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Influence of slippage parameter on swept-gain amplifiers

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A transit-time broadened swept-gain laser amplifier is investigated theoretically. General values of the slippage parameter, which measures the relative values of the time of transit of the radiating sources across the device and the pulse width of the sources, are considered. The sources in this calculation are two-level atoms, but the result should be directly applicable to the free-electron laser (FEL). Both the linear and nonlinear operation are considered, and the results are in much better agreement with FEL experiment than are the corresponding FEL calculations. A formula is found which predicts maximum output power for all values of slippage parameter.

Recently there has been a renewed theoretical interest in swept-gain laser amplifiers,[1] since they have close parallels in the free-electron laser (FEL).[2] In swept-gain laser amplifiers, the sources, at any position z, have a time dependence $\rho(t-z/v)$, where t is time, ρ is the density of the sources, and v is the velocity at which the gain is swept. In cases in which the width of the source, i.e., the width of $\rho(t)$, is comparable to the relaxation times of the source, which is the case in the FEL, the gain process is characterized by "laser lethargy."[3,4] In a lethargic amplifier, long relaxation times lead to inefficient radiation and low gain for the leading edge of the optical pulse. If $v=c$, where c is the velocity of light, it has been shown that the gain decreases asymptotically to zero in the limit of large z independently of conditions of loss or saturation. Hence, the condition of gain exceeding losses is unachievable. One can achieve nonzero gain[4] by setting $v<c$, where c is the velocity of light. The trailing edge of the pulse is built up by the spontaneous emission by the induced polarization into the vacuum. This coherent spontaneous emission has infinite gain, and compensates for the low gain on the leading edge of the optical pulse. Hence, a finite overall gain is achieved.

With such a complex radiation process, many of the properties of the radiating system turn out to be insensitive to the details of the radiating source. In previous work,[5–7] two-level atom models, configured to have properties that are similar to the FEL (i.e., swept-gain configurations[1]) have been shown to give results that are largely indistinguishable from comparable FEL predictions.[7] Atomic models are simpler than the corresponding cases in FEL's, and can sometimes give analytic results.[6] Atomic calculations are also of interest as a means by which FEL results that arise through the radiation process can be distinguished from those due to the novel free-electron gain medium.

In the present work, earlier results are generalized to cover small-signal gain and energy extraction for finite slippage parameter.[8,9] In the rest frame of the optical pulse, the individual radiating sources move at a finite velocity, which causes them to slip relative to the optical pulse over a time t_0. In the atomic models and in the FEL, t_0 denotes the transit time of the sources across the space-time regime in which the optical pulse is confined (a light guide[5,7] is assumed for the atomic case). In addition, the pulse of radiating sources can have a finite duration μ_0 of its own. The optical pulse built up from the sources must be zero at any position z and retarded time $\mu=t-z/c$ (t is time in the laboratory frame) if there are no radiating sources for all $z'<z$, $\mu'=t-z'/c$. Consider, for simplicity, the case $v=c$ in which the first radiating source appears at $\mu=0$ for all z. Then the pulse duration is strictly limited to the interval $0<\mu<\mu_0+t_0$. Now follow Ref. 9 in defining a slippage parameter $s=2t_0/\mu_0$. Then, as $s\rightarrow\infty(\mu_0<<t_0)$ the optical pulse width is determined by t_0, and as $s\rightarrow 0(\mu_0>>t_0)$, it is determined by μ_0. The limit $\mu_0\rightarrow\infty$ describes continuous pumping. One expects to achieve conventional laser action in that limit, which is shown later to be true (not all cases in which conventional laser gain is expected turn out to be so straightforward[10]).

Now define $1/\beta=1/v-1/c$, and let the atoms arrive statistically at a position z with a uniform probability over the interval $z/\beta\leqslant\mu\leqslant z/\beta+\mu_0$, which is equivalent,[11] to a uniform pump pulse of duration μ_0. Only novel features involving μ_0 are described here; otherwise the discussion follows precisely the earlier ones.[5–7] The dimensionless time reads $\eta=(\mu+z/\beta)/t_0$, the dimensionless length reads $\zeta=z/\alpha' t_0$, where α' is defined in Eq. (6.4) of Ref. 12, and the electric field (measured in units of the Rabi frequency) is made dimensionless by multiplying by t_0. Expand the electromagnetic field as a linear superposition of supermodes,[13] denoted $f_k(\eta)\exp(g_k\zeta)$, which are pulses of fixed temporal shape that grow exponentially in ζ. Then keep only the term with highest achieved gain g, which should dominate for large ζ, and drop the superfluous index k.

