Proton–hydrogen-atom scattering in a nearly resonant laser field

H. Bachau* and Robin Shakeshaft
Physics Department, University of Southern California, Los Angeles, California 90089-0484
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We consider proton scattering from hydrogen atoms in the presence of a laser beam that resonantly (or nearly resonantly) excites the hydrogen atoms from the $1s$ to the $2p$ state. The laser beam propagates in a direction perpendicular to the proton beam, and it is linearly polarized, with polarization either parallel (longitudinal) or perpendicular (transverse) to the direction of incidence of the proton. We allow the collision to couple the $1s$, $2s$, and $2p$ states and we treat the interaction of the laser with the atom in the two-state rotating-wave approximation. We have calculated the integrated cross section, $\sigma(2s)$, for excitation of the $2s$ state. We find that the laser enhancement of $\sigma(2s)$ is small for longitudinal polarization, but for transverse polarization $\sigma(2s)$ varies rapidly with laser intensity, and in the intensity range 10^9–10^{10} W/cm^2 $\sigma(2s)$ is of the order of 100 times larger than its value in the field-free case.

Suppose that hydrogen atoms are initially prepared in the $2p$ state and are then bombarded with ions. The integrated cross section for the atoms to undergo a transition to the $2s$ state would be *infinite* in the limit where spontaneous emission and the splitting of the $2s$-$2p$ degeneracy are neglected. This divergence of the cross section is a consequence of the fact that the collision-induced coupling between the $2s$ and $2p$ states is, at large distances, of the form of a nonoscillatory dipole.

We envisage a slightly different experiment in which hydrogen atoms are initially in their ground state, but are resonantly (or nearly resonantly) excited to the $2p$ state by a picosecond laser pulse. The laser is turned on adiabatically on the time scale of atomic motion and it induces the hydrogen atoms into a dressed state, a time-dependent linear superposition of $1s$ and $2p$ states. While the laser is on, the atoms are bombarded by protons, and after the collision is over the laser is turned off. (For a proton impact energy of 100 keV, the collision duration is of order 10^{-15} s, which is much less than the pulse time.) We have calculated the cross section $\sigma(2s)$ for the atoms to undergo a transition to the $2s$ state, and we have studied the behavior of $\sigma(2s)$ with varying field strength and polarization of the laser. The purpose of this paper is to report our results. We note that $\sigma(2s)$ is not infinite, because the long-range dipole coupling between the $2s$ and $2p$ states *oscillates* at a small frequency close to the Rabi frequency.

The laser pulse is of sufficiently short duration for spontaneous emission from the $2p$ state to be neglected. However, ionization of the excited atoms by the laser, which we neglect, becomes significant as the field strength increases. We calculated the ionization rate from the $2p$ state using Fermi's golden rule and found the lifetime of the $2p$ state to be (in seconds)

$$\tau_{2p} \approx 10^{-3}/I\ (\text{W/cm}^2) \ , \tag{1}$$

where $I \equiv cE_0^2/8\pi$ is the intensity of the field and E_0 is the amplitude; the field $\vec{E}(t)$ is treated classically and is of the form

$$\vec{E}(t) = \hat{\epsilon} E_0 \cos(\omega t + \eta) \ , \tag{2}$$

where $\hat{\epsilon}$, ω, and η are, respectively, the polarization, frequency, and phase of the field. The lifetime τ_{2p} becomes equal to the pulse time, 10^{-12} s, at an intensity of about 10^9 W/cm^2; to avoid depleting the excited atoms at higher intensities one would have to use a shorter pulse.

