Magnetic field effects on electron heat transport in laser-produced plasmas

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The classical treatment of thermal heat transport in the presence of magnetic fields has been modified to include effects associated with steep temperature gradients by extending the one-dimensional model of Shvarts *et al.* [Phys. Rev. Lett. **47**, 247 (1981)] to three dimensions. The effects of magnetic field inhibition are described in terms of the parameter $\beta_0=\omega_c\tau_{ei}$, where ω_c is the electron gyrofrequency and τ_{ei} the (thermal) electron-ion collision time. The model has been applied to plasmas whose zeroth-order distribution functions (f_0) are Maxwellian, and solutions have been obtained for the components of the heat flux across the magnetic field, parallel and perpendicular to the temperature gradient. It is found that it is only for small β_0 ($\lesssim 0.2$) that the anisotropic portion of the distribution function ($\vec{f}_1$) is limited, according to the prescription of Shvarts *et al.*, to δf_0 where δ is an *ad hoc* cutoff parameter of value approximately unity; for higher values of β_0, a strong reduction of both components of the heat flux occurs due to the inhibition of the more energetic heat-carrying electrons in the distribution, and the classical Braginskii results are valid (in the sense that $f_1 \lesssim f_0$ for heat-carrying electrons). The sensitivity of the results to the parameter δ is examined. For parameters typical of glass-laser-generated plasmas, strong inhibition may occur for magnetic fields as small as 100 kG.

I. INTRODUCTION

Thermal transport in laser-produced plasmas is a topic of great importance to laser fusion, and has attracted considerable attention in the literature. In one-dimensional situations, and where moderately steep temperature gradients are believed to occur, heat fluxes considerably smaller than those predicted by the classical theories of Spitzer[1] and Braginskii[2] have been inferred experimentally; such results are usually parametrized in terms of a "flux limiter" f,[3] with the heat flux q given as a multiple f of the "free-streaming flux" q_F, defined here as

$$q_F=n_e kT(kT/m)^{1/2}\ , \tag{1}$$

where n_e, T, and m are the electron number density, temperature, and mass, respectively, and k is Boltzmann's constant. The absorption fractions observed in various short-wavelength experiments (laser wavelength $\lambda \lesssim 1$ μm) dominated by inverse bremsstrahlung, for example, are generally consistent with a flux limiter of the order of 0.03,[4–7] although there are exceptions.[8]

Many attempts have been made to understand the physical basis of reduced heat fluxes in terms of microscopic processes such as ion-acoustic turbulence,[9–11] or in terms of kinetic theory alone.[12–16] In particular, the Fokker-Planck calculations performed by Bell *et al.*[12] indicated values of f of the order of 0.1. A good review of some of the experimental evidence for reduced heat fluxes is given by Kruer.[17]

In two-dimensional situations, the existence of large magnetic fields (~ 1 MG), perpendicular to the plane containing the temperature and density gradients, has been known for some time,[18] and various authors have modeled the reduction of thermal conductivity caused by such fields[19–24] using either Braginskii's theory[2] or a subset of his equations. In such calculations it is assumed that the localization of electron orbits provided by the magnetic fields will guarantee the validity of Braginskii's model, even in situations where the Braginskii model is applied outside its domain of validity (electron mean free path $\gtrsim 0.01$ of the temperature scale length[25]).

It has always been a possibility that magnetic fields could be the cause of the experimentally observed "flux inhibition," in which case a particular value of f inferred from an experiment would be related to the value of β_0 [$=\omega_c\tau_{ei}$, where ω_c is the electron gyrofrequency and τ_{ei} is the (thermal) electron-ion collision time] in the region of magnetic field inhibition. There is currently little evidence for this explanation, unfortunately, since to our knowledge no experimental correlations have been reported between inferred values of f and either measured or calculated magnetic fields. It is still useful, however, when given some calculated heat flux q, to introduce the term "effective flux limiter" to describe the quantity q/q_F. It will be seen that in typical parameter regimes effective flux limiters of less than 0.1 are implied by modest magnetic fields.

Recent experiments and simulations performed at Los Alamos, for 10-μm laser radiation, have illustrated the role of magnetic fields in enhancing lateral energy transport along the target surface and away from the focal spot.[26–28] The transport mechanism here is essentially collisionless, the dominant heat flux being the convective flux associated with the current. Our results are not expected to apply to these experiments, since the collisionless regime is outside the scope of this paper.

In simulations of a typical two-dimensional laser-produced plasma, there may be spatial regions of large magnetic fields where Braginskii's theory is applicable, and other regions of small or zero magnetic field where some additional treatment (e.g., flux limiting) is required to prevent unphysically large heat fluxes. While the use of a flux limiter in magnetic-field-free calculations and the use of Braginskii's equations in magnetic field calculations are both commonplace in hydrodynamic simulations, a consistent treatment of both effects in a two-dimensional fluid code has to the best of our knowledge not been reported in the literature. As a step towards this end, this paper develops a simple model which attempts to combine these two effects.

A number of problems arise when attempting flux limiting in a two-dimensional fluid code. In one dimension it is standard practice to take a smooth transition between the classical Spitzer-Härm (SH) flux $q_{\rm SH}$ (Ref. 1) and the limited flux fq_F according to the equation[3]

$$q^{-1}=q_{\rm SH}^{-1}+(fq_F)^{-1}\,; \qquad (2)$$

this is normally implemented by decreasing the classical conductivity $\kappa_{\rm SH}$ to κ' where

$$\kappa'=\kappa_{\rm SH}[1+(\,|\,q_{\rm SH}\,|\,/fq_F)]^{-1} \qquad (3)$$

and solving the thermal diffusion equation using

$$\vec{q}=-\kappa'\vec{\nabla}T\,. \qquad (4)$$

In two-dimensional hydrodynamic codes such as SAGE,[7] these same equations are used in the absence of magnetic fields. This treatment ensures that the heat-flux vector $\vec{q}$ is always directed down the temperature gradient. This condition, while appearing physically plausible, will not necessarily always apply; for example, in plasmas where the electron mean free path is sufficiently long that the heat flux at a point is not given in terms of locally defined variables, $\vec{q}$ and $\vec{\nabla}T$ need not be parallel.

The issue of the relative orientation of $\vec{q}$ and $\vec{\nabla}T$ is of particular importance for the coronas of spherical targets irradiated by short-wavelength laser radiation, where the fraction of energy carried by long-mean-free-path electrons is believed to be small and where lateral heat fluxes may help to smooth out nonuniformities in the laser energy deposition. For experiments with a high degree of irradiation uniformity it is reasonable to neglect magnetic fields, although the level of uniformity necessary for this simplification has not been quantitatively determined. Even in the absence of magnetic fields, care must be taken with the numerical implementation of Eqs. (2)–(4) in a two-dimensional calculation. A large temperature gradient in the radial direction implies a large value of $q_{\rm SH}$, a small value of κ' [from Eq. (3)], and therefore a reduction of the heat flux in the lateral direction, even if the lateral component of the temperature gradient is small. It may appear that radial inhibition implies lateral inhibition, but the radial and lateral directions (unlike the direction of the temperature gradient) have no intrinsic physical significance. As an illustration, consider what would result in a two-dimensional code from introducing separate flux limiters for the two coordinate directions, limiting q_r to f_rq_F and q_θ to $f_\theta q_F$ in spherical (r,θ) geometry, for example. In the strongly flux-limited case the heat-flow vector would always be directed at an angle of $\pm\tan^{-1}(f_r/f_\theta)$ to the computational grid, regardless of the direction of the temperature gradient. An alternative approach might be, for example, to seek a theory which includes an additional component of heat flow proportional to the density gradient.

In the presence of magnetic fields the situation appears to be more complicated, as a result of at least three effects: (a) the heat flow in the plane perpendicular to the magnetic field is no longer directed parallel to the temperature gradient; (b) diffusive (i.e., collisional) transport is inhibited; and (c) lateral transport at low densities is dominated by the (collisionless) convection associated with a lateral current.[26–28] This third effect may not enhance symmetry of drive, since the convected energy is redeposited in the plasma in localized regions near magnetic-field nulls. In some respects, however, the transport problem becomes more tractable in the presence of magnetic fields, since long-mean-free-path electrons are better confined.

