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Laser-frequency division and stabilization

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A novel optical interferometric technique is proposed for stabilizing and measuring a laser frequency in terms of an rf standard. In preliminary studies, a sensitive optical dual-frequency modulation scheme allows locking a laser to an optical cavity and the cavity in turn to a radio frequency reference with a noise level of 60×10^{-3} Hz or 2 parts in 10^{10}. In principle, the laser frequency ω_0 can acquire the stability of the rf standard and being locked to a high-order multiple n of the rf frequency ω_1 facilitates the optical–rf division $\omega_0/n=\omega_1$.

The current method for measuring an optical frequency relative to the primary time standard, the cesium beam standard at ~ 9.2 GHz, utilizes a complex frequency synthesis chain involving harmonics of laser and klystron sources. The method has been extended recently to the visible region,[1] to the 633-nm He-Ne laser locked to a molecular iodine line, with an impressive accuracy of 1.6 parts in 10^{10}. With the new definition of the meter, the distance traversed by light in vacuum during the fraction 1/299 792 458 of a second, the speed of light is now fixed and both time and length measurements can be realized with the same accuracy as an optical-frequency measurement. In view of the complexity of optical-frequency synthesis, these developments set the stage for originating complementary techniques for stabilizing and measuring laser frequencies which are more convenient.

This paper reports a sensitive optical interferometric technique, dual-frequency modulation (DFM), for measuring and stabilizing a laser frequency by comparison, in a single step, to a rf standard. Conversely, a low-noise rf source can be stabilized by a laser frequency reference. Preliminary measurements discussed below give a resolution of 2 parts in 10^{10}, but optimized devices should have a resolution between 10^{-12} and 10^{-15}. The method may be competitive with the optical frequency synthesis chain in accuracy and its simplicity suggests its convenient use in metrology, high-precision optical spectroscopy, and gravity wave detection.[2]

The principle of the technique rests on phase locking the mode spacing $c/2L$ of an optical cavity to a radio frequency standard and simultaneously phase locking a laser to the nth order of the same cavity. When these two conditions are satisfied, the optical frequency ω_0 and the radio frequency ω_1 are simply related,

$$\omega_0=n\omega_1 \ , \tag{1}$$

neglecting for the moment diffraction and phase-shift corrections. The idea of locking a laser to a cavity is of course a well-established subject,[3,4] but the concept of phase locking an optical cavity to a radio frequency source is new. Interferometric rf-optical frequency comparisons of lower sensitivity have previously been performed by Bay, Luther, and White[5] using a related idea based on amplitude modulation (AM) rather than frequency modulation, as will be discussed below.

To introduce the DFM technique, first consider a single-frequency modulation scheme. An electro-optic phase modulator driven at ω_1 generates a comb of optical frequencies $\omega_0\pm m\omega_1$ which are compared to cavity modes of frequency $n\sigma$, where σ is the cavity free spectral range, $m=0,1,2,\ldots$, and n is a large integer $\sim 10^6$ (see Fig. 1). The cavity response perturbs the balanced phase relationships between the sidebands and transforms frequency modulation into intensity modulation at ω_1. A photodetector, viewing the cavity either in reflection or transmission, then generates an error signal at the heterodyne beat frequency ω_1. This signal yields a null when the laser frequency ω_0 equals $n\sigma$ and the radio frequency ω_1 matches the mode spacing σ. In this circumstance, the comb of optical frequencies all resonate with their corresponding cavity modes. The difficulty with this approach is that the error signal depends not only on the rf detuning $\delta=\omega_1-\sigma$ but also on the optical detuning $\Delta=\omega_0-n\sigma$. Detailed analysis as well as experiment show that the error signal is proportional to $\delta\Delta$ and thus vanishes when $\Delta=0$, independent of δ, a feature which prevents direct locking of the cavity to the rf source and realizing Eq. (1).

