External bremsstrahlung spectra generated by the complete absorption of beta particles from ${}^{90}_{38}Sr \rightarrow {}^{90}_{39}Y^{+} + e^{-}$ in thick targets of Cu, Cd, Ta, and Pb

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Experimental data are presented for the spectral shapes for beta-electron—nucleus bremsstrahlung produced by beta particles from ${}^{90}_{38}Sr \rightarrow {}^{90}_{39}Y^{+} + e^{-}$ in thick targets of Cu, Cd, Ta, and Pb over the photon-energy range from 40 to 500 keV. The experimental data are compared with the numerical calculations based on the recent formulation of Pratt *et al.* and with the Born-approximation theory of Bethe and Heitler. Though small discrepancies exist in certain restricted energy region, the overall agreement between the theory of Pratt *et al.* and experiment appears to be good. The experimental data agree with Bethe-Heitler theory within experimental uncertainties for Cu, and a systematic positive deviation is found for Cd, Ta, and Pb.

I. INTRODUCTION

The interest in bremsstrahlung has been longstanding, beginning with the realization that an accelerated charged particle can radiate; its physical importance continues, for interpreting and modeling laboratory fusion and astrophysical plasmas, designing or controlling biological exposure to radiation, etc. Yet the process has been relatively less studied and a certain mismatch has persisted between the efforts of theory and experiment.[1] Recent calculations[2,3] have clearly brought out the importance of the contribution of bremsstrahlung to the gamma-ray energy deposition in bulk media.

An exhaustive review of information on atomic field bremsstrahlung is given by Koch and Motz[4] and recently by Pratt.[1] Exact numerical screened calculations of the bremsstrahlung process have been done by Pratt and his co-workers,[5] resulting in an extensive tabulation of the photon energy spectrum from 1 keV to 2 MeV.[6] These calculations provide what is expected to be the best prediction of the photon energy spectrum or singly differential cross section $d\sigma/dk$. The experimental measurements of the external bremsstrahlung (EB) produced by monochromatic electrons in thin targets have been reported.[7–11] A comparison of theory and experiment is given by Tseng and Pratt[5] and Lee *et al.*[12] These comparisons show that simpler analytical approximations[13–15] are at best of qualitative validity and exact screened calculations agree satisfactorily with experimental data. In the case of thick target bremsstrahlung which is the usual laboratory situation experimental coverage is still too sparse. The earlier experimental investigators[16–19] have compared bremsstrahlung spectral shapes generated by beta particles and electrons in thick targets with the spectral shapes obtained from analytical approximations[13–15] and found agreement only for low-Z elements and concluded that these analytical approximations are inadequate to explain the experimental results for the medium- and high-Z elements at the higher energy end of the spectrum. There are discrepancies among the results of various investigators. Some, for example, Liden and Starfelt[16] investigated the EB produced in Cu, Al, Fe, Sn, and Pb by ${}^{32}P$ beta rays and found there was reasonable agreement between the Born-approximation theory of Bethe and Heitler[14] and experiment in the case of low-Z elements while in the case of high-Z elements and at high photon energies the experimental values exceeded the theory by $\sim$90% to 95%. Bussolate[17] measured the EB spectrum using ${}^{90}Y$ beta rays in Ag and Pb and also reported that the disagreement between the experiment and Bethe-Heitler theory increased both with Z of the target and photon energy. For lead at 2 MeV, the experimental results were higher by a factor of 5 compared to theory. In his investigation of EB spectra excited by beta rays from ${}^{90}_{38}Sr \rightarrow {}^{90}_{39}Y^{+} + e^{-}$ in Cu, Cd, and Pb targets Mudhole[18] reported that the experimental values exceeded Bethe-Heitler theory by 10% at 1 MeV and by 250% at 2.0 MeV for lead.