We work in the laboratory frame and neglect recoil of the target atom so that the atom remains at rest. The incident proton is treated as a classical particle which moves with constant velocity $\vec{v} = v\hat{v}$ and impact parameter $\vec{b} = b\hat{b}$ relative to the target proton. The relative coordinate of the incident and target protons is $\vec{R}(t) = \vec{b} + \vec{v}t$. Since the incident proton is so massive, its trajectory will not be appreciably affected by the laser, and we can neglect the coupling between the laser and incident proton. In any case, the dimensionless parameter, $evE_0/\hbar\omega^2$, governing the strength of the coupling between the laser and incident proton is small over most of the region of interest; for a proton impact energy of 100 keV, this parameter does not reach unity until laser intensities of about 10^{14} W/cm^2. We take the laser beam to be perpendicular to $\vec{v}$, and we take the polarization to be linear; $\hat{\epsilon}$ is either parallel to $\vec{v}$ (longitudinal polarization) or perpendicular to $\vec{v}$ (transverse polarization). With W_0 and W_1 the energies of the ground and first excited states of the atom, the detuning of the laser from resonance is $\Delta\omega = \omega - W_{10}/\hbar$, where $W_{10} = W_1 - W_0$. We assume that $|\Delta\omega| << W_{10}/\hbar$. In treating the interaction of the laser with the atom, we work in the two-state rotating-wave approximation,[1] with the $1s$ state coupled only to the $2p$ state; the polarization of the $2p$ state is the same as that of the laser. Under the influence of the laser, the atom adiabatically evolves into one of two dressed states represented by the vectors[1]

$$|\phi_\pm(t)\rangle = (2\cosh\mu)^{-1/2}(e^{\pm\mu/2}e^{i\omega t/2}|1s\rangle \pm e^{\mp\mu/2}e^{-i\omega t/2-i\theta}|2p\rangle)\exp\left[-i\left(\frac{W_0+W_1}{\hbar}\pm\Omega\right)t/2\right] \ , \tag{3}$$

where, with $\vec{r}$ the electron coordinate,

$$\sinh\mu = \hbar\Delta\omega/|\Lambda| \ , \tag{4}$$

$$\Lambda = (eW_{10}/\hbar\omega)E_0 e^{i\eta}\langle 1s|(\hat{\epsilon}\cdot\vec{r})|2p\rangle \equiv |\Lambda|e^{i\theta} \ , \tag{5}$$

$$\Omega = (|\Lambda/\hbar|^2 + \Delta\omega^2)^{1/2} \ . \tag{6}$$

The atom evolves into $|\phi_+(t)\rangle$ or $|\phi_-(t)\rangle$ according to whether $\Delta\omega > 0$ or $\Delta\omega < 0$. In Eq. (3), $|1s\rangle$ is the normalized vector representing the bare atomic ground state. With $|2p_m\rangle$ the normalized vector representing the bare $2p$ atomic state with angular momentum projection $\hbar m$ along $\vec{v}$, the vector $|2p\rangle$ is $|2p_0\rangle$ for longitudinal polarization, and $|2p_-\rangle$ for transverse polarization, where

$$|2p_\pm\rangle = 2^{-1/2}(|2p_1\rangle \pm |2p_{-1}\rangle) .$$

In defining the laser-atom coupling Λ we have made the dipole approximation, and we can set $W_{10}/\hbar\omega$ equal to unity since $\Delta\omega$ is very small; we thereby obtain $|\Lambda| \approx 0.7449 eE_0 a_0$. Note that Ω is the (generalized) Rabi frequency. Note also that we have neglected the ac Stark shift of the energy levels.

We approximate the exact state vector, $|\psi(t)\rangle$, of the electron by the following truncated expansion in orthonormal basis vectors:

$$|\psi(t)\rangle = a_+(t)|\phi_+(t)\rangle + a_-(t)|\phi_-(t)\rangle + e^{-iW_1 t/\hbar}[a_{2s}(t)|2s\rangle + a_{2\bar{p}}(t)|2\bar{p}\rangle + a_{2p_+}(t)|2p_+\rangle] , \quad (7)$$

where $|2\bar{p}\rangle$ is $|2p_0\rangle$ for transverse polarization and $|2p_-\rangle$ for longitudinal polarization, and where $|2s\rangle$ represents the atomic $2s$ state. Note that reflection symmetry in the $\vec{v}$-$\vec{b}$ plane implies $a_{2p_+}(t) = 0$ in the case of longitudinal polarization. With the electron-laser interaction incorporated in the dressed states, we determined the coefficients $a_+(t)$, etc., by solving the standard coupled-state equations

$$i\hbar \frac{d\vec{a}(t)}{dt} = \underline{M}(t)\vec{a}(t) , \quad (8)$$

where $\vec{a}(t)$ is the column vector with elements $a_+(t)$, etc., and where $\underline{M}(t)$ is composed of matrix elements of the interaction

$$V(t) = e^2/R(t) - e^2/|\vec{r} - \vec{R}(t)|$$

between the atom and the projectile. Equation (8) was solved for $\Delta\omega > 0$ subject to the boundary condition $a_+(-\infty) = 1$ and with the other coefficients set equal to zero at $t = -\infty$.

