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Processes involved in pion capture in hydrogen-containing molecules

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A systematic analysis is presented of the possible elementary processes which determine the fate of negative pions stopped in hydrogen-containing samples with use of a phenomenological description in comparison with the available experimental information on pion capture by hydrogen in gas mixtures.

Pion capture by bound hydrogen has proved to be a unique information source on the electronic state of hydrogen in various substances. The W probability, that a negative pion stopped in a sample with bound hydrogen will be absorbed by a proton, is closely related to the properties of the chemical bond of hydrogen.^{1,2} The method used in the pionic hydrogen experiments is described in Refs. 1–4; the results are summarized in reviews.^{1–3} The experimental data have been analyzed using the model of large mesic molecules proposed by Ponomarev.^{1,2}

In order to clarify the roles of the various elementary processes, considered at that time to be possibly involved in the formation and decay of pionic hydrogen atoms in condensed substances, Petrukhin and co-workers carried out a systematic experimental investigation of pion capture by protons in gas mixtures $H_2 + {}_Z X$ (Refs. 5 and 6), $C_m H_n + {}_Z X$ (Refs. 7 and 8), and ${}^3He + {}_Z X$ (Ref. 9), where ${}_Z X$ is an atom with atomic number Z . The results led to the conclusion that the formation and the decay of pionic hydrogen atoms are dominated by collisional processes.

Recently, the role of new, noncollisional elementary processes has been considered^{10–12} for describing the pion capture by bound hydrogen: (i) the direct formation of pionic hydrogen via radiative atomic capture of pions in bound hydrogen atoms (thereby avoiding the molecular state); (ii) the direct nuclear capture of the pion by the proton from the molecular state ${}_Z X H \pi^-$ without forming a $p\pi^-$ atom; and (iii) an “inner” transfer of the pion from the $p\pi^-$ atom to atom ${}_Z X$ via tunneling through the bond ${}_Z X-H$. In the following we shall attempt a systematic analysis of the gas-mixture data using a general, model-free, phenomenological description of pion capture with all possible elementary processes, including (i)–(iii).

The phenomenological model generally used for the analysis of the experimental data^{3–9} describes the pion capture by a proton in the consecutive steps of Coulomb capture of the pion in a mesic molecular orbit by probability P , the transition from molecular to $p\pi^-$ atomic state by probability Q , and finally, the nuclear capture of the pion by a proton with probability R . Thus, the probability that a stopped pion will be absorbed by a proton is²

$$W = PQR \quad (1)$$

In a general case, including the processes (i)–(iii) Eq. (1) should be modified to

$$W = (PQ + D)R + PE \quad (2)$$

where E is the probability of process (ii), the nuclear capture of the pion by the proton from the molecular orbit, and D is the probability of process (i), the radiative transition of the pion from the continuum to a $p\pi^-$ atomic bound state, i.e., that of the radiative atomic capture of the pion in a hydrogen atom (see Fig. 1).

The only principal assumption used in our considerations is the independence of the various channels depicted in Fig. 1; we will not use any specific feature of the model of large mesic molecules^{1–3} or of the molecular model suggested in Refs. 10–12.

In the following the rates of the collisional processes are assumed to be proportional to the concentrations of the species involved. This assumption has theoretical^{13–15} and experimental^{9,16} support. The atomic capture probabilities depend on the properties of the corresponding atoms; we shall assume a dependence on atomic number Z without at-