In dimensionless units,[5] the transit time is unity, the duration of the pump pulse is $\eta_0=2/s$, and the supermodes are defined by the equation

$$-\frac{1}{\beta_0}\frac{df}{d\eta}+af=\frac{1}{\eta_0}\int_{\eta_l}^{\eta_u}\sin\theta(\eta,\eta')d\eta' \ , \tag{1}$$

where

$$\theta(\eta,\eta')=\int_{\eta'}^{\eta}f(\eta'')d\eta'' \ . \tag{2}$$

Equation (1) is obtained by generalizing the sine-Gordon equation (see, e.g., Ref. 7) and then substituting the supermode.[5–7] In Eq. (1), the small-signal regime is obtained by setting $a=g$, where g is the achieved gain, and expanding the sine to first order in the angle [there are factors left out of Eq. (1) that cancel when this expansion is made[7]]. The nonlinear regime is obtained by letting $a=\kappa$, where κ is the loss [the factors left out of Eq. (1) are unity in this case[7]]. Restricting the discussion to $\beta_0>0(v<c)$, where β_0 is the

scaled value of β, sets the limits of integration in Eq. (1) to

$$\begin{array}{llll} & 0<\eta<\eta_0 & \eta_l=0 & \eta_u=\eta \\ \eta_0<1 & \eta_0<\eta<1 & \eta_l=\eta-\eta_0 & \eta_u=\eta \\ & 1<\eta<1+\eta_0 & \eta_l=\eta-\eta_0 & \eta_u=1 \\ & \eta<1 & \eta_l=0 & \eta_u=\eta \\ \eta_0>1 & 1<\eta<\eta_0 & \eta_l=\eta-1 & \eta_u=\eta \\ & \eta_0<\eta<1+\eta_0 & \eta_l=\eta-1 & \eta_u=\eta_0 \end{array}$$

The electromagnetic field must vanish at the trailing edge, since there are no radiators that can give rise to a field at this time. Hence,

$$f(\eta=1+\eta_0)=0 \ . \tag{3}$$

In the small-signal regime, Eq. (3) makes $f(\eta)$ an eigenvector of Eq. (1), whose eigenvalue is $a=g$. In the nonlinear regime, Eq. (3) fixes the initial value $f(\eta=0)$. Except for $\eta=0(s\rightarrow\infty)$, there seem to be no simple closed solutions[5] or useful transformations[6] to these equations, but the results are readily found numerically. The small-signal results are given in Fig. 1. There the achieved gain g is plotted as a function of the velocity β_0 for $\eta_0=0, 0.50, \ldots, 2.50$ (the case $\eta_0=0$ is determined analytically). As slippage decreases (η_0 increases) the optimum gain decreases, shifts to larger β_0, and the range of β_0 over which near optimum gain is found increases. Since $\beta_0\rightarrow\infty$ corresponds to $v\rightarrow c$ (i.e., the scizzor velocity of the gain approaches the speed of light), Fig. 1 shows a passage to conditions described by conventional laser theory. For $\eta_0>>1$ the optimum gain approaches $g=1/2\eta_0$, which is the value expected from conventional gain analysis[14] of a transit-time broadened laser amplifier.

The nonlinear calculation determines the extraction efficiency ζ, where

$$\zeta=\frac{1}{2\eta_0}\int_0^{\eta_0}[1-\cos\theta(\eta_0+\eta',\eta')]d\eta' \ .$$

In Fig. 2, the efficiency is plotted as a function of $1/\beta_0$. The plot is laid out to facilitate comparison with Fig. 5 of Ref. 9, insofar as the vertical axis is proportional to the output power of the device and the horizontal axis is proportional to the desynchronism.[9] The case computed is $s=1$, which corresponds approximately to the case $s=1.2$ in Ref. 9; uncertainties in experimental pulse shapes make more

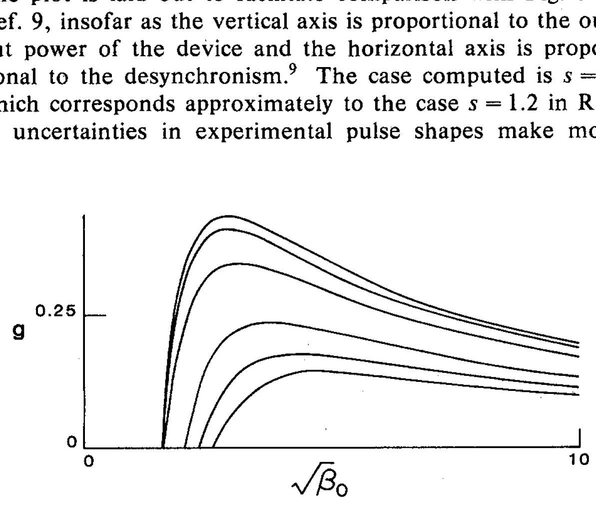

FIG. 1. Achieved gain as a function of gain velocity [inversely proportional to delay (Refs. 4–7) or desynchronism (Ref. 8) in FEL problems]. Cases are (top to bottom) $\eta_0=0(s=\infty)$, $\eta_0=0.5(s=0.25)$, $\eta_0=1(s=0.5)$, $\eta_0=1.5(s=1.333)$, $\eta_0=2.0(s=1)$, $\eta_0=2.5(s=0.8)$.

