Coupled optical bistable systems

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The coupling of two bistable systems is considered. The coupling is shown to modify the behavior of the output such that it exhibits multistability. Moreover, the resulting system is shown to exhibit bistability for parameters for which the uncoupled systems have monostable behavior.

Optical bistability continues to occupy a very prominent place in optics,[1] and one is discovering newer systems that exhibit bistability as well as new phenomena in the behavior of nonlinear systems contained in cavities. However, most of the work[1,2] to date has been on the nonlinearities of systems in a single cavity. In this Brief Report we present the results of our investigations on coupled ring cavities. In particular, we show that the coupling results in the multistable behavior of the output as a function of the input. An important characteristic of the coupled system is a bistability threshold that is lower than that for a single cavity.

We schematically show, in Fig. 1, our coupled bistable system—the two bistable systems are contained in another ring cavity which provides the overall feedback and which couples the two bistable systems. The dynamical equations for single-ring cavity problems are very well known from the work of Bonifacio, Gronchi, and Lugiato.[3] The dynamical equations for our coupled system are

$$\frac{\partial S_i}{\partial t}=\frac{\mu_i}{\hbar}\epsilon_i D_i-[\gamma_{\perp i}+i(\omega_i-\omega_0)]S_i,\quad i=1,2\ , \tag{1}$$

$$\frac{\partial D_i}{\partial t}=\frac{-\mu_i}{2\hbar}(\epsilon_i S_i^*+\epsilon_i^* S_i)-\gamma_{\parallel i}\left[D_i-\frac{N_i}{2}\right],\quad i=1,2\ , \tag{2}$$

$$\frac{\partial \epsilon_i}{\partial t}-c\frac{\partial \epsilon_i}{\partial z}=-g_i S_i,\quad i=1,2\ , \tag{3}$$

where S_i, D_i, and ϵ_i refer, respectively, to the polarization, inversion, and the field in the ith cavity, ω_0 is the frequency of the incident field, and ω_i is the atomic transition frequency in the ith cavity. The boundary conditions, in terms of the transmission of each mirror and the detuning factor θ_i of each cavity, can be written as

$$\epsilon_1(0,t)=T_1^{1/2}F(t)+R_1\epsilon_1(L_1,t-\Delta t_1)e^{-i\theta_1 T_1}\ ; \tag{4}$$

$$G(t)=T_1^{1/2}\epsilon_1(L_1,t)\ ; \tag{5}$$

$$\epsilon_2(0,t)=T_2^{1/2}G(t)+R_2\epsilon_2(L_2,t-\Delta t_2)e^{-i\theta_2 T_2}\ ; \tag{6}$$

$$H(t)=T_2^{1/2}\epsilon_2(L_2,t)\ ; \tag{7}$$

$$F(t)=T_3^{1/2}\epsilon_i(t)+T_2^{1/2}R_3\epsilon_2(L_2,t-\Delta t_3)e^{-i\theta_3 T_3}\ ; \tag{8}$$

$$\epsilon_t(t)=T_3^{1/2}H(t);\ \ \Delta t_i=(2l_i+L_i)/c\ . \tag{9}$$

The boundary conditions (5), (6), and (8) provide for the mutual coupling of the two cavities. The above dynamical equations are expected to yield a very wide variety of phenomena, which should be much richer than one had in the case of a single cavity.[1] Therefore, in this Brief Report we concentrate on the simplest, but remarkable aspects of coupled bistable systems.

We use the mean-field approximation[3] for each cavity and take the steady-state limit. Thus Eqs. (1) and (2) are solved for S and D in terms of the local electric field. These solutions are used in (3), which is then integrated, and resulting solutions are substituted in the boundary conditions (4)–(9) to obtain the output field. A long calculation then shows that the transmission of the coupled system is related to the input by

$$\begin{aligned} y=x\Bigg[&\left(\frac{T_2}{T_3}\right)^{1/2}[(1+iR_1\theta_1)(1+iR_2\theta_2)-R_3(1-i\theta_3T_3)]+\frac{\alpha_2L_2}{\sqrt{T_2T_3}}\frac{1}{1+\delta_2^2+|x|^2}(1-i\delta_2)(1+iR_1\theta_1)\\ &+\frac{\alpha_1L_1}{\sqrt{(T_1T_3)}}(1-i\delta_1)\frac{(T_2/T_1)^{1/2}[(1+i\theta_2R_2)+(\alpha_2L_2/T_2)(1-i\delta_2)(1/1+\delta_2^2+|x|^2)]}{1+\delta_1^2+|(T_2/T_1)^{1/2}x[(1+iR_2\theta_2)+(\alpha_2L_2/T_2)(1-i\delta_2)(1/1+\delta_2^2+|x|^2)]|^2}\Bigg]\ ,\\ \delta_i=&(\omega_i-\omega_0)\ ; \end{aligned} \tag{10}$$