The DFM technique (Fig. 1) overcomes this problem by using two phase modulators driven at frequencies ω_1 and ω_2, respectively. Dual-frequency modulation creates, in lowest order, sidebands at $\omega_0\pm\omega_1$, $\omega_0\pm\omega_2$, and $\omega_0\pm\omega_1\pm\omega_2$. A photodetector views the cavity in reflection and two error signals are derived: one at ω_2 and the other at the *intermodulation* frequency $\omega_1\pm\omega_2$. The first signal at ω_2 allows locking the laser to the reference cavity as described elsewhere[3,4] and is independent of ω_1 tuning. The second signal at $\omega_1\pm\omega_2$ allows locking the cavity to the rf reference. This signal varies directly with the rf detuning $\delta=\omega_1-\sigma$ and provides the desired null at the rf resonance condition

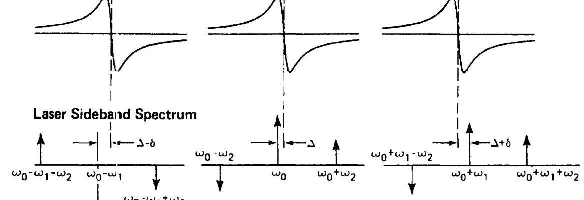

FIG. 1. Schematic representation (not to scale) of the dual-frequency modulation sidebands of a laser relative to the dispersive part of the interferometer line-shape function g, Eq. (7).

$\omega_1 = \sigma$, while being independent of laser detuning Δ. To derive this result, we write for the double phase modulated light wave incident on the cavity

$$E_i = E_0 \exp(i\omega_0 t + i\beta_1 \sin\omega_1 t + i\beta_2 \sin\omega_2 t) \ . \tag{2}$$

Taking only the zero- and first-order sidebands of the Fourier decomposition

$$e^{i\beta \sin\omega t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{i\omega t} \sim J_0(\beta) + J_1(\beta) 2i \sin\omega t \ , \tag{3}$$

the reflected light waves

$$E_r(\omega) = g(\Delta) E_i \tag{4}$$

take the form

$$E_r = E_0 e^{i\omega_0 t} \{ J_0(\beta_1) J_0(\beta_2) g(\Delta) + 2i J_0(\beta_1) J_1(\beta_2) \sin\omega_2 t + e^{i\omega_1 t} [J_0(\beta_2) J_1(\beta_1) g(\Delta + \delta) + 2i J_1(\beta_1) J_1(\beta_2) \sin\omega_2 t]$$
$$- e^{-i\omega_1 t} [J_0(\beta_2) J_1(\beta_1) g(\Delta - \delta) + 2i J_1(\beta_1) J_1(\beta_2) \sin\omega_2 t] \} \ , \tag{5}$$

where $g(\Delta)$ is the cavity line-shape function.

Equation (5) assumes that $\omega_2 >> \Gamma$ so that light at $\omega_0 \pm \omega_2$ and $\omega_0 \pm \omega_1 \pm \omega_2$ falls outside the fringe linewidth Γ and is totally reflected with $g = 1$. The detected heterodyne beat signal at $\omega_1 \pm \omega_2$ is given by

$$E_r E_r^*(\omega_1 \pm \omega_2) = 4|E_0|^2 J_0(\beta_1) J_1(\beta_1) J_0(\beta_2) J_1(\beta_2) \sin\omega_2 t$$
$$\times \{ \mathrm{Im}[g(\Delta + \delta) - g(\Delta - \delta)] \cos\omega_1 t + \mathrm{Re}[g(\Delta + \delta) + g(\Delta - \delta) - 2g(\Delta)] \sin\omega_1 t \} \ . \tag{6}$$

Since the coefficient of $\sin\omega_1 t$ in (6) is ~ 0, only the $\cos\omega_1 t$ term contributes and the DFM signal is directly proportional to the difference in cavity phase shifts of the $\omega_0 + \omega_1$ and $\omega_0 - \omega_1$ sidebands. With the further assumption that the line-shape function g can be approximated by a Lorentzian[6]

$$g(\Delta) = \frac{\Delta(\Delta - i\Gamma)}{\sqrt{R}\,(\Delta^2 + \Gamma^2)} \ , \tag{7}$$

the error signal simplifies to

$$E_r E_r^*(\omega_1 \pm \omega_2) = -4|E_0|^2 J_0(\beta_1) J_0(\beta_2) J_1(\beta_1) J_1(\beta_2)$$
$$\times \frac{\delta}{\Gamma} [\sin(\omega_1 + \omega_2)t + \sin(\omega_1 - \omega_2)t] \ . \tag{8}$$

By detecting the beat either at $\omega_1 + \omega_2$ or $\omega_1 - \omega_2$ in a double balanced mixer, an error signal proportional to the rf detuning δ can be derived for locking an optical cavity to an rf standard, or conversely an rf source to a cavity. Second, the error signal is independent of laser detuning Δ and thus optical frequency jitter. Third, there is no background signal. Fourth, the DFM signal has an excellent signal-to-noise ratio since

$$4 J_0(\beta_1) J_1(\beta_1) J_0(\beta_2) J_1(\beta_2) = 0.45$$

at $\beta_1 = \beta_2 = 1$.