A fundamental treatment of the two-dimensional transport problem in the presence of moderately steep temperature gradients, including both strong and weak magnetic field limits, would require, for example, a two-dimensional Fokker-Planck treatment. Ideally, self-consistent models for the source of heated electrons,[16] the generation of the magnetic field, and the energy loss to hydrodynamic motion would also be included. To date, such treatments have been computationally prohibitive. However, it was shown by Shvarts *et al.*[14] that, in one dimension, a simple local treatment, in which the anisotropic portion of the distribution function (f_1) is bounded from above by the isotropic Maxwellian distribution function (f_0), leads to results similar in many respects to those obtained by Bell *et al.*[12] from Fokker-Planck simulations. This correspondence encourages us to extend this simple local model to two and three dimensions, including magnetic fields. While some questions will remain unanswered in the absence of a full Fokker-Planck treatment, this approach provides some insight into the respective roles of magnetic fields and kinetic effects in "inhibiting" thermal conduction.

In Sec. II we start from the Boltzmann equation for a Lorentz plasma, with the collision operator including just electron-ion collisions, and use a moment expansion in which the distribution function $f(\vec{v})$ is expressed as the sum of isotropic and anisotropic components, $f_0(v)$ and $\vec{f}_1(v)$, respectively. The conventional treatment, which gives $\vec{f}_1(v)$ as a linear response to the temperature and density gradients ($\vec{\nabla}T$, $\vec{\nabla}n_e$) and the electric field ($\vec{E}$), is modified by requiring in addition that the components of $\vec{f}_1$ parallel and perpendicular to the temperature gradient should be bounded by some number of order unity times f_0. The current ($\vec{J}$) and heat flux ($\vec{q}$) are then obtained from the bounded $\vec{f}_1(v)$.

In one-dimensional calculations a zero-current condition is usually used to determine $\vec{q}$ as a function of $\vec{\nabla}T$. In two dimensions, solutions may also be determined for $\vec{J}=\vec{0}$, but this is an unnecessarily restrictive condition. In typical laser-produced plasmas, where the displacement current is neglected, the magnetic field is given from Faraday's law as the time integral of $\vec{\nabla}\times\vec{E}$, and the current is obtained from Ampère's law as $(4\pi)^{-1}\vec{\nabla}\times\vec{B}$. The magnetic field and current generally change on hydrodynamic time scales which are long in comparison with the time scales associated with electron thermal transport. For the transport calculation, therefore, the current should be treated as given, with $\vec{q}$ to be determined as a function of $\vec{\nabla}T$ and $\vec{J}$. Ohm's law, instead of giving $\vec{J}$ as a function of $\vec{E}$ and $\vec{\nabla}T$, gives $\vec{E}$ as a function of $\vec{J}$ and $\vec{\nabla}T$.

Typical solutions for various cases are given in Sec. III. The main result is that the transition between flux limitation and magnetic-field inhibition occurs at modest values of β_0, of the order of 0.1, even for large temperature gradients.

II. CALCULATIONAL METHOD

We start with the Boltzmann equation for the electron distribution function $f(\vec{r},\vec{v},t)$:

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\vec{\nabla}f-\frac{e}{m}\left[\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right]\cdot\frac{\partial f}{\partial\vec{v}}=C\ . \tag{5}$$

$\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, $-e$ and m the electron charge and mass, respectively, c the speed of light, and C the collision operator. We consider for simplicity a coordinate system (x,y,z) with the z axis locally aligned along the magnetic field, as indicated in Fig. 1, and we use spherical polar coordinates (v,θ,ϕ) in velocity space: $\vec{v}\equiv v\vec{\Omega}$. (In single-beam laser-plasma interactions, the magnetic field is of course oriented azimuthally about the laser beam;[18] effects associated with the curvature of this magnetic field are outside the scope of this paper.) The normalization of f is chosen such that the total electron number density n_e is given by

$$n_e=(4\pi)^{-1}\int f(\vec{v})v^2\,dv\,d\Omega,\quad d\Omega\equiv\sin\theta\,d\theta\,d\phi\ . \tag{6}$$

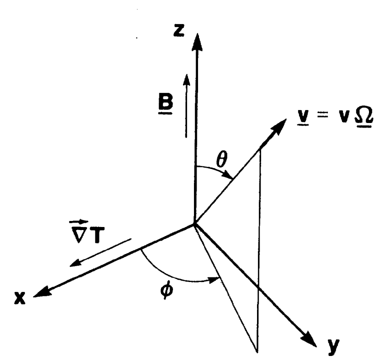

FIG. 1. Coordinate system. The z axis is taken along the magnetic field, and spherical coordinates about this axis (v,θ,ϕ) are used to describe velocity space.

In Fig. 1, the temperature gradient $\vec{\nabla}T$ is shown in the x direction; this will apply to all of the illustrative calculations of Sec. III, but initially $\vec{\nabla}T$ and the current $\vec{J}$ are arbitrary.

We use the first two terms of a moment expansion for f,[29]

$$f(\vec{r},\vec{v},t)=f_0(\vec{r},v,t)+\vec{f}_1(\vec{r},v,t)\cdot\vec{\Omega}\ , \tag{7}$$

where f_0 and $\vec{f}_1$ will be referred to as the isotropic and anisotropic components of f:

$$f_0=(4\pi)^{-1}\int f\,d\Omega\ , \tag{8}$$

$$\vec{f}_1=(3/4\pi)\int f\vec{\Omega}\,d\Omega\ . \tag{9}$$

Equations for f_0 and $\vec{f}_1$ are obtained by substituting Eq. (7) into Eq. (5) and integrating Eq. (5) over $d\Omega$ and $\vec{\Omega}\,d\Omega$, respectively. We find

$$\frac{\partial f_0}{\partial t}+\frac{v}{3}\vec{\nabla}\cdot\vec{f}_1-\frac{e}{m}\frac{1}{3v^2}\frac{\partial}{\partial v}(v^2\vec{E}\cdot\vec{f}_1)=C_0 \tag{10}$$

and

$$\frac{\partial\vec{f}_1}{\partial t}+v\vec{\nabla}f_0-\frac{e}{m}\vec{E}\frac{\partial f_0}{\partial v}-\omega_c\hat{z}\times\vec{f}_1=\vec{C}_1\ , \tag{11}$$

where $\omega_c=eB/mc$ is the electron gyrofrequency and $\hat{z}$ is a unit vector in the z direction. The collision integrals are defined by

$$C_0=(4\pi)^{-1}\int C\,d\Omega\ , \tag{12}$$

$$\vec{C}_1=(3/4\pi)\int C\vec{\Omega}\,d\Omega\ . \tag{13}$$

In obtaining Eqs. (10) and (11) use was made of the equation

$$\frac{\partial f}{\partial\vec{v}}=\vec{\Omega}\frac{\partial f_0}{\partial v}-\frac{1}{v}(\vec{\Omega}\cdot\vec{f}_1)\vec{\Omega}+\frac{1}{v}\vec{f}_1+\left[\vec{\Omega}\cdot\frac{\partial f_1}{\partial v}\right]\vec{\Omega}\ . \tag{14}$$

Here as in Refs. 1, 2, 14, and 29, it is assumed that f_0 is known: Eq. (10) will therefore not concern us further, and Eq. (11) will be used to calculate $\vec{f}_1$ in terms of f_0.