α_i represents the absorption coefficient for the atomic system in the ith cavity. The parameters y and x are related to the incident field ϵ_i and the transmitted field ϵ_T by

$$x=\frac{\mu_2}{\hbar}\frac{\epsilon_t}{\sqrt{T_2T_3\gamma_{\parallel}\gamma_{\perp}}},\qquad y=\frac{\epsilon_i\mu_2}{\hbar\sqrt{\gamma_{\parallel}\gamma_{\perp}}}\ . \tag{11}$$

It is easy to verify that (10), in the limit $\alpha_1L_1=0$, $T_3=1$

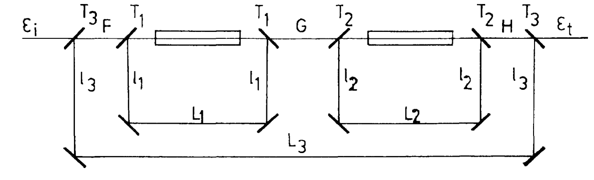

FIG. 1. Schematic diagram of the coupled bistable systems.

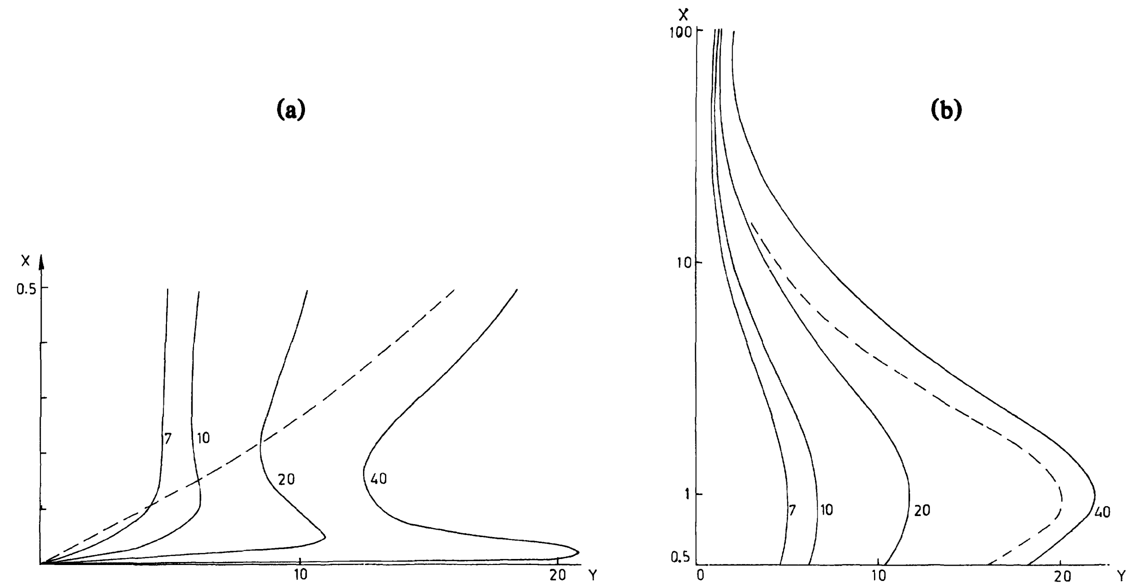

FIG. 2. The transmitted amplitude $|x|$ as a function of the incident field for absorptive case $\theta=0$, $\omega_0=\omega_i$, $T_1=T_2=T_3=0.01$. Different curves are labeled by the value of $\alpha_2 L_2/\ T_2=\alpha_1 L_1/\sqrt{T_1 T_3}=\alpha_2 L_2/\sqrt{T_2 T_3}$. The dashed curve is for empty first cavity ($\alpha_1 L_1=0$). Part (a) [(b)] is for small (large) values of the output field.