The probability for a transition to the $2s$ state at a given impact parameter is $P(2s) = |a_{2s}(\infty)|^2$. For large b, we make a multipole expansion of $V(t)$, with the leading term the dipole term, namely, $-e^2(\vec{r}\cdot\vec{R})/R^3$. If we retain only the dipole term, and include this term only to leading order in perturbation theory, we obtain the following useful approximation to $P(2s)$, applicable[2] for $b >> a_0$:

$$P(2s) \approx (9e^{-|\mu|}/2\cosh\mu)(\hbar\bar{\Omega}/mv^2)^2 K_\nu^2(\bar{\Omega}b/2v)f . \quad (9)$$

Here $\bar{\Omega} = \Omega + |\Delta\omega|$, $f = (\hat{\epsilon}\cdot\hat{v})^2 + (\hat{\epsilon}\cdot\hat{b})^2$, $K_\nu(x)$ is the modified Bessel function, and $\nu = 0$ or 1, according to whether the polarization is longitudinal ($\parallel$) or transverse ($\perp$). The physical mechanism underlying Eq. (9) is a two-step process in which first the laser excites the atom from the $1s$ to the $2p$ state, and then the collision induces a transition from the $2p$ to the $2s$ state. Note that for $|x| << 1$ we have $K_0(x) \approx -\ln x$ and $K_1(x) \approx 1/x$, so that over the range of impact parameters $a_0 << b << 2v/\bar{\Omega}$ we have that $P_\parallel(2s)$ is smaller than $P_\perp(2s)$ by a factor of order $(x\ln x)^2$ (for $f \approx 1$), where $x = \bar{\Omega}b/2v$ and where the subscript on $P(2s)$ indicates the polarization. This difference can be understood as follows. The collision-induced dipole coupling between the $2s$ and $2p$ states is proportional to (for $\Delta\omega > 0$)

$$-(e^2/R^3)\langle 2s|(\vec{r}\cdot\vec{R})|2p\rangle e^{-i\bar{\Omega}t/2} = \pm 3(e^2/R^3)(\hat{\epsilon}\cdot\vec{R})e^{-i\bar{\Omega}t/2}$$

Now the duration of the collision is of order b/v and so if $x << 1$, the oscillatory term $\exp(-i\bar{\Omega}t/2)$ can be set equal to unity during the collision. For transverse polarization $\hat{\epsilon}\cdot\vec{R} = \hat{\epsilon}\cdot\vec{b}$ and the coupling is even in t, but for longitudinal polarization $\hat{\epsilon}\cdot\vec{R} = vt$ and the coupling is odd in t. Hence, the time average of the coupling over the collision duration vanishes for longitudinal polarization (but not for transverse polarization). It follows that the average coupling is stronger in the transverse case and hence $P_\perp(2s)$ is