Therefore it seems appropriate to study experimentally the thick-target bremsstrahlung and compare with the exact screened calculations of Pratt *et al.* and with the predictions of the Born-approximation theory of Bethe and Heitler. In this paper we present thick-target bremsstrahlung spectra generated by beta particles from ${}^{90}_{38}Sr \rightarrow {}^{90}_{39}Y + e^{-}$ in Cu, Cd, Ta, and Pb, and a comparison between the experiment and theory over the photon energy range from 40 to 500 keV.

II. EXPERIMENTAL DETAILS

The experimental setup is shown in Fig. 1. The detector was a 5-mm-thick, 44.5-mm-diam NaI(Tl) crystal supplied by Bicron Corporation and mounted on an RCA 6199 photomultiplier tube. The detector was surrounded by cylindrical lead blocks lined with aluminum to minimize the production of x rays and bremsstrahlung from the shielding material. The output of the detector was fed to the ND 100 multichannel analyzer. The beta source was sandwiched between two Mylar aluminized foils of 0.7 mg/cm^2 thickness mounted in a Perspex holder of diameter 20 mm and 10 mm height. The beta activity was

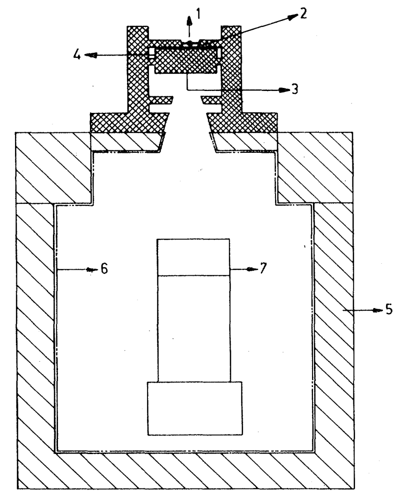

FIG. 1. Experimental setup. 1, position of the source; 2 and 3, target positions; 4, Perspex sheet; 5, lead shielding; 6, aluminum lining; 7, NaI(Tl) detector.

determined to be 22.7±2.0 μCi. The source and the target were supported by a Perspex stand to minimize scattering of beta particles and production of bremsstrahlung from the apparatus and the surroundings. The beta isotope of ${}^{90}_{38}\mathrm{Sr}\rightarrow{}^{90}_{39}\mathrm{Y}+e^-$ (the endpoint energies of beta spectrum are 546 and 2274 keV) emits two groups of beta particles both being first forbidden unique transitions. The targets of elements Cu, Cd, Ta, and Pb of size 20 mm×20 mm and thickness sufficient to stop all 2270-keV beta particles were prepared. A Perspex sheet was placed in between the source and the detector with thickness sufficient to stop all beta particles of ${}^{90}_{38}\mathrm{Sr}\rightarrow{}^{90}_{38}\mathrm{Y}^++e^-$. The spectrometer was calibrated with sources of standard gamma isotopes in the energy region of interest. The stability, resolution, and linearity of the response of the detector was tested. The targets were placed in position 2 and sufficient counts were recorded and the targets were placed in position 3 and the counts were recorded for the same time. The difference of these two counts gives the external bremsstrahlung generated in the target folded by the detector response. Compared to the bremsstrahlung generated in the target materials of interest in the present study, the bremsstrahlung generated in the Perspex material used as a beta stopper was negligible.