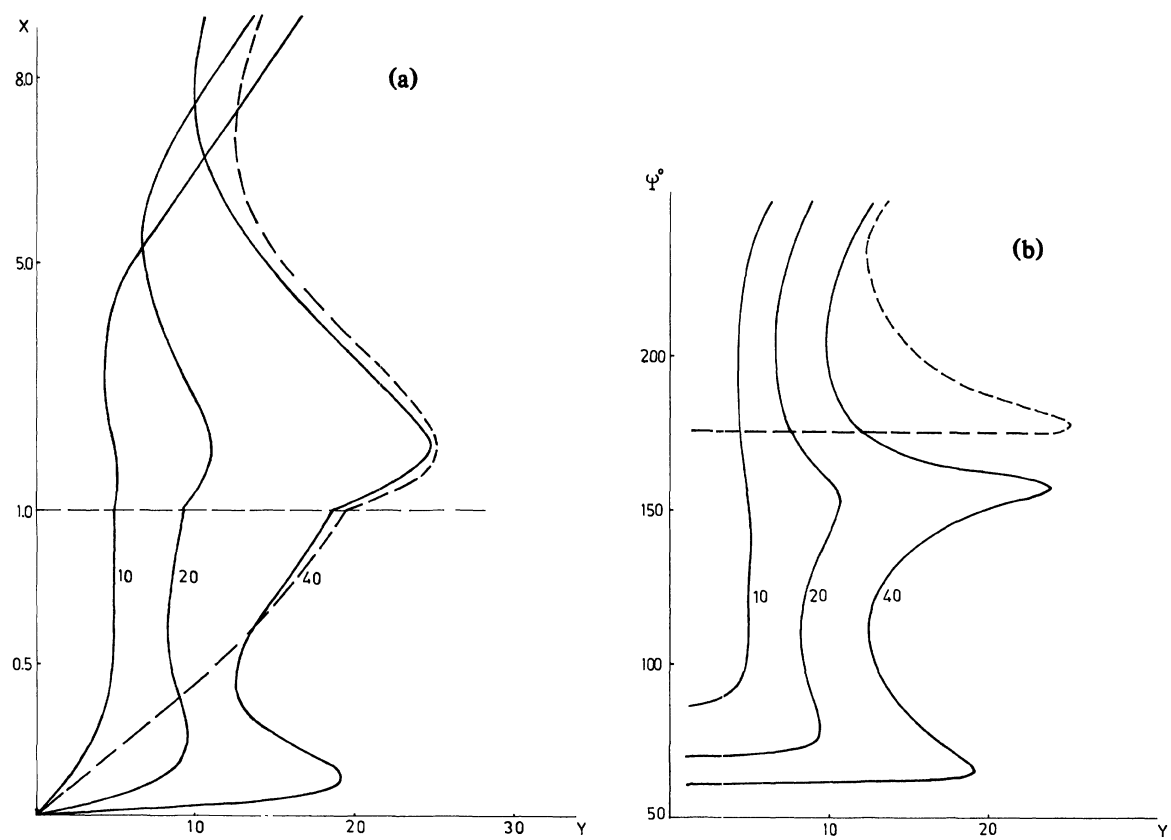

FIG. 3. (a) Same as in Fig. 2, but now $\theta=1$; $\delta_1=\delta_2=2$. (b) The phase ψ of the transmitted amplitude as a function of the incident field for $\delta_1=\delta_2=2$, $\theta=1$, and $T_1=T_2=T_3=0.01$.

$= T_1$ ($\alpha_2 L_2 = 0$, $T_2 = T_3 = 1$), leads to the standard result for the single cavity.

The details of the results that follow from (10) for a number of system parameters are given in Figs. 2–4. For nonzero θ and δ, x is complex $-x = |x| e^{-i\psi}$. The result for the purely absorptive case is shown in Fig. 2. Assuming identical cavities with $\alpha_1 L_1 = \alpha_2 L_2$, $T_1 = T_2 = T_3 = 0.01$, we find that the present coupled system can exhibit a wide range of multistable[4] behavior. In fact there is the possibility of tristability.[5] A remarkable aspect of the coupled system is the existence of bistability for $\alpha L / T = 7$. Thus, bistability can exist even for $\alpha L / T$ values, which are less than those for which bistability exists for a single cavity. This could be quite significant for materials like GaAs, where purely absorptive bistability has been difficult to see.[6] When there is no overall feedback ($T_3 = 1$), then the output of the first cavity feeds the second cavity, and in such a case, the behavior of the coupled system is simple—the output of the total system shows bistability for the cavity 1, while the second cavity remains on the lower branch. When the first cavity is switched to the upper branch, then the second cavity again shows bistability. Therefore, for $T_3 = 1$, the total system would exhibit double hysteresis loops. Results with cavity detuning are shown in Fig. 3. The multistable behavior of both amplitude and phase is to be noted. Here, one has the possibility of the system switching from the uppermost branch to the lowermost branch. Note that the condition on $\alpha L / T$ for the occurrence of bistability in single cavity[3] is

