1$, so that last ones hardly contribute. As a consequence, Figs. 3 and 5 can be used directly. Figure 3 exhibits an exceptionally flat spectrum right above threshold that corresponds to an evaluation of t_e dramatically larger than T_e (see Fig. 5). Such a behavior above threshold was in fact observed experimentally[37] and suggested to come from the likely existence of a distribution of the type $m=5$. For energies lower than the $1s$ threshold, the recombination spectrum comes from various captures. It cannot be scaled on Z as the individual $j(\hbar\omega)$'s were, because of the differentiation of the threshold energies. An example of total emissivity coefficients is given in Fig. 6 for Al^{13+}. The most important effect of the simultaneous involvment of successive captures is that the spectrum is not as flat above each threshold I_{nl} as if it were due only to the capture into this n,l subshell. This is illustrated in Fig. 6, and it would have consequences on the ratio $t_e/T_e=-[kT_e d(\ln J(\hbar\omega))/d(\hbar\omega)]^{-1}$ of the type drawn in Fig. 5. To that respect, the non-Maxwellian character of the spectrum becomes less pronounced.

Total power losses $P=\int_0^\infty j(\omega)d\omega$ are never as different from each other—depending on whether $m=2$ or $m=5$—as the emissivity coefficients themselves can be, because of cancellation effects. Unlike for thermal conductivity, which is comparatively much reduced for $m=5$ because of the dominant influence of the tail of the distribution,[27,38] the emission power losses are little affected by the value of m. In a hydrogenic plasma, the bremsstrah-

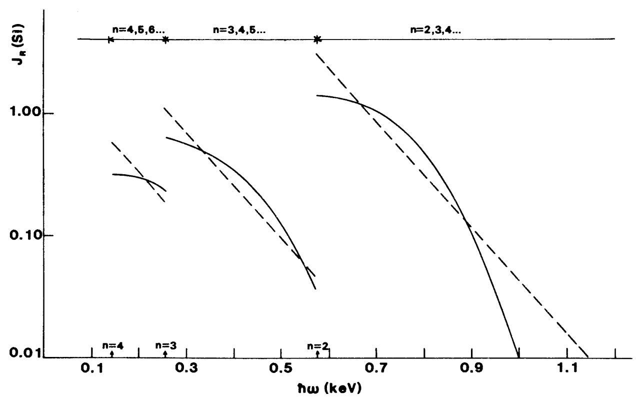

FIG. 6. Total radiative-recombination emissivity coefficient $J_R(\hbar\omega)=\sum_n j_R^n(\hbar\omega)$ vs photon energy for an Al^{13+} plasma at $T_e=100$ eV (see Fig. 1).

lung power loss is larger by around 12% for $m=5$ than $m=2$, and the radiative power loss is smaller by around 15%. At this point, we want to mention a recent study[39] on the total radiative power losses in the Maxwellian case. It shows that general trends are well obtained from the hydrogenic situation, and that an analytical generalized expression of the power loss can be found for the nonhydrogenic ions as well. We could expect conclusions of a similar type in a non-Maxwellian plasma as well. Moreover, it is likely that the power losses are modified from $m=5$ to $m=2$ by around the same amount as indicated above, even in highly ionized but not hydrogenic cases.

When considering again the photon-energy—resolved spectrum, one can give a few general remarks. Suppose that it is possible to isolate experimentally the emission due to a simple definite capture, the actual distribution can be traced back because of relation (2). Conversely, if the free electrons of the plasma are predicted to follow the distribution of the type $\exp[(-v/v_0)^5]$, use of relation (2) or of curves of the type (3) can be made to deduce the kinetic temperature. For bremsstrahlung, the connection between the distribution and the temperature is less immediate because of the integral in relation (1). However, it is still possible to deduce the kinetic temperature T_e from an experimental spectrum evidencing a non-Maxwellian character inasmuch as it is recorded over a wide photon-energy range. Within a translation along the coordinate axis, the experimental $\ln j(\omega)$ curve fits indeed the calculated curve of Fig. 2 only for the adequate abscissa choice $\hbar\omega/kT_e$.

When dealing with hydrogenic plasma following distributions of the type $\exp[-(v/v_0)^m]$, the emissivity coefficients for bremsstrahlung or radiative recombination into each n,l subshell can be rigorously represented in one general convenient way for hydrogenic plasmas. Because of a simple scaling, universal curves can be established (see Figs. 2 and 3). Another set of general curves (Figs. 4 and 5), still qualitatively appropriate for nonhydrogenic plasmas, gives the emissivity coefficients in comparison with their Maxwellian values, and the error made on the kinetic temperature if a current temperature diagnostic were uncarefully applied.

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