We now assume that only electron-ion Coulomb scattering contributes to $\vec{C}_1$:

$$\vec{C}_1=-\nu(v)\vec{f}_1\ , \tag{15}$$

where the velocity-dependent collision frequency $\nu(v)$ and the mean free path of thermal electrons (λ_T) are given by[30]

$$\begin{aligned}
&\nu(v)=4\pi e^4 Z n_e(\ln\Lambda)/(m^2v^3)\ ,\\
&\nu(v)=v/\lambda(v)=\tau_{ei}^{-1}(v)\ ,\\
&\lambda(v)=\lambda_T(v/v_T)^4\ ,\\
&v_T=\sqrt{3}v_0\ ,\\
&v_0=(kT/m)^{1/2}\ .
\end{aligned} \tag{16}$$

Here Z is the ionic charge number and $\ln\Lambda$ is the Coulomb logarithm. [Note that the λ_T defined here, and the λ of Ref. 12, are equal to $(\frac{9}{4})$ times the λ_0 used in

Ref. 14. The $v_{\rm th}$ used in Ref. 14 is $\sqrt{2}v_0$.] We also drop the time derivative in Eq. (11), assuming a quasistatic state. Then, substituting Eq. (15) into Eq. (11) we find[30]

$$\vec{f}_1-\beta\hat{z}\times\vec{f}_1=-\frac{1}{\nu}\left[v\vec{\nabla}f_0-\frac{e}{m}\vec{E}\frac{\partial f_0}{\partial v}\right], \tag{17}$$

where

$$\beta(v)=\omega_c\tau_{ei}(v)=\beta_0(v/v_0)^3 \tag{18}$$

and the electron Hall parameter β_0 is defined as $\beta(v_0)$. The strong velocity dependence of $\beta(v)$ should be noted.

Equation (17) may be inverted to yield

$$\vec{f}_1(v)=-\frac{1}{\nu(1+\beta^2)}[1+(\beta^2\hat{z}\hat{z}\cdot)+(\beta\hat{z}\times)]\times\left[v\vec{\nabla}f_0-\frac{e}{m}\vec{E}\frac{\partial f_0}{\partial v}\right], \tag{19}$$

which may be conveniently rewritten as

$$f_{1i}(v)=-\frac{v}{\nu(1+\epsilon_i^2\beta^2)}\left[\vec{\nabla}_i f_0-\beta\epsilon_i\vec{\nabla}_{i'}f_0-\frac{e}{m}(E_i-\beta\epsilon_iE_{i'})\frac{1}{v}\frac{\partial f_0}{\partial v}\right], \tag{20}$$

where

$$i=\{x,y,z\}, \quad i'=\{y,x,z\}, \quad \epsilon_i=\{1,-1,0\}. \tag{21}$$

The use of this notation considerably simplifies the algebraic manipulations. Note that if Eq. (20) were substituted into Eq. (10), an equation would be obtained which (in principle) would permit the determination of f_0. Such a treatment would be computationally prohibitive, as discussed above, and is outside the scope of this paper.

The importance of the parameter β is evident from the high-velocity scaling of Eq. (20). In the absence of magnetic fields ($\beta=0$), $v/\nu\sim v^4$, and $f_{1i}(v)$ exceeds $f_0(v)$ at large v. In the presence of magnetic fields, however, $(v/\nu)(1+\epsilon_i^2\beta^2)^{-1}\sim v^{-2}$, and the contribution to $f_{1i}(v)$ from $\nabla_i f_0$ becomes small at large v. The contribution to $f_{1i}(v)$ from the $\vec{\nabla}_{i'}f_0$ term scales as v.

So far, we have followed Braginskii's treatment. Now we make use of the prescription of Shvarts *et al.*[14] We assume that the ith component of Eq. (20) is satisfied for $v\le v_i^*$, and that

$$f_{1i}(v)=\delta_i f_0(v)\quad (v\ge v_i^*). \tag{22}$$

The cutoff velocities v_i^* are to be determined (by iteration), and $f_{1i}(v)$ is required to be continuous at $v=v_i^*$.

The quantities δ_i are arbitrary parameters in this model. In one dimension, a single parameter δ is used for the ratio of f_1 to f_0. Shvarts *et al.*[14] presented results for values of δ in the range 0.75—1.0. Shkarofsky[30] used $\delta=1$. In the one-dimensional Fokker-Planck calculations of Bell *et al.*[12] it was found that the ratio f_1/f_0 was less than but close to unity. Matte and Virmont[15] pointed out that f_1 equals $3f_0$ in the extreme case of a beam, and $1.5f_0$ for a semi-isotropic distribution; in their Fokker-Planck calculations, the ratio f_1/f_0 was typically <1.5 and always <2. In the Fokker-Planck calculations of Albritton,[16] the ratio f_1/f_0 ranged from 0.70 to 1.4 (at the point in space of maximum heat flux and at the velocity below which 90% of the heat is carried). In one dimension, the condition $\delta\le 1$ ensures that the truncated distribution function $(f_0+\vec{f}_1\cdot\vec{\Omega})$ is non-negative; in two dimensions this is guaranteed by taking $\delta\le 1/\sqrt{2}=0.71$. In view of these considerations, we have chosen $\delta_i=0.67$ for each i in the illustrative calculations presented below; it should be recognized, however, that there is an uncertainty involved in this choice of δ_i, and that other reasonable choices might differ from this value, possibly by up to a factor of 2. The sensitivity of the results to the choice of δ_i will be discussed quantitatively below.

The validity of separately limiting the three components of $\vec{f}_1$ in Eq. (22) may be questioned as being a procedure dependent on the choice of coordinate system. However, in the configuration considered in this paper (Fig. 1), with the x direction taken along $\vec{\nabla}T$, the three coordinate directions have clear physical significance. In the presence of a strong magnetic field, heat flow in the z direction is unaffected, cross-field heat flow in the x direction is strongly reduced, and the "Righi-Leduc" heat flow perpendicular to both the magnetic field and the temperature gradient is reduced by a smaller factor.

We now specialize to the case where $f_0(v)$ is a Maxwellian:

$$f_0(v)=A_0\exp(-\eta^2/2), \tag{23}$$

where

$$\eta=v/v_0 \tag{24}$$

and

$$A_0=n_e/[\sqrt{2}v_0^3\Gamma(\tfrac{3}{2})]. \tag{25}$$

Other possibilities could equally well be treated; for example, Shkarofsky[30] considered two-temperature distribution functions, with one containing a term in $\exp(-\eta^5)$ to represent a plasma heated by nonlinear inverse bremsstrahlung as suggested by Langdon.[31] Our model will break down in the limit where $f_0(v)$ is determined nonlocally by long-mean-free-path electrons. If we consider $\vec{f}_1$ to represent the response of a plasma, initially in equilibrium with f_0 Maxwellian, to the application of perturbative forces ($\vec{\nabla}T$ and $\vec{J}$), we are assuming that these forces do not significantly change f_0.

Substituting Eq. (23) into Eq. (20), and introducing the following dimensionless quantities (of obvious physical significance):

$$\vec{D}_T=-\lambda_T(\vec{\nabla}T)/T, \quad \vec{D}_n=-\lambda_T(\vec{\nabla}n_e)/n_e, \quad \vec{D}_E=\lambda_T eE/kT, \tag{26}$$

we obtain

$$f_{1i}(\eta)=\frac{f_0(\eta)\eta^4}{9(1+\beta_0^2\eta^6\epsilon_i^2)}[D_{n,i}-\beta_0\eta^3\epsilon_i D_{n,i'}+\tfrac{1}{2}(D_{T,i}-\beta_0\eta^3\epsilon_i D_{T,i'})(\eta^2-3)-(D_{E,i}-\beta_0\eta^3\epsilon_i D_{E,i'})] \quad (\eta\le\eta_i^*\equiv v_i^*/v_0) \tag{27a}$$

$$f_{1i}(\eta)=\delta_i f_0(\eta) \quad (\eta\ge\eta_i^*) . \tag{27b}$$

The current $\vec{J}$ and the heat flux $\vec{q}$ are determined from integrating the appropriate moments of $\vec{f}_1$ over velocity:

$$\vec{J}=-\frac{e}{3}\int_0^\infty v^3\vec{f}_1 dv , \tag{28}$$

$$\vec{q}=\frac{m}{6}\int_0^\infty v^5\vec{f}_1 dv . \tag{29}$$

It will be noted from the form of Eqs. (27)–(29) that $\vec{J}$ and $\vec{q}$ are linear tensor functions of $\vec{D}_n$, $\vec{D}_T$, $\vec{D}_E$, and $\vec{\delta}$; the last contribution arises from normalized velocities η above η_i^*, and depends on the other three quantities only indirectly through the solutions obtained for η_i^*.