$$\left(\frac{\alpha L}{T} + 1 - \Delta\theta\right)^2 \left(\frac{\alpha L}{2T} - 4 + 4\Delta\theta\right) > 27 \frac{\alpha L}{2T} (\Delta + \theta)^2 , \qquad (12)$$

and thus for $\alpha L / T = 10$, $\theta = 1$, $\delta = 2$, no bistability is there for the single cavity case. However, for our coupled system, we obtain bistability for these values of the parameters. For comparison we also show the results for the case when, say, the first cavity is empty. Finally, in Fig. 4, we show how the multistable behavior of the coupled system changes when one cavity exhibits absorptive bistability and the other cavity shows dispersive bistability. In conclusion, we have shown how the coupled bistable systems could lead to new features that can have interesting applications.

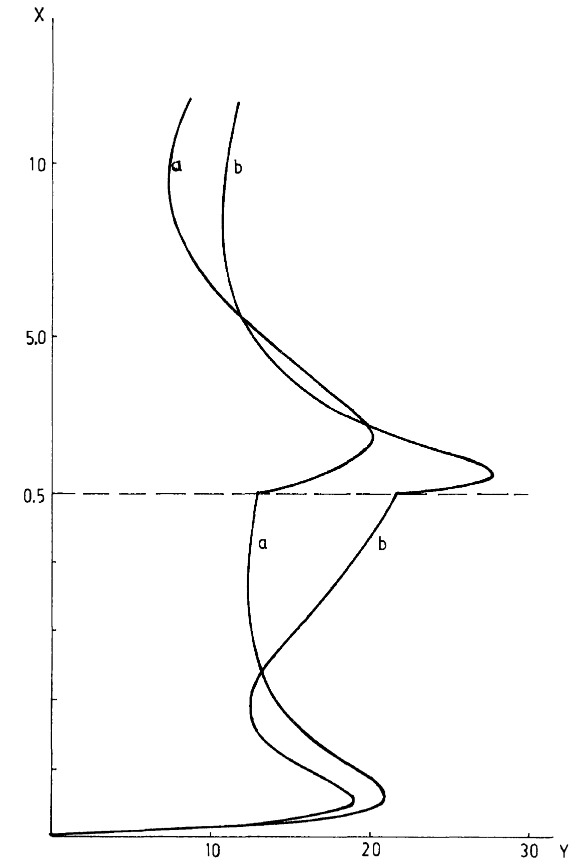

FIG. 4. The transmitted amplitude $|x|$ as a function of the incident field for $T_1 = T_2 = T_3 = 0.01$, $\alpha_1 L_1 / T = \alpha_2 L_2 / T = 40$, $\theta_3 = 1$. Here, curve (a) [(b)] occurs when the first cavity exhibits absorptive (dispersive, $\theta_1 = 1, \delta_1 = 2$) bistability and the second cavity exhibits dispersive $\theta_2 = 1, \delta_2 = 2$ (absorptive) bistability.

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[1]See, for example, the proceedings of two recent conferences on this subject: (a) *Proceedings of the International Conference on Optical Bistability, Asheville, North Carolina, 1980*, edited by C. M. Bowden, M. Ciftan, and H. R. Robl (Plenum, New York, 1981); (b) *Proceedings of the International Conference on Optical Bistability, Rochester, New York, 1983*, edited by C. M. Bowden, H. M. Gibbs, and S. L. McCall (Plenum, New York, 1984).

[2]A recent paper [J. S. Satchell, C. Parigger, and W. J. Sandle, Opt. Commun. **47**, 230 (1983)] discusses a different type of coupling between bistable devices and obtains results which are different from ours. For example, these authors find only bistable behavior for the output as a function of the input.

[3]Cf. R. Bonifacio, M. Gronchi, and L. A. Lugiato, in Ref. 1(a), p. 31.

[4]Within the mean-field approximation, the branch with the negative slope is expected to be unstable.

[5]Tristability has also been reported in a single ring cavity in situations (such as those in the limit of large αL), where the spatial structure is important and where both absorptive and dispersive effects are included [H. J. Carmichael and J. A. Hermann, Z. Phys. B **38**, 366 (1980)]. Similarly, lasers with saturable absorbers, and with injected coherent signals also exhibit tristability, cf. P. Mandel, Z. Phys. B **33**, 205 (1979).

[6]H. M. Gibbs, S. L. McCall, T. N. C. Venkatesan, A. Passner, A. C. Gossard, and W. Weigmann, in Ref. 1(a), p. 109.