A prototype DFM standard (Fig. 2) has been constructed to verify the principle and study resolution and systematic errors. A homemade cw dye ring laser containing an intracavity ammonium dihydrogen phosphate (ADP) crystal for laser phase locking to an external cavity emits 30 mW of light at 5900 Å. The beam is focused through a homemade $LiTaO_3$ traveling wave electro-optic phase modulator,[7] which generates FM sidebands at $\omega_1 = 298.667\,120$ MHz $= c/2L$. The rf at ω_1 is generated by a Fluke 6071A synthesizer locked to a Vectron CO-247 10-MHz quartz oscillator with a stability of one part in 10^{10} per day. The beam then passes through a second phase modulator, an ADP crystal resonantly driven at $\omega_2 = 19.400$ MHz. The mode-matched cavity is a 50-cm confocal interferometer with an Invar spacer, is acoustically and thermally isolated and has a measured linewidth half-width at half maximum (HWHM) $\Gamma/2\pi = 75$ kHz (finesse = 2000). An optical circulator both isolates the laser from the cavity and directs the reflected light, the signal, to a high-speed photodiode (Motorola MRD-510). The detector photocurrent contains rf beats at ω_2 and $\omega_1 \pm \omega_2$ which are amplified and then separately filtered. The error signal at ω_2 is sent to an FM sideband servo which locks the dye laser to the nth cavity fringe so that $\omega_0 = n\sigma$ with a short-term error $\Delta/2\pi \sim 300$ Hz rms.[4] The DFM signal at $\omega_1 - \omega_2 = 279.267$ MHz is coherently detected in a double-balanced mixer. The error signal [Eq. (8)] controls the cavity length L via a piezo so that the resonance condition $\sigma = \omega_1$ is satisfied. Ignoring mirror phase shifts and diffraction, the resonances $\omega_0 = n\sigma$ and $\sigma = \omega_1$ yield $\omega_1 = \omega_0/n$, and an optical-rf frequency divider is realized.

The resolution or short-term frequency stability of the prototype has been measured by opening the cavity-rf servo loop, integrating the error signal with a 0.1-sec time con-

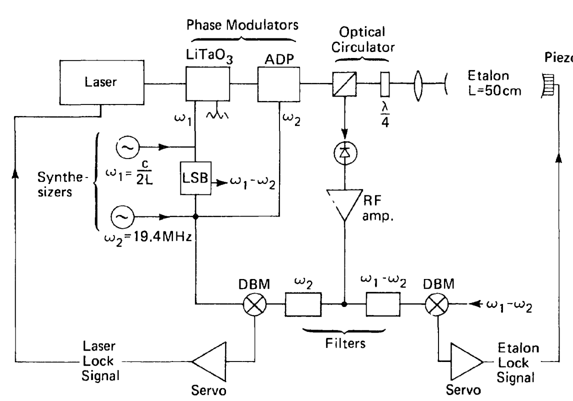

FIG. 2. Block diagram of an optical-frequency divider showing two servo loops where the laser is locked to a reference cavity and the cavity to a radio frequency standard. The $LiTaO_3$ modulator is driven at ω_1 and the ADP modulator at ω_2. LSB denotes a mixer and filter which generates the difference frequency $\omega_1 - \omega_2$ for the double balanced mixer (DBM) in the cavity-rf servo.

stant, and recording it on an oscilloscope. Figure 3 shows a peak-to-peak noise level ~ 0.4 Hz or an rms noise level $\Delta\nu_{rms} \sim 60$ mHz (60×10^{-3} Hz). This represents an rms fractional frequency deviation (Allan variance) $\sigma_y = 2\pi\Delta\nu_{rms}/\omega_1 = 2\times10^{-10}$. For comparison, the earlier AM technique of Bay *et al.*[5] developed noise levels $\Delta\nu_{rms} \approx 100$ Hz for a 100-sec integration time, which implies that our signal-to-noise ratio is 50000 times greater.