From this point on, the algebra is straightforward but tedious, and the details are relegated to the Appendix. When the integrals for $\vec{J}$ and $\vec{q}$ are performed, the results have the form

$$\vec{J}=\vec{J}(\vec{\eta}^*;\vec{D}_n,\vec{D}_T,\vec{D}_E,\vec{\delta}) , \tag{30}$$

$$\vec{q}=\vec{q}(\vec{\eta}^*;\vec{D}_n,\vec{D}_T,\vec{D}_E,\vec{\delta}) . \tag{31}$$

Since we are treating $\vec{J}$ as given, we may invert Eq. (30) to give

$$\vec{D}_E=\vec{D}_E(\vec{\eta}^*;\vec{D}_n,\vec{D}_T,\vec{J},\vec{\delta}) . \tag{32}$$

Using Eq. (32) to eliminate $\vec{D}_E$ from the right-hand side of Eq. (27a), we obtain

$$f_{1i}(\eta)=f_{1i}(\eta,\vec{\eta}^*;\vec{D}_T,\vec{J},\vec{\delta}) \quad (\eta<\eta_i^*) . \tag{33}$$

The density gradient $\vec{D}_n$ disappears at this point as a consequence of $f_0(v)$ being Maxwellian.

The iterative solution for η_i^* and f_{1i} is based on Eq. (33). Given an iterate $\vec{\eta}^{*(n)}$, an iterate for the distribution function is obtained, for all η, as

$$f_{1i}^{(n+1)}(\eta)=f_{1i}(\eta,\vec{\eta}^{*(n)};\vec{D}_T,\vec{J},\vec{\delta}) . \tag{34}$$

The next iterate for η_i^*, namely $\eta_i^{*(n+1)}$, is set as the lowest value of η for which $|f_{1i}^{(n+1)}|>\delta_i f_0(\eta)$, and $f_{1i}^{(n+1)}(\eta)$ is then set to $\pm\delta_i f_0(\eta)$, depending on its sign, for $\eta>\eta_i^{*(n+1)}$. For the first iteration, $\eta_i^{*(0)}=\infty$. In practice, convergence to η_i^* is very fast—typically eight decimal places in four or five iterations.

Finally, the heat flux $\vec{q}$ is given from Eqs. (29) and (34) as

$$\vec{q}=\vec{q}(\vec{D}_T,\vec{J},\vec{\delta}) . \tag{35}$$

It is, of course, not necessary to use Eq. (31) for $\vec{q}$.

III. ILLUSTRATIVE RESULTS

The magnetic-field-induced modifications to the heat flux are presented in terms of the Hall parameter β_0 for thermal electrons $[\beta_0=\omega_c/\nu(v_0)=\omega_c\tau_{ei}(v_0)]$. The collision frequency for electrons with velocity v_0 is

$$\nu_0=\nu(v_0)=\frac{4\pi e^4(\ln\Lambda)(Z+1)n_e}{m_e^{1/2}(kT)^{3/2}} \tag{36}$$

giving

$$\beta_0=(5.10\times10^{21})BT^{3/2}/[n_e(Z+1)] , \tag{37}$$

where n_e is measured in cm^{-3}, T in keV, and B in MG. [If the average electron-ion momentum-transfer collision time τ_e given by Braginskii[2] had been used in place of $\tau_{ei}(v_0)$, β_0 would be higher by a factor of $(9\pi/2)^{1/2}=3.8$.] The effect of electron-electron angular-scattering collisions is approximated here by using $(Z+1)$ in place of Z, and the Coulomb logarithm is taken to be 10. For a typical plasma of $Z=4$ and $T=1$ keV, we obtain the useful relationship

$$\beta_0\simeq(10^{21}/n_e)B , \tag{38}$$

or, at the critical density for 1-μm Nd:glass laser radiation, $\beta_0\simeq B$.

Magnetic fields of the order of a megagauss have been observed in the coronas of laser-fusion targets through Faraday rotation,[32,33] while fields of the order of 0.1 MG are harder to detect and are often considered unimportant. However, even values of β_0 as low as 0.1 are sufficient to modify the heat flux significantly, because the β corresponding to the electrons which carry the bulk of the heat is at least an order of magnitude higher. (For moderate intensities $\gtrsim10^{14}$ W/cm^2, $T_e\sim2$ keV would also be a reasonable estimate for the coronal temperature, and $B=0.1$ MG would then imply $\beta_0=0.3$.) The region of greatest importance for magnetic-field-induced transport inhibition is found at electron densities just above critical, between the absorption and ablation regions; the magnetic field here is probably submegagauss, since the observed magnetic fields appear to maximize in the subcritical region (around 0.2 times critical).[33]

We will restrict ourselves to the geometry of Fig. 1, with $\vec{B}$ aligned in the z direction and $\vec{\nabla}T$ in the x direction. The current, heat flux, and electric field will all lie in the x-y plane, and we will refer to their components parallel to the x and y axes as longitudinal and transverse, respectively. We will write the normalized temperature gradient $D_{T,x}$ as the ratio of the (thermal) electron mean free path to the temperature scale length,

$$D_{T,x}=\lambda_T/L_x=-\lambda_T\hat{x}\cdot(\vec{\nabla}T)/T , \tag{39}$$

and we will restrict our attention to positive L_x (temperature gradient in the negative x direction). In all cases the heat flux $\vec{q}$ will be expressed relative to the free-streaming flux q_F [$=n_e m v_0^3$ from Eqs. (1) and (16)]; the ratio q/q_F may be thought of as the effective flux limiter. Unless otherwise stated, we will use $\delta_i=0.67$ to limit the f_{1i}.

A. Zero current ($J_x=J_y=0$)

In general two-dimensional situations the current is to be treated as a known quantity, as discussed above, and contributions to the heat flux will arise from the current as well as the temperature gradient. In order to isolate these two contributions, we will commence by considering the zero-current case; this also enables comparison to be made with the one-dimensional case where $\vec{J}=0$. The case where a finite current is specified will be considered in Sec. III C below.

The normalized heat fluxes q_x/q_F and q_y/q_F in the current-free case are given in Fig. 2 as functions of β_0 for $\lambda_T/L_x=0.1$. Here, as elsewhere, the solid lines denote q_x/q_F and the dashed lines q_y/q_F, for bounded $\vec{f}_1$. For the purposes of comparison, the thin lines denote the same quantities for unbounded $\vec{f}_1$ (Braginskii's results[2]). The Braginskii result for $\beta_0=0$ (off scale) is $q_x/q_F=0.57$.

We note that for $\beta_0 \gtrsim 0.2$, there is little difference between the bounded and unbounded results for either component. Therefore, even for a relatively small magnetic field, there is no need to invoke a flux limiter.

The asymptotic behavior of $\vec{q}$ at large β_0 is suggested from the form of Eq. (20) or Eq. (27): $q_x \sim \beta_0^{-2}$ and $q_y \sim \beta_0^{-1}$. From Fig. 2, there is a strong reduction of q_x/q_F (to 0.04) even for $\beta_0=0.2$. At higher values of β_0 the transverse component exceeds the longitudinal component. The transverse heat flux has a peak at very low β_0 (0.03 in the Braginskii case, 0.1 in the bounded case); it should maximize when the β of heat-carrying electrons [$\sim\beta_0(v/v_0)^3$] is of the order of π, on the basis of a simple physical picture where these electrons traverse half a Larmor orbit between collisions. Indeed, taking $\beta_0=0.1$ and $v/v_0 \simeq 3.2$ (see Fig. 3), we find $\beta \simeq 3$.

Figure 3 shows, plotted as functions of v/v_0, the x and y components of $\vec{f}_1/f_0$ (upper graphs) and $(v/v_0)^5\vec{f}_1$ (lower graphs), for various values of β_0. The same parameters of Fig. 2 apply: i.e., $\lambda_T/L_x=0.1$ and $\vec{J}=0$.