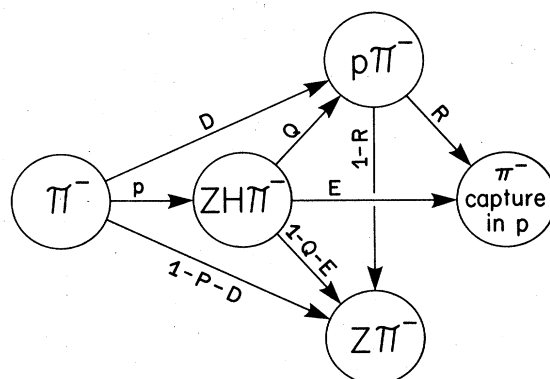


FIG. 1. Fate of a slow π^- meson in a compound $({}_Z X)_m H_n$. The pion is captured in a molecular state ${}_Z X H \pi^-$ or in atomic orbits of $p\pi^-$ and ${}_Z X \pi^-$ with probabilities D , P , and $(1-P-D)$, respectively. The pionic molecule may decay via pion capture in a proton or by pion transition to atomic orbits $p\pi^-$ and ${}_Z X \pi^-$. The pion can be transferred from $p\pi^-$ to ${}_Z X$, or absorbed by a proton.

tempting a detailed description. By definition, $W(\text{H}_2) = 1$.

In a $({}_Z\text{X})_m\text{H}_n$ compound, probabilities P and D can be expressed using the α_m , α_r , and α_z cross sections of pion capture in the ${}_Z\text{XH}\pi^-$, $p\pi^-$, and ${}_Z\text{X}\pi^-$ states and the N_{H} and N_z densities of H and ${}_Z\text{X}$ atoms in the sample:

$$P = \frac{\alpha_m N_{\text{H}}}{(\alpha_m + \alpha_r) N_{\text{H}} + \alpha_z N_z} = \frac{A_m}{1 + A_z C_z} \quad (3)$$

and

$$D = \frac{\alpha_r N_{\text{H}}}{(\alpha_m + \alpha_r) N_{\text{H}} + \alpha_z N_z} = \frac{A_r}{1 + A_z C_z} \quad (4)$$

where $A_m = \alpha_m / (\alpha_m + \alpha_r)$, $A_r = \alpha_r / (\alpha_m + \alpha_r)$, $A_z = \alpha_z / (\alpha_m + \alpha_r)$ are the cross sections relative to $(\alpha_m + \alpha_r)$ and $C_z = N_z / N_{\text{H}} = m/n$ is the atomic concentration of ${}_Z\text{X}$ in the sample.

The E probability of nuclear capture from ${}_Z\text{XH}\pi^-$ in a proton and the Q transition probability are independent from the C_z concentration but may depend on Z due to the ${}_Z\text{X}-\text{H}$ chemical bond.

The R probability of nuclear capture by proton from a $p\pi^-$ atom can be expressed in terms of the cross sections β_x of the following competing reactions: pion capture in proton via radiative process within the $p\pi^-$ atom (β_r); pion capture in a proton in collisions of $p\pi^-$ with hydrogen (β_{H}) or with ${}_Z\text{X}$ (β_z); pion transfer from $p\pi^-$ to ${}_Z\text{X}$ via tunneling through bond ${}_Z\text{X}-\text{H}$ (λ_t); pion transfer in collisions $p\pi^- + {}_Z\text{X}$ (λ_z):

$$R = \frac{\beta_{\text{H}} N_{\text{H}} + \beta_z N_z + \beta_r}{\beta_{\text{H}} N_{\text{H}} + \beta_z N_z + \beta_r + \lambda_t + \lambda_z N_z} \\ = \frac{\beta_{\text{H}} + \beta_z C_z + \beta_r / N_{\text{H}}}{\beta_{\text{H}} + (\beta_z + \lambda_z) C_z + (\beta_r + \lambda_t) / N_{\text{H}}} \quad (5)$$

We see from Eq. (5) that including noncollisional processes with β_r and λ_t in a system of collisional ones leads to a virtual density dependence of the R probability. However, there is extensive experimental evidence that the W probability of pion capture in protons does not depend on the N_{H} density of hydrogen but depends sensitively on the C_z concentration of the other constituent.¹⁻³ For example, the W probability measured in HD and $\text{H}_2 + \text{D}_2$ (Ref. 17) or in C_2H_6 (Ref. 18) gases did not change within the experimental error when the density was increased by two orders of magnitude. Thus, the cross section of both the radiative nuclear capture in the $p\pi^-$ atom (β_r) and the "inner" transfer of pions (λ_t) must be negligibly small as compared to the other cross sections in (5). The smallness of β_r is in agreement with the theoretical picture² of mesoatomic processes. As to the "internal" transfer, it should proceed within the pionic molecule¹⁰⁻¹² before the actual formation of the $p\pi^-$ atom. According to Ponomarev,^{1,2} however, the $p\pi^-$ system is neutral and relatively small, and it leaves the molecule with a high kinetic energy after its formation.