III. UNFOLDING PROCEDURE

In order to reduce the spectra recorded by the crystal into those produced in the targets, the following method was adopted. Correction for the finite resolution of the scintillation spectrometer was made using the expression given by Palmer and Laslett.[20] The EB distribution was corrected for the Compton continuum present, superimposed on the full energy distribution. This was done following the method of Liden and Starfelt.[21] The escape of iodine x rays from the detector following photoelectric absorption near the surface becomes important in the region just above the K-edge energy of iodine. This correction is about 35% around 35 keV and negligible above 200 keV. Correction to this effect was done using the values given by Axel.[22] For illustration, the EB pulse height distribution for Ta and Pb is shown in Figs. 2(a) and 2(b), respectively. From the figures it can be seen that the EB spectra of lead and tantalum have peaks around 75 and 60 keV corresponding to their K x rays, respectively, and for the purpose of stripping, these peaks were assumed to have resulted from a single energy corresponding to the average K x-ray energies. The extended target-detector geometric factor was estimated. The intrinsic efficiency of the crystal was determined for normal incidence of photons on the crystal. The unfolded spectrum was corrected for gamma detection (photo peak) and geometric efficiency of the crystal. Corrections were applied for absorption of photons in the target, aluminium can of the NaI(Tl) crystal, and Perspex sheet used for stopping of beta particles. The average depth of production of bremsstrahlung in the target was taken as $R/4$ where R is the range of electrons in the target following Brich and Marshall.[23] The major errors in the present measurement were statistical errors; error in determining intrinsic and geometrical detection efficiencies, photofractions, and energy resolution of the detector. Uncertainty in the full energy peak detection efficiency was about 3%, while the experimentally measured photofractions were uncertain by 4% in the entire energy range covered in the present work. The total error involved in the correction for energy resolution, iodine escape peak, and Compton contribution was around 6%. Uncertainties due to statistics in recording the data were less than 6% at the highest energy studied for each target. The total uncertainty in the measurement was 12% at 40 keV and 18% at 500 keV. This did not include the 10% uncertainty in the measurement of beta activity of the source, since the latter affected the EB photon yield and not the photon spectral distribution. As a result the EB spectral distributions were normalized at 260 keV to avoid error due to uncertainty in the beta activity of the source.

IV. THEORY

The single-collision EB cross sections from the Born approximation theory of Bethe and Heitler and from exact screened calculations of Pratt *et al.* were used as basic data to arrive at the thick-target bremsstrahlung spectra generated by the complete absorption of beta particles in thick targets and is given below.

Bethe and Heitler[14] investigated theoretically the process for the production of EB which arises from the interaction of fast-moving electrons with the Coulomb fields of target nuclei. This theory was developed in the Born approximation independently by Beth and Heitler,[14] Sauter,[14] and Racah.[14] They calculated the cross section $\sigma(W_e,k)$ for the production of an EB photon of energy between k and $k+dk$ integrated over all photon directions by an electron of energy W_e (including the rest mass energy m_0c^2) when it interacts with the Coulomb field of target nucleus of atomic number Z and atomic weight A. The expression for the cross section $\sigma(W_e,k)$ is given below:

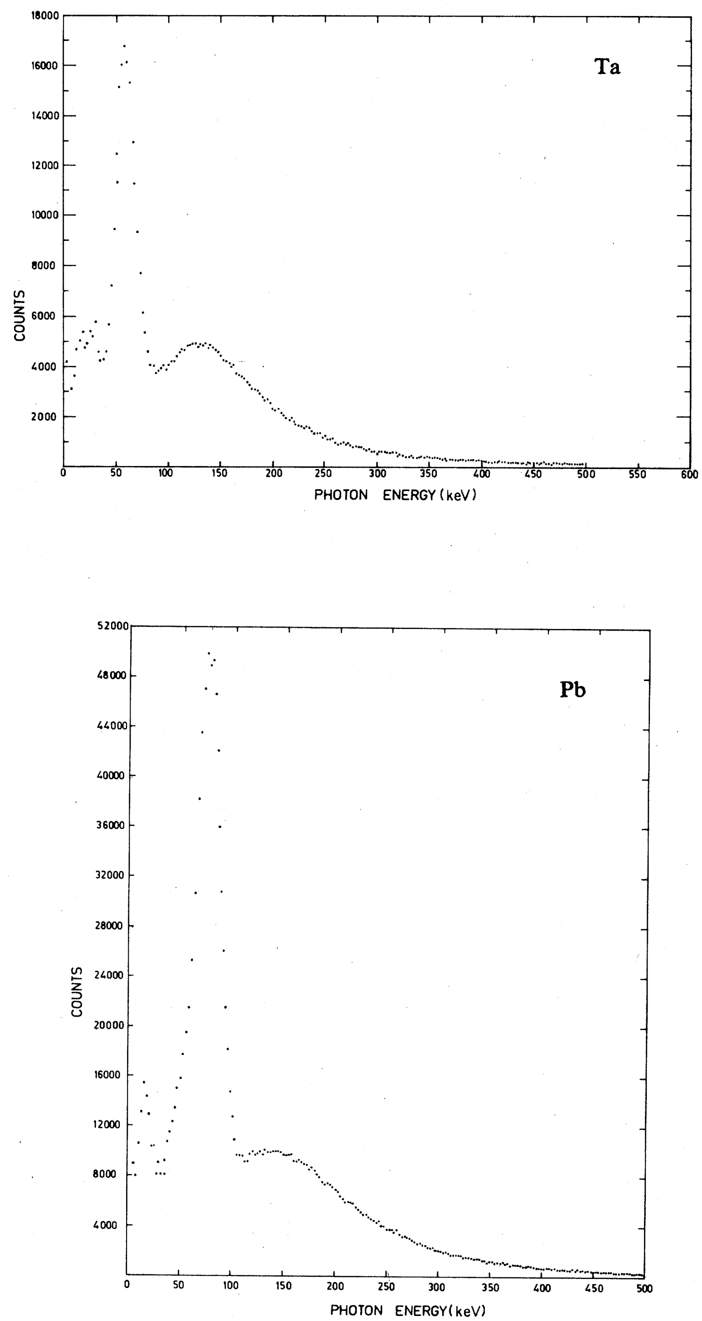

FIG. 2. Experimental pulse height distributions for Ta and Pb targets.

$$\sigma(W_e,k)=\frac{Z^2r_0^2}{137}\frac{dk}{k}\frac{P}{P_e}\left\{\frac{4}{3}-2W_eW\left[\frac{P_e^2+P^2}{P_e^2P^2}\right]+\frac{\epsilon_e W}{P_e^3}+\frac{\epsilon W_e}{P^3}-\frac{\epsilon_e\epsilon}{P_eP}\right.$$
$$+L\left\{\frac{8}{3}\frac{W_eW}{P_eP}+k^2\frac{(W_e^2W^2+P_e^2P^2)}{P_e^3P^3}\right.$$
$$\left.\left.+\frac{k}{2P_eP}\left[\left[\frac{W_eW+P_e^2}{P_e^3}\right]\epsilon_e-\left[\frac{W_eW+P^2}{P^3}\right]\epsilon+\frac{2kW_eW}{P_e^2P^2}\right]\right\}\right\}, \tag{1}$$