The theoretical shot-noise-limited resolution of the DFM technique can be estimated as $\Delta\nu_{rms} = \Gamma/(2\pi\sqrt{N\tau})$ where N is the number of photoelectrons/sec and τ is the integration time. For $N = 10^{15}$ sec^{-1} and $\tau = 0.1$ sec, $\Delta\nu_{rms} = 10^{-7}\,\Gamma/2\pi$ or 7.5 mHz. Our prototype is thus within a factor of 10 of the shot-noise limit.

Optimized DFM standards can be expected to have a resolution 10^2–10^5 higher. For example, the rf sideband spacing ω_1 can be increased to equal an integral multiple of the cavity mode spacing $c/2L$. Because of the periodicity of the cavity line-shape function g, the short-term stability $\Delta\nu_{rms}$ will be unchanged, but the fractional frequency stability improves with increasing ω_1. Operating at $\omega_1 = 29.8667$ GHz so that the sidebands interact with the $(n+100)$th and $(n-100)$th cavity modes would give a short-noise-limited resolution $\sigma_y = \Delta\nu_{rms}/(29.9\text{ GHz}) = 2.5\times10^{-13}$ for $\tau = 0.1$ sec. This value far exceeds the stability of current commercial standards. Secondly, superpolished mirrors developed for ring laser gyros can increase the finesse from 2000 to >20000, reducing Γ and $\Delta\nu_{rms}$ by another factor of 10. Third, increasing the cavity length L reduces Γ proportionality. Use of a gravitational wave interferometer would reduce Γ by several additional factors of 10. Optimization of the DFM technique thus offers the possibility of microwave frequency standards with microhertz linewidths or hertz stability in the optical region.

Systematic errors that can affect the accuracy of rf-optical frequency comparisons fall into two classes: (1) instrumental effects that shift the lock point of the DFM system off the center of the cavity resonance, and (2) corrections to Eq. (1) due to cavity mirror reflection and diffraction phase shifts. Residual AM in the phase modulator is well known to introduce an instrumental offset in FM servo systems. FM polarization spectroscopy[8] and AM servo techniques[9] can reduce the offset but the DFM technique solves this problem more decisively, in a new way. Because AM at the intermodulation frequency $\omega_1 \pm \omega_2$ is the *product* of the AM factors at ω_1 and ω_2, AM in our system can be only $(10^{-4})^2$ or 10^{-8}, a negligible level. A second kind of instrumental offset arises from the interference of light scattered off pairs of optical surfaces between the laser and detector. Fringes of low contrast result which cause significant offsets that are easily recognized, measured, and corrected in the DFM technique because they are periodic in laser tuning and generate a background rf signal at $\omega_1 \pm \omega_2$.

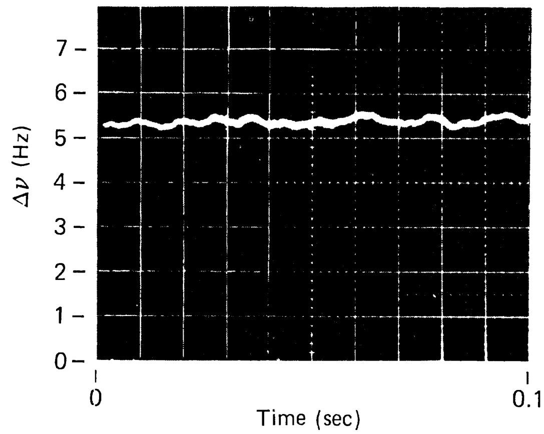

FIG. 3. Oscilloscope photograph of the experimental DFM error signal at $\omega_1 - \omega_2$ corresponding to Eq. (8). The vertical scale was calibrated at 1 Hz/div by detuning ω_1 by 5 Hz.

The second class of systematic errors due to cavity mirror reflection and diffraction phase shifts has been analyzed by Bay *et al.*[5] 12 years ago and by Layer, Deslattes, and Schweitzer.[10] They suggest that these optical-frequency corrections can be measured to $\sim 10^{-10}$–10^{-11}. Recent improvements in mirror quality indicate that these limits can now be exceeded, and new methods of measuring the phase-shift correction are also being examined. The DFM technique should, therefore, have a systematic error comparable to the current accuracy of 1.6×10^{-10} of the optical-frequency synthesis chain.[1]

We conclude that both the long- and short-term stability of the optical-frequency divider can be made competitive with, if not superior to, current techniques. In view of the large effort currently directed at optical-frequency synthesis and new microwave frequency standards, we propose that the relatively simple and promising technique of DFM should now be pursued.

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