For $\beta_0=0$, a limit of f_{1x}/f_0 is needed to avoid large values of this ratio. No limit is needed for the higher values of β_0 shown, since the magnetic field introduces a maximum for f_{1x}/f_0 in the Braginskii theory and f_{1x} is well behaved throughout the whole velocity range. This maximum, and the minimum corresponding to the low-velocity return current, both decrease in amplitude as β_0 increases.

In our model it is always necessary to limit f_{1y}/f_0 at some velocity. For low values of β_0 (e.g., $\beta_0=0.2$), a strong transverse flow of high-velocity electrons is partially limited. As β_0 increases (e.g., $\beta_0=0.6$), the cutoff point moves to higher velocities and the electrons which carry the bulk of the energy flow are unaffected.

The areas under the lower graphs [of $(v/v_0)^5 f_{1i}(v)$] are proportional to the heat fluxes in the respective directions.

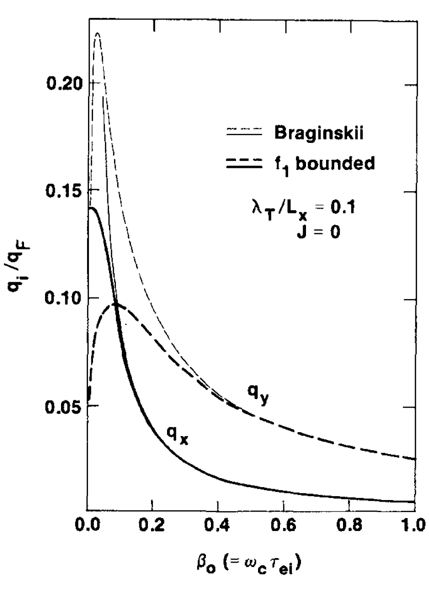

FIG. 2. Dependence of heat flux on β_0 ($=\omega_c\tau_{ei}$) for $\lambda_T/L_x=0.1$, $\vec{J}=0$, and cutoff parameters $\delta_x=\delta_y=0.67$. Solid curves q_x; dashed curves q_y. Light curves: Braginskii theory.

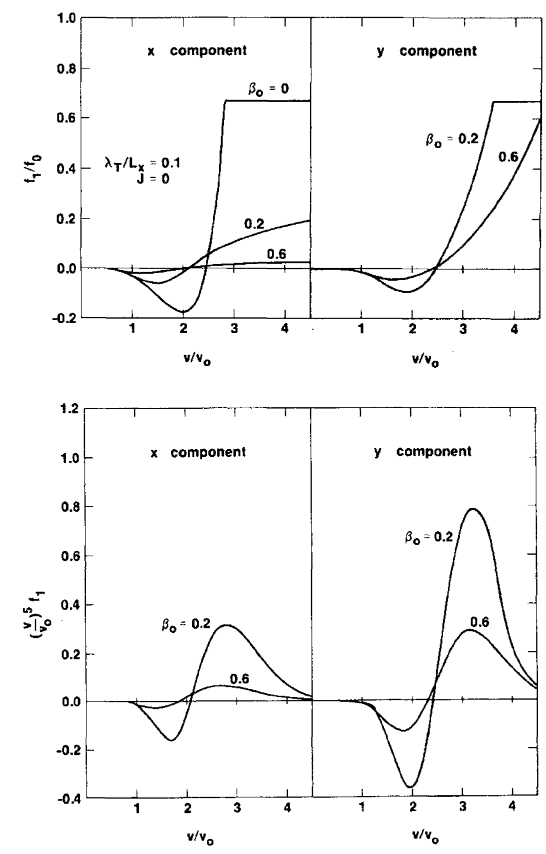

FIG. 3. Distribution functions corresponding to Fig. 2, for $\lambda_T/L_x=0.1$, $\vec{J}=0$, and cutoff parameters $\delta_x=\delta_y=0.67$. Upper plots f_{1i}/f_0, lower plots normalized heat flow $A_0^{-1}(v/v_0)^5 f_{1i}$, for $i=x,y$; $v_0=(kT/m)^{1/2}$.

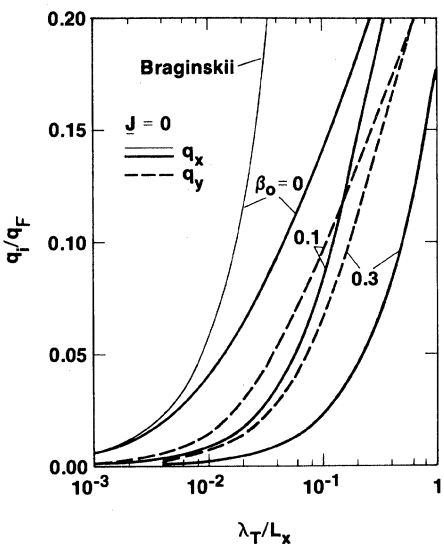

FIG. 4. Dependence of heat flux on λ_T/L_x for $\vec{J}=0$ and $\delta_x=\delta_y=0.67$, for various β_0. Solid curves q_x; dashed curves q_y. Light curve: Braginskii (or Spitzer-Härm) theory for $\beta_0=0$.

In each case, the maximum occurs at $v/v_0 \sim 3$. The integrated transverse flux is clearly greater than the longitudinal flux, and both decrease with increasing β_0.

In Fig. 4, the heat fluxes q_x/q_F and q_y/q_F are shown as functions of λ_T/L_x, for various values of β_0. The classical Braginskii[2] or Spitzer-Härm[1] result for $\beta_0=0$ is also included, and would of course be a straight line on a linear-linear plot. There is a region in the figure ($\lambda_T/L_x \sim 0.1$, $0.1 \lesssim \beta_0 \lesssim 0.3$) where both components of the heat flux are of the order of a few percent (3—10 %) of the free-streaming value. This value of λ_T/L_x is typical of what may occur in laser-produced plasmas at moderately high intensities, and it is arguable that the inhibition commonly observed can be explained by very modest values of magnetic field. It must, however, be noted that the effective flux limiter implied by Fig. 4 is a strong function of both λ_T/L_x and β_0, both of which quantities vary spatially; magnetic field inhibition might therefore lead to a greater diversity of experimentally inferred flux limiters than has to date been observed.

Figure 4 includes values of λ_T/L_x up to 1.0, but the regime of validity of this theory probably does not extend beyond $\lambda_T/L_x \simeq 0.1$,[14] at least in the magnetic-field-free case. Beyond this limit, the heat flux is dominated by nonlocal contributions from electrons whose mean free path exceeds the temperature-gradient scale length L_x. Conversely, for smaller λ_T/L_x or for larger β_0, the nonlocal contribution decreases.

B. Zero longitudinal current and zero transverse electric field ($J_x=E_y=0$)

Up to this point the electric field has not been treated as an independent parameter. There are situations, however, where one might expect the electric field in the direction of the current to be small. This is handled by a straightforward modification of the algebra used to obtain Eq. (32) from Eq. (30), since any two of the quantities $\{J_x,J_y,E_x,E_y\}$ may be determined from the others. Here we take the transverse electric field E_y to be zero, and calculate the self-consistent current J_y. (The density gradient is also assumed to be zero.)

Results are shown in Fig. 5 (case B), for $\lambda_T/L_x=0.1$, and with β_0 the independent variable. Curves for the previous zero-current case A, taken from Fig. 2, are given for comparison. The dotted line is the normalized current ($-J_y/J_0$ where $J_0=n_e e v_0$) calculated in case B. Comparison with Braginskii's results (the thin curves) again shows that above $\beta_0 \sim 0.2$ there is no need for a flux limiter.

The increase of transverse heat flow in case B is clearly due to the current contribution. The current maximizes at $\sim 3\%$ of the free-streaming current, at $\beta_0 \simeq 0.1$, and decreases to $\sim 1\%$ at $\beta_0 \simeq 1.0$. The longitudinal heat flow is slightly reduced, due to a small cross-coupling between the current in the y direction and the heat flow in the x direction. The curious features occurring on the graphs of J_y and q_y at $\beta_0 \simeq 0.04$ correspond to a minimum in the cutoff velocity η_y^* ($=3.0$).