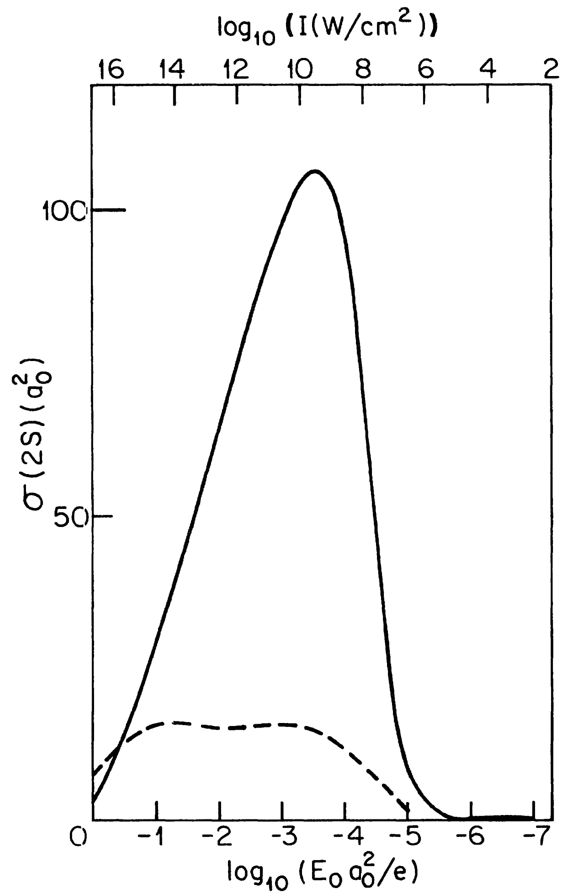

FIG. 1. The integrated cross section, $\sigma(2s)$, vs the amplitude E_0 (or intensity I) of the field. The proton impact energy is 100 keV, the detuning $\Delta\omega$ is 5.44×10^{-4} eV, and the phase θ is π. The solid and dashed curves refer to transverse and longitudinal polarization, respectively.

TABLE I. The integrated cross section $\sigma_\perp(2s)$ in atomic units vs the phase θ of the laser-atom coupling, for a proton impact energy of 100 keV, a laser intensity of 3.5×10^6 W/cm^2, and a laser detuning of 5.44×10^{-4} eV.

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
$\sigma(2s)$	9.60	9.64	9.65	9.64	9.61	9.57	9.56	9.57

larger than $P_\parallel(2s)$. The integrated cross section for $2s$ excitation is

$$\sigma(2s)=\int d^2bP(2s) \quad . \tag{10}$$

Note that within the approximation of Eq. (9), $P(2s)$ is independent of both the sign of the detuning $\Delta\omega$ and the phase θ of the laser-atom coupling; this is not true for the exact $P(2s)$.

In Fig. 1, we show $\sigma(2s)$ versus the field strength for the proton impact energy equal to 100 keV and for the detuning and phase of the coupling equal to 5.44×10^{-4} eV and π, respectively. The results were obtained by solving Eq. (8) numerically. Except for very low or very high intensities, Eq. (9) is accurate for $b > b_0$, where b_0 is about $20a_0$ in the transverse case and somewhat larger in the longitudinal case since in the latter case the $2s$-$2\bar{p}$ coupling is relatively strong at smaller impact parameters. We see that $\sigma_\perp(2s)$ can be greatly enhanced by the laser. In the intensity range 10^9–10^{10} W/cm^2, $\sigma_\perp(2s)$ is roughly a factor of 100 times larger than its value in the absence of a laser.[3] The rise and fall of $\sigma_\perp(2s)$ as the laser intensity increases can be understood as follows. We see from Eq. (3) that the $2p$ population induced by the laser is $e^{-|\mu|}/(2\cosh\mu)$. At very low intensities $|\mu|$ is large ($|\mu|\rightarrow\infty$ as $I\rightarrow0$), and thus the laser induced $2p$ population is small, and the cross section is the same as in the field-free case. [Equation (8) reduces to the field-free equations for $I\approx0$.] As the laser intensity rises, the $2p$ population also rises and becomes roughly constant, equal to $\frac{1}{2}$, at and above the intensity I_p, at which $\sigma_\perp(2s)$ peaks. One would expect $\sigma_\perp(2s)$ to be constant for $I > I_p$ were it not for the fact that the main contribution to $\sigma_\perp(2s)$ comes from the range of impact parameters $b\lesssim v/\overline{\Omega}$; for greater impact parameters the $2s$-$2p$ coupling, which oscillates at frequency $\overline{\Omega}$, averages to zero over the collision duration, b/v. Since $\overline{\Omega}$ increases with increasing intensity (roughly as the square root of I), the significant range of impact parameters decreases, and this tends to reduce the cross section. Consequently, $\sigma_\perp(2s)$ rises (because the $2p$ population rises) and then falls (because the significant range of b diminishes) as I increases. In contrast, $\sigma_\parallel(2s)$ rises and then levels off at the intensity I_p and stays roughly constant over a wide range of I. The reason that $\sigma_\parallel(2s)$ exhibits a plateau for $I > I_p$ is as follows. Recall that if the factor $\exp(-i\overline{\Omega}t)$ is set equal to unity, the longitudinal $2s$-$2p$ coupling is odd in t and it therefore averages to zero over the collision duration. However, the $\sin(\overline{\Omega}t)$ component of $\exp(-i\overline{\Omega}t)$ combines to make the longitudinal $2s$-$2p$ coupling even in t so that it no longer averages to zero for $b\lesssim v/\overline{\Omega}$. Furthermore, with $|t|\lesssim b/v$, the magnitude of $\sin(\overline{\Omega}t)$ grows as $\overline{\Omega}$ increases. Hence, while the range of significant b decreases with increasing I, the magnitude of the (time-averaged) longitudinal $2s$-$2p$ coupling increases; that the two effects combine to make $\sigma_\parallel(2s)$ roughly constant for $I > I_p$ can be seen by substituting $P_\parallel(2s)$ from Eq. (9) into Eq. (10) for $\sigma_\parallel(2s)$, setting the factor $e^{-|\mu|}/(2\cosh\mu)$ equal to $\frac{1}{2}$, and changing the integration variable from $\vec{b}$ to $\overline{\Omega}\vec{b}$; the resulting expression is independent of the field. Note that Eq. (9) is strictly valid only for $b >> a_0$, but we have just used it over the entire range of b; the relative error is, however, fairly small. We could not make the same argument for $\sigma_\perp(2s)$ using Eq. (9) for $P_\perp(2s)$ since the integral over b would diverge at the lower limit $b=0$.