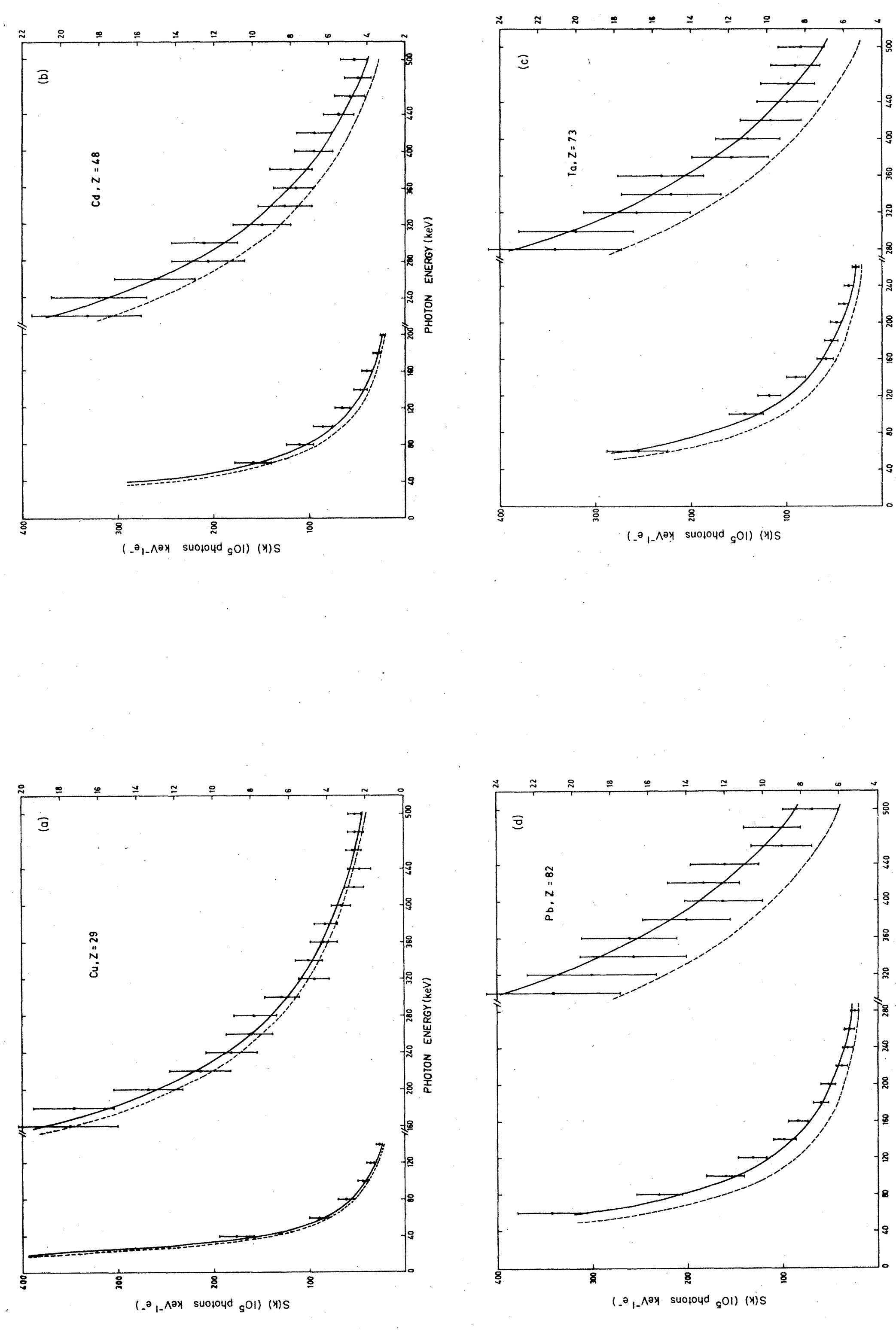

FIG. 3. Experimental EB spectral distributions for the complete absorption of beta particles from ${}^{90}_{38}\mathrm{Sr} \rightarrow {}^{90}_{39}\mathrm{Y}^{+} + e^{-}$ in targets of Cu, Cd, Ta, and Pb are compared with the exact screened calculations of Pratt *et al.* (solid curve) and with Elwert corrected Bethe-Heitler theory (dotted curve). Error bars represent the rms values of uncertainties at various points.

where

$$\epsilon_e=\ln\left[\frac{W_e+P_e}{W_e-P_e}\right],\quad \epsilon=\ln\left[\frac{W+P}{W-P}\right],$$

$$L=2\ln\left[\frac{W_eW+P_eP-1}{k}\right],$$

$$r_0=\frac{e^2}{m_0c^2}=2.818\times10^{-13}\text{ cm}.$$

W_e,P_e and W,P are the initial and final values of electron energy and momentum in units of m_0c^2 and m_0c, respectively, and k is the photon energy in m_0c^2 unit.