C. Nonzero transverse current ($J_x=0$, $J_y \neq 0$)

Here the transverse current J_y is specified. We show in Fig. 6 the two components of the heat flux as functions of β_0, again for $\lambda_T/L_x=0.1$, for three values of J_y/J_0 ($0,-0.01,-0.03$). The curves for $J_y=0$ are taken from Fig. 2. As in Fig. 5, the transverse heat flux is increased due to the current contribution, the increase being almost independent of β_0. The longitudinal flux is again slightly decreased, and in one of the cases shown it changes sign.

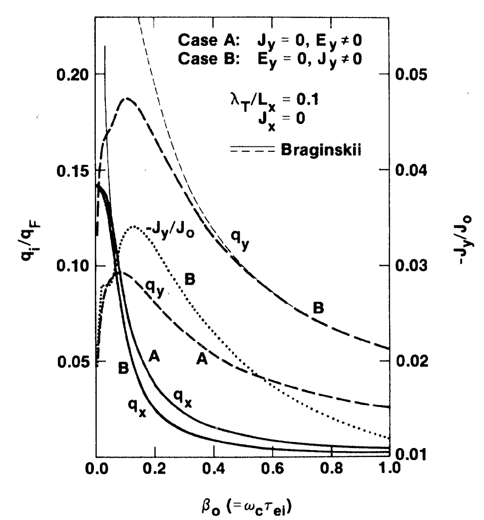

FIG. 5. Dependence of heat flux on β_0 for $\lambda_T/L_x=0.1$, $\delta_x=\delta_y=0.67$, and $J_x=E_y=0$ (case B). Curves for $\vec{J}=0$ are from Fig. 2 (case A). Dotted curve: ($-J_y/J_0$) for case B. Light curves: Braginskii theory for case B.

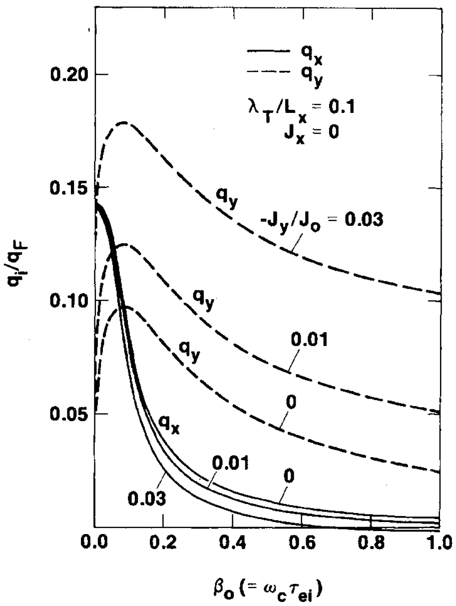

FIG. 6. Dependence of heat flux on β_0 for $\lambda_T/L_x=0.1$, $J_x=0$, and various J_y. Solid curves q_x, dashed curves q_y. $\delta_x=\delta_y=0.67$.

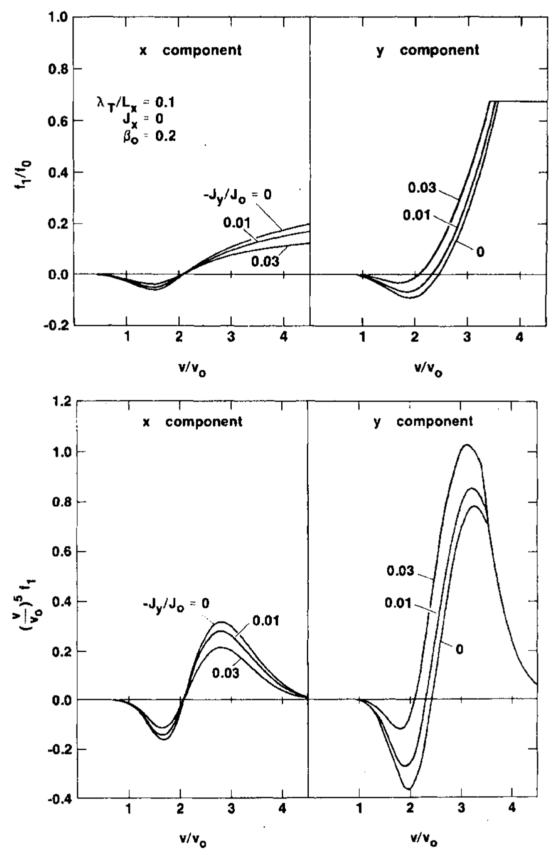

FIG. 7. Distribution functions corresponding to Fig. 6, for $\lambda_T/L_x=0.1$, $\beta_0=0.2$, $J_x=0$, and J_y specified. Upper plots f_{1i}/f_0, lower plots normalized heat flow $A_0^{-1}(v/v_0)^5 f_{1i}$, for $i=x,y$; $v_0=(kT/m)^{1/2}$. $\delta_x=\delta_y=0.67$.

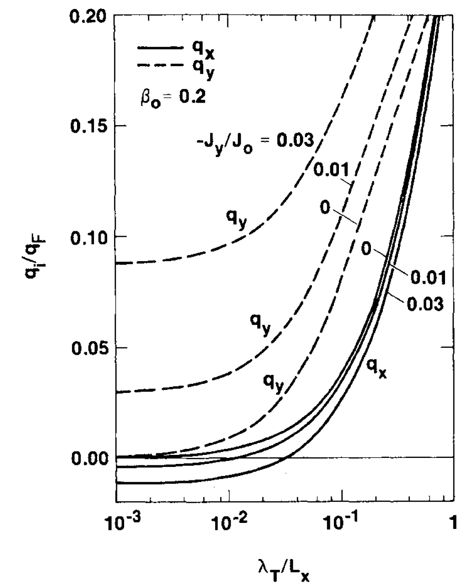

FIG. 8. Dependence of heat flux on λ_T/L_x for $\beta_0=0.2$, $J_x=0$, and J_y specified. Solid curves q_x, dashed curves q_y. $\delta_x=\delta_y=0.67$.

The effect on the distribution function of varying J_y is shown in Fig. 7, for the case $\beta_0=0.2$. For larger values of $-J_y/J_0$, the positive high-velocity contributions to f_{1y} and $v^5 f_{1y}$ increase, and the negative low-velocity contributions decrease, both leading to an increase in the heat flow in the y direction. In the x direction, both the positive and negative velocity contributions to f_{1x} decrease in magnitude, and the resultant q_x is almost unchanged.

Finally, in Fig. 8, we show q_i/q_F ($i=x,y$) as functions of λ_T/L_x, for $\beta_0=0.2$ and $-J_y/J_0=(0,0.01,0.03)$. Regardless of λ_T/L_x, q_y depends strongly on J_y, while q_x varies less with J_y.

D. Sensitivity of results to the cutoff parameter

It has been pointed out above that the quantities δ_i are arbitrary parameters of the model, and that, within probably a factor of 2, 0.67 is a reasonable value to use. We now examine the sensitivity of our results to the value of δ, taking $\delta_x=\delta_y=\delta$ here.

Curves obtained under the conditions of Fig. 2 for values of δ ranging from 0.33 to ∞ are plotted in Figs. 9 and 10, for q_x and q_y, respectively. The upper series of curves correspond to a steep temperature gradient ($\lambda_T/L_x=0.1$, as in Fig. 2), and the lower series to a gentler gradient ($\lambda_T/L_x=0.01$). The curves with $\delta=\infty$ correspond to the Braginskii limit. For larger values of δ or β_0 the distribution function is seen to be affected less by the cutoff, leading to results closer to the Braginskii limit.