In Fig. 1 we have shown $\sigma(2s)$ for a specific phase θ of the coupling Λ, and hence a specific phase η of the field. However, over the intensity range considered $\sigma(2s)$ is not very sensitive to θ, and if we were to average $\sigma(2s)$ over θ, the results would not be much different from those shown. This is because the approximate expressions for $P(2s)$ given by Eq. (9) are independent of θ and work well in the region $b >> a_0$, which is the region providing the dominant contribution to $\sigma(2s)$. In Table I, we give the values of $\sigma_\perp(2s)$ for several different values of the phase, at a field intensity of 3.5×10^6 W/cm^2. Note that the measurable cross section involves an average over θ.

Lasers with the pulse duration and frequency considered here do currently exist with powers of up to about 10^6 W/cm^2, and higher powers might be realized in the future. It might be possible to observe the strong enhancement of $\sigma_\perp(2s)$ by the laser. However, the present calculation may be only qualitatively accurate owing to the simple treatment of the laser field, and the neglect of ionization and the ac Stark shift.

Finally, we take note of related work[4] on electron impact excitation of H$(2s)$ and He$(2s)$ in a nonresonant laser field. Electron impact excitation of H$(2s)$ in a resonant field has also been investigated,[5] but the intensity dependence of the cross section was not studied and no numerical results were given.

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*Permanent address: Laboratoire des Collisions Atomiques (Equipe de Recherche No. 260), Universite de Bordeaux I, 33400, Talence, France.

[1]M. H. Mittleman, *Theory of Laser-Atom Interactions* (Plenum, New York, 1982).

[2]In the transverse case, Eq. (9) breaks down when $\hat{\epsilon}\cdot\hat{b}\approx 0$, for then $f\approx 0$.

[3]For theoretical estimates of proton-hydrogen atom cross sections in the field-free case see, e.g., R. Shakeshaft, Phys. Rev. A **18**, 1930 (1978).

[4]S. Jetzke, F. H. M. Faisal, R. Hippler, and H. O. Lutz, Z. Phys. A **315**, 271 (1984); see also, F. H. M. Faisal, in *Coherence and Correlation in Atomic Physics,* edited by H. Kleinpoppen and U. F. Williams (Plenum, New York, 1980), p. 479; in this paper the variation of the cross section with laser frequency was studied, and near to resonant frequencies interesting structure was revealed. However, the laser-atom coupling was treated in first-order perturbation theory, and Rabi flopping between levels was therefore neglected.

[5]J. I. Gersten and M. H. Mittleman, Phys. Rev. A **13**, 123 (1976).