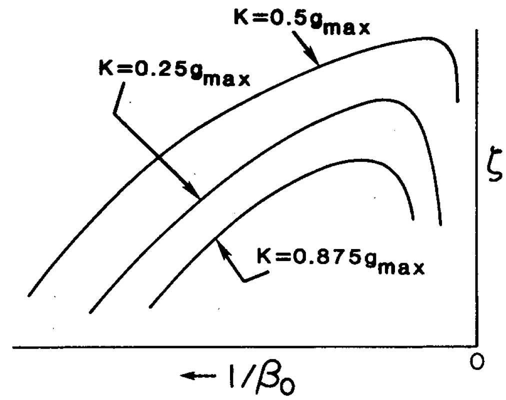

FIG. 2. Power tuning curve: extraction efficiency ζ vs $1/\beta$ (increasing to left) for $\kappa=0.5$, 0.75, and $0.875g_{max}$ (g_{max} is the maximum of the curve $s=1$ of Fig. 1). Vertical axis is proportional to power out, horizontal axis to desynchronism (negative by convention in the operating region of the FEL).

precise comparisons irrelevant. The different values of the losses are labeled relative to g_{max}, where g_{max} is the largest gain (as a function of β_0 for constant s) achieved in Fig. 1.

The various values of the loss span the likely operating conditions of the FEL,[2] and yield tuning curves with similar shapes. These curves are in good agreement with the shape of the experimental curve reported in Fig. 5 of Ref. 9, and are in substantial disagreement with the shape of the theoretical curve. The likely reason for the disagreement between FEL and atomic theory lies in the instabilities[15,16] that are predicted by FEL theory in regions where the theory and experiment disagree (see, e.g., Fig. 6 of Ref. 9). These instabilities cannot occur in the atomic case, and are an example in which the novel character of the free-electron medium might give results that differ from the atomic model. The theoretical findings in Fig. 2 suggest that these instabilities are not occurring in this experiment.

In the limit of zero slippage, Eqs. (1) and (2) have a solution in terms of a time-independent field, and the atomic model becomes formally analogous to the molecular-beam maser.[14] In that case, ζ_{max}, the maximum value of ζ (as a function of β_0 for constant s), can be written as

$$\zeta_{max}=\sin^2[\mathrm{sinc}^{-1}(\kappa/g_{max})^{1/2}] \ , \tag{4}$$

where $\mathrm{sinc}(x)=\sin(x)/x$. Equation (4) is found to fit to within about 10% the maxima of Fig. 2, several other test cases (mostly at $s=0.75$) and the $s=\infty$ curve in Fig. 2 of Ref. 5 (there the efficiency is denoted η, and $k=0.43\kappa/g_{max}$). Hence, Eq. (4) seems to apply approximately for all values of slippage. As an estimate of output power it is far superior to conventional laser formulas that are not even qualitatively accurate.[6] The threshold $g=\kappa$ fixes the end points of the power tuning curve. The curve ζ vs β_0 is roughly symmetrical, which allows an estimate of the value of β_0 that gives ζ_{max}. Since Eq. (4) gives ζ_{max}, the power tuning characteristics for all s can be estimated from small-signal calculations.

In summary, the effect of finite slippage parameter in swept-gain amplifiers has been considered. The small-signal gain is seen to approach continuous-wave (cw) operating values as slippage parameter decreases, which is expected. The nonlinear results are in better agreement with FEL experiment than are the FEL theoretical results. This suggests that the instabilities[15,16] that are predicted in FEL theory, but which cannot occur in the atomic case, have not yet occurred in the experiment. Finally, the atomic case is seen to reduce to molecular-beam maser rather than to a conventional laser in the cw limit. This is reasonable since the swept-gain model used here, the molecular-beam maser, and the FEL are dominated by phase-coherent interactions, while conventional lasers are not. The energy extraction formula from theory of the molecular-beam maser is found empirically to give reasonably accurate results for arbitrary values of the slippage parameter. This permits an estimate of device performance from a knowledge of small-signal gain.

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