In Fig. 9 it will be observed that the Braginskii results are obtained in all cases for $\beta_0 \gtrsim 0.2$, even for the smallest value of δ used (0.33). However, in the case $\lambda_T/L_x=0.1$ and in the limit of $\beta_0 \to 0$, the calculated heat flux depends

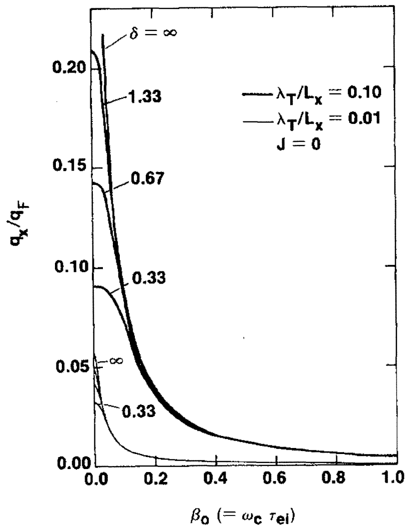

FIG. 9. Dependence of heat flux q_x on β_0 for the zero-current case. The various curves illustrate the sensitivity of the results to the cutoff parameter δ $(=\delta_x=\delta_y)$, for $\lambda_T/L_x=0.1$ (solid curves) and $\lambda_T/L_x=0.01$ (light curves).

significantly on the value of δ. This is an uncertainty inherent in the model of Shvarts *et al.*[14] Quantitatively, in the "reasonable" range of δ (0.33—1.33), where δ varies by a factor of 4, the heat flux at $\beta_0=0$ varies by a factor of 2.3 for $\lambda_T/L_x=0.1$ and a factor of 1.5 for $\lambda_T/L_x=0.01$. The variation in heat flux is less than the variation in δ probably because not all of the distribution function is affected by the cutoff. In the case of $\lambda_T/L_x=0.01$, the cutoff parameter affects the results only for very small magnetic fields.

Similar observations may be made from Fig. 10. While δ varies by a factor of 4 between 0.33 and 1.33, the peak value of q_y varies by a smaller factor: 2.0 for $\lambda_T/L_x=0.1$ and 1.2 for $\lambda_T/L_x=0.01$. The position of the peak varies by factors of 1.9 ($\lambda_T/L_x=0.1$) and 1.5 ($\lambda_T/L_x=0.01$). We may conclude from Figs. 9 and 10 that the results of our model are dependent on, but not strongly sensitive to, the parameter δ.

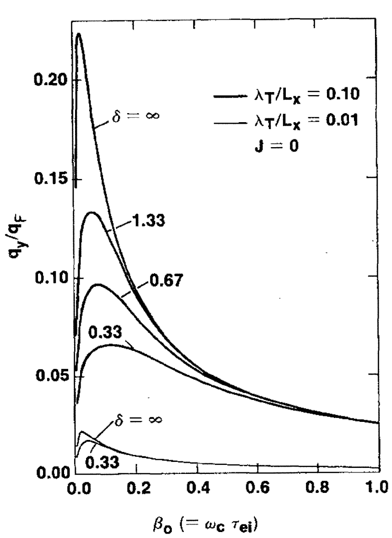

FIG. 10. As Fig. 9, but for q_y instead of q_x.

IV. SUMMARY

We have investigated the relationship between flux limiting and magnetic-field-induced transport inhibition, using a simple model which describes the transition between these two regimes in a physically reasonable way and yields some useful insight. We have found that even for steep temperature gradients ($\lambda_T/L_x\sim0.1$) a modest magnetic field is sufficient to reduce the anisotropic portion of the distribution function ($\vec{f}_1$) to a level where flux limiting is not required, simultaneously reducing the heat flux parallel to the temperature gradient to a few percent of the free-streaming value. Additionally, the transverse heat flux maximizes at small values of β_0 (~0.1), where β_0 is the Hall parameter $\omega_c\tau_{ei}$ for thermal electrons, because the Hall parameter for the more energetic heat-carrying electrons is then of the order of π. These results are not strongly dependent on the cutoff parameter used to limit $\vec{f}_1$. The model could clearly be extended to treat non-Maxwellian distribution functions (f_0), for example two-temperature distribution functions as were considered by Shkarofsky.[30]

While it would be unwise to advocate classical magnetic field inhibition as the primary explanation for the small flux limiter inferred from experiments, it is clear that these magnetic field effects deserve more careful consideration. For parameters corresponding to typical Nd:glass irradiation experiments, strong inhibition may occur for fields as small as a hundred kilogauss, an order of magnitude smaller than the megagauss fields which have been observed. At shorter wavelengths magnetic field effects are probably less (since the collision time τ_{ei} at the critical density scales as the square of the laser wavelength), but could still be significant.

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APPENDIX

We give here the algebraic details involved in obtaining Eq. (35) from Eq. (27). All of the integrals with respect to η may be written in terms of the following quantities:

$$\theta_i(m)=\int_0^{\eta_i^*}\frac{\eta^m e^{-\eta^2/2}}{1+\beta_0^2\eta^6\epsilon_i^2}d\eta\ , \tag{A1}$$

$$\theta_i^*(m)=\int_{\eta_i^*}^{\infty}\eta^m e^{-\eta^2/2}d\eta\ . \tag{A2}$$

Inserting Eq. (27) into Eq. (28) we find the current $\vec{J}(\vec{\eta}^*;\vec{D}_n,\vec{D}_T,\vec{D}_E,\vec{\delta})$:

$$\vec{J}=J_0\vec{D}_J\ , \tag{A3}$$

where

$$J_0=n_e e v_0\ , \tag{A4}$$

$$D_{J,i}=-A_1^{-1}\left(\tfrac{1}{9}[D_{n,i}\theta_i(7)-\beta_0\epsilon_i D_{n,i'}\theta_i(10)]+\tfrac{1}{18}\{D_{T,i}[\theta_i(9)-3\theta_i(7)]-D_{T,i'}\beta_0\epsilon_i[\theta_i(12)-3\theta_i(10)]\}\right.$$
$$\left.-\tfrac{1}{9}[D_{E,i}\theta_i(7)-\beta_0\epsilon_i D_{E,i'}\theta_i(10)]+\delta_i\theta_i^*(3)\right)\ , \tag{A5}$$

and

$$A_1=3\sqrt{2}\Gamma(\tfrac{3}{2})\ . \tag{A6}$$

For notational consistency we have introduced the normalized current $\vec{D}_J$. Note that all the cross terms involving i' include the factor $\beta_0\epsilon_i$, and that the arguments of θ_i in these terms are higher by 3 because of the η^3 dependence of β.

Treated as an equation for $\vec{D}_E$, Eq. (A5) has the form

$$a_iD_{E,i}-\epsilon_ib_iD_{E,i'}=c_i\ , \tag{A7}$$

which when inverted yields

$$D_{E,i}=(a_{i'}c_i+\epsilon_ib_ic_{i'})/(a_ia_{i'}+\epsilon_i^2b_ib_{i'})\ . \tag{A8}$$

The normalized electric field $\vec{D}_E(\vec{\eta}^*;\vec{D}_n,\vec{D}_T,\vec{D}_J,\vec{\delta})$ is therefore given as

$$D_{E,i}=D_{n,i}+[(\delta_{i,3}-\tfrac{3}{2})D_{T,i}-\epsilon_i\delta_{i,4}D_{T,i'}]$$
$$+(A_1\delta_{i,1}D_{J,i}+A_1\epsilon_i\delta_{i,2}D_{J,i'})$$
$$+[\delta_{i,1}\delta_i\theta_i^*(3)+\epsilon_i\delta_{i,2}\delta_{i'}\theta_{i'}^*(3)]\ , \tag{A9}$$

where

$$\delta_{i,1}=9\theta_{i'}(7)S_i\ ,$$
$$\delta_{i,2}=9\beta_0\theta_i(10)S_i\ ,$$
$$\delta_{i,3}=S_i[\theta_i(9)\theta_{i'}(7)+\epsilon_i^2\beta_0^2\theta_i(10)\theta_{i'}(12)]/2\ , \tag{A10}$$
$$\delta_{i,4}=S_i\beta_0[\theta_i(12)\theta_{i'}(7)-\theta_i(10)\theta_{i'}(9)]/2\ ,$$
$$S_i=[\theta_i(7)\theta_{i'}(7)+\epsilon_i^2\beta_0^2\theta_i(10)\theta_{i'}(10)]^{-1}\ .$$