The Born approximation used in the Bethe-Heitler theory (BH) renders it less accurate for high-Z materials and for high momentum transfer. Several correction factors have been suggested to modify BH theory to make it more realistic and accurate. Elwert[24] gave a correction factor taking into account the effects of Coulomb field towards the low-energy end of the spectrum. This correction factor is known as the Elwert factor f_E and is a function of the target atomic number Z, initial and final values of electron energy, and momentum before and after the emission of photon. It is given as

$$f_E=\frac{(W/P)[1-\exp(-2\pi Z\alpha W_e/P_e)]}{(W_e/P_e)[1-\exp(-2\pi Z\alpha W/P)]}. \quad (2)$$

This factor is valid only if $(Z/137)(1/\beta-1/\beta_0)\ll1$, where $\beta_0=P_e/W_e$, $\beta=P/W$. This requirement forbids the use of the Elwert factor at the high-frequency limit.

Bethe and Heilter[14] gave the expression $n(W_e,k)$ for the spectral distribution of EB when an electron of energy W_e is completely absorbed in a target with N atoms per unit volume. It is given as

$$n(W_e,k)=N\int_W^{W_e}\frac{\sigma(W,k)f_E}{-dW/dx}dW,$$

where $-dW/dx$ is the energy loss per unit path length of the target and W is the electron energy corresponding to the photon energy k. Improved and more accurate dW/dx values were taken from the recent tabulations of Berger and Seltzer.[25] While calculating the theoretical EB distribution in different elements under consideration the expression $\sigma(W_e,k)$ in formula (1) was evaluated for different electron energies. At these energies the multiplicate factor given in Eq. (2) was also separately calculated.

Similarly, to calculate the theoretical EB distribution in different elements from exact screened calculations of Pratt *et al.*, $\sigma(W,k)$ values were taken from the available tabulations[6] and interpolated using Lagrange's interpolation formula for the required energies. In these calculations electron wave functions have been obtained in partial-wave series by numerically integrating the Dirac equation for Kohn-Sham, Hartree-Fock-Slater, and Thomas-Fermi potentials.

In the case of a beta emitter with an endpoint energy W_{max} the bremsstrahlung spectrum is given as $S(k)$, the number of photons of energy k per unit energy interval per beta disintegration:

$$S(k)=\frac{\int_W^{W_{max}}P(W_e)n(W_e,k)dW_e}{\int_1^{W_{max}}P(W_e)dW_e},$$

where $P(W_e)dW_e$ is the beta spectrum of the isotope under study. $S(k)$ was evaluated by numerical integration for various photon energies by using strontium-90 and yttrium-90 beta spectrums. It may be mentioned that these distributions ignore the contribution of electron-electron bremsstrahlung.

V. RESULTS AND DISCUSSION

In Fig. 3 we have plotted the bremsstrahlung spectrum. The error bars shown represent the total error. It is evident from Fig. 3 that the experimental points show good agreement with exact numerical calculations of Pratt *et al.*, although there are small discrepancies between theory and experiment in the energy region of 80–120 keV. The agreement between the Born-approximation theory of Bethe and Heitler and experimental points appear to be good within experimental uncertainties in the case of Cu. In the case of Cd there are disagreements in the majority of the points and experimental values lies above the theoretical points. For high-Z elements Ta and Pb there are systematic differences between experimental data and Bethe-Heitler theory. This is in contrast to some of the earlier experimental findings[18,26] which show good agreement in the low-energy region for medium- and high-Z elements. The Born-approximation theory of Bethe and Heitler[14] appears to underestimate the bremsstrahlung process in the case of medium- and high-Z elements throughout the energy region studied. In the case of the element Cd the deviation between Bethe-Heitler theory and the theory of Pratt *et al.* is 13.8% at 100 keV and 15.8% at 500 keV. For Ta the deviation between Bethe-Heitler theory and the theory of Pratt *et al.* is 22% at 100 keV and 26% at 500 keV and for Pb the deviation is 25% at 100 keV and 30% at 500 keV. The breakdown of the theory is due essentially to the use of oversimplified Born-approximation wave functions in estimating the cross section for the bremsstrahlung process. The theory of Pratt *et al.*, where electron wave functions have been obtained in partial-wave series by numerically integrating the Dirac equation, describes the experimental data very well in the energy range studied, i.e., from 40 to 500 keV in the atomic number range $Z=29-82$.

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