Inserting Eq. (A9) into Eq. (27a) we obtain $f_{1i}(\eta,\vec{\eta}^*;\vec{D}_T,\vec{D}_J,\vec{\delta})$, for $\eta\le\eta_i^*$:

$$f_{1i}(\eta)=\frac{f_0(\eta)\eta^4}{9(1+\beta_0^2\eta^6\epsilon_i^2)}$$
$$\times[\tfrac{1}{2}(\eta^2-\gamma_3)D_{T,i}-\tfrac{1}{2}\epsilon_i(\beta_0\eta^5-\gamma_4)D_{T,i'}$$
$$-9\gamma_1A_1D_{J,i}-9\epsilon_i\gamma_2A_1D_{J,i'}$$
$$-9\gamma_1\delta_1\theta_i^*(3)-9\epsilon_i\gamma_2\delta_{i'}\theta_{i'}^*(3)]\ , \tag{A11}$$

where

$$\gamma_1=S_i[\theta_{i'}(7)+\epsilon_i^2\beta_0^2\eta^3\theta_{i'}(10)]\ ,$$
$$\gamma_2=S_i\beta_0[\theta_i(10)-\eta^3\theta_i(7)]\ ,$$
$$\gamma_3=\theta_i(9)\gamma_1+\epsilon_i^2\beta_0\theta_{i'}(12)\gamma_2\ , \tag{A12}$$
$$\gamma_4=-\theta_{i'}(9)\gamma_2+\beta_0\theta_i(12)\gamma_1\ .$$

Finally, inserting Eq. (A11) into Eq. (29), we obtain the heat flux $\vec{q}(\vec{D}_T,\vec{D}_J,\vec{\delta})$, normalized to the free-streaming flux q_F $(=mn_ev_0^3)$:

$$q_i/q_F=A_2\left(\{\tfrac{1}{18}[\theta_i(11)-\phi_3]D_{T,i}-\tfrac{1}{18}\epsilon_i[\theta_i(14)\beta_0-\phi_4]D_{T,i'}\}-(A_1\phi_1D_{J,i}+\epsilon_iA_1\phi_2D_{J,i'})\right.$$
$$\left.+\{[\theta_i^*(5)-\phi_1\theta_i^*(3)]\delta_i-\epsilon_i\phi_2\theta_{i'}^*(3)\delta_{i'}\}\right)\ , \tag{A13}$$

where

$$\phi_1=S_i[\theta_{i'}(7)\theta_i(9)+\epsilon_i^2\beta_0^2\theta_{i'}(10)\theta_i(12)]\ ,$$
$$\phi_2=S_i\beta_0[\theta_i(10)\theta_i(9)-\theta_i(12)\theta_i(7)]\ ,$$
$$\phi_3=\theta_i(9)\phi_1+\epsilon_i^2\beta_0\theta_{i'}(12)\phi_2\ ,$$
$$\phi_4=-\theta_{i'}(9)\phi_2+\beta_0\theta_i(12)\phi_1\ ,$$

and

$$A_2=1/[6\sqrt{2}\Gamma(\tfrac{3}{2})]\ . \tag{A14}$$

[1]L. Spitzer and R. Härm, Phys. Rev. **89**, 977 (1953).
[2]S. I. Braginskii, in *Reviews of Plasma Physics,* edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 1, p. 205.
[3]R. C. Malone, R. L. McCrory, and R. L. Morse, Phys. Rev. Lett. **34**, 721 (1975).
[4]M. D. Rosen, D. W. Phillion, V. C. Rupert, W. C. Mead, W. L. Kruer, J. J. Thomson, H. N. Kornblum, V. W. Slivinsky, G. J. Caporaso, M. J. Boyle, and K. G. Tirsell, Phys. Fluids **22**, 2020 (1979).
[5]W. C. Mead, E. M. Campbell, K. G. Estabrook, R. E. Turner, W. L. Kruer, P. H. Y. Lee, B. Pruett, V. C. Rupert, K. G. Tirsell, G. L. Stradling, F. Ze, C. E. Max, and M.D. Rosen, Phys. Rev. Lett. **47**, 1289 (1981).
[6]W. Seka, R. S. Craxton, J. Delettrez, L. Goldman, R. Keck, R. L. McCrory, D. Shvarts, J. M. Soures, and R. Boni, Opt. Commun. **40**, 437 (1982).
[7]R. S. Craxton and R. L. McCrory, Laboratory for Laser Energetics Report No. 108, 1980 (unpublished).
[8]C. Garban-Labaune, E. Fabre, C. E. Max, R. Fabbro, F. Amiranoff, J. Virmont, M. Weinfeld, and A. Michard, Phys. Rev. Lett. **48**, 1018 (1982).
[9]R. J. Bickerton, Nucl. Fusion **13**, 457 (1973).
[10]W. M. Manheimer, Phys. Fluids **20**, 265 (1977).
[11]D. R. Gray, J. D. Kilkenny, M. S. White, P. Blyth, and D. Hull, Phys. Rev. Lett. **39**, 1270 (1977).
[12]A. R. Bell, R. G. Evans, and D. J. Nicholas, Phys. Rev. Lett. **46**, 243 (1981).
[13]R. J. Mason, Phys. Rev. Lett. **47**, 652 (1981).
[14]D. Shvarts, J. Delettrez, R. L. McCrory, and C. P. Verdon, Phys. Rev. Lett. **47**, 247 (1981).
[15]J. P. Matte and J. Virmont, Phys. Rev. Lett. **49**, 1936 (1982).
[16]J. R. Albritton, Phys. Rev. Lett. **50**, 2078 (1983).
[17]W. L. Kruer, Comments Plasma Phys. **5**, 69 (1979).
[18]J. A. Stamper, K. Papadopoulos, R. N. Sudan, S. O. Dean, E. A. McLean, and J. M. Dawson, Phys. Rev. Lett. **26**, 1012 (1971).
[19]J. B. Chase, J. M. LeBlanc, and J. R. Wilson, Phys. Fluids **16**, 1142 (1973).
[20]D. G. Colombant, K. G. Whitney, D. A. Tidman, N. K. Winsor, and J. Davis, Phys. Fluids **18**, 1687 (1975).
[21]R. S. Craxton and M. G. Haines, Phys. Rev. Lett. **35**, 1336 (1975).
[22]J. P. Christiansen and N. K. Winsor, J. Comput. Phys. **35**, 291 (1980).
[23]R. S. Craxton and R. L. McCrory, Laboratory for Laser Energetics Report No. 99, 1980 (unpublished).
[24]G. J. Pert, J. Comput. Phys. **43**, 111 (1981).
[25]D. R. Gray and J. D. Kilkenny, Plasma Phys. **22**, 81 (1980).
[26]D. W. Forslund and J. U. Brackbill, Phys. Rev. Lett. **48**, 1614 (1982).
[27]M. A. Yates, D. B. van Hulsteyn, H. Rutkowski, G. Kyrala, and J. U. Brackbill, Phys. Rev. Lett. **49**, 1702 (1982).
[28]J. U. Brackbill and S. R. Goldman, Commun. Pure Appl. Math. **36**, 415 (1983).
[29]I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynski, *The Particle Kinetics of Plasmas* (Addison-Wesley, Reading, Mass., 1966).
[30]I. P. Shkarofsky, Phys. Rev. Lett. **42**, 1342 (1979).
[31]A. B. Langdon, Phys. Rev. Lett. **44**, 575 (1980).
[32]J. A. Stamper and B. H. Ripin, Phys. Rev. Lett. **34**, 138 (1975); J. A. Stamper, E. A. McLean, and B. H. Ripin, *ibid.* **40**, 1177 (1978).
[33]A. Raven, O. Willi, and P. T. Rumsby, Phys. Rev. Lett. **41**, 554 (1978).