Neglecting β_r and λ_t , and using relative cross sections $B_z = \beta_z / \beta_{\text{H}}$ and $\Lambda_z = \lambda_z / \beta_{\text{H}}$, Eq. (5) can be written as

$$R = \frac{1 + B_z C_z}{1 + (B_z + \Lambda_z) C_z} \quad (6)$$

The study of pion capture by ${}^3\text{He}$ in mixtures of ${}^3\text{He}$ with noble gases⁹ is the simplest possible check of our model as in that case we have neither transfer ($\lambda_z = 0$, i.e., $R = 1$), nor chemical bond effects ($Q = 1$, $E = 0$). Thus, from

(2)-(4)

$$W({}^3\text{He} + {}_Z\text{X}) = P + D = \frac{1}{1 + A_z C_z} \quad (7)$$

in agreement with the experimental data.⁹ [In (7) P denotes the probability of Auger capture of π^- in ${}^3\text{He}$.]

In the mixtures of H_2 with noble gases^{5,6} there is no chemical bond between ${}_Z\text{X}$ and H, i.e., $E + Q = 1$. Using Eqs. (2)-(6), the probability of pion capture in proton is

$$W(\text{H}_2 + {}_Z\text{X}) \\ = \frac{1}{1 + A_z C_z} \left[(1 - A_m E) \frac{1 + B_z C_z}{1 + (B_z + \Lambda_z) C_z} + A_m E \right] \quad (8)$$

The experimental $W(\text{H}_2 + {}_Z\text{X})$ values measured for all noble gases ${}_Z\text{X}$ in a wide concentration range $0 \leq C_z \leq 2$ have been fitted with a satisfactory agreement by the simple function⁶

$$W_{\text{exp}}(\text{H}_2 + {}_Z\text{X}) = \frac{1}{1 + \bar{A}_z C_z} \frac{1}{1 + \bar{\Lambda}_z C_z} \quad (9)$$

where the \bar{A}_z and $\bar{\Lambda}_z$ parameters have been found to be proportional to $(Z^{1/3} - 1)$:

$$\bar{A}_z = (7.1 \pm 0.1)(Z^{1/3} - 1) \quad (10a)$$

$$\bar{\Lambda}_z = \bar{A}_z C_z^{1/3} \quad (10b)$$

\bar{A}_z in (9) is obviously an estimation of A_z in (8). As to the square brackets in (8), its reciprocal value can be transformed to the form

$$[\dots]^{-1} = \frac{1 + (B_z + \Lambda_z) C_z}{1 + (B_z + A_m E \Lambda_z) C_z} \quad (11)$$

This expression cannot have the quasilinear $(1 + \bar{\Lambda}_z C_z)$ form in the region $0 \leq C_z \leq 2$ required by (9) unless both B_z and $A_m E$ are negligibly small. We conclude that in $\text{H}_2 + {}_Z\text{X}$ mixtures in the collisions $p\pi^- - {}_Z\text{X}$ the collisional transfer process dominates over the pion capture in proton, and no pion capture is observed from the $\text{H}_2\pi^-$ molecular state. The first statement implies $B_z = 0$, the second one $A_m = 0$ or $E = 0$. Which ever is small of A_m and E , it should be small in other hydrogen-containing chemical systems as well because they are related mainly to the hydrogen atom itself. However, strong chemical effects have been observed in the W pion capture probability in various substances,¹⁻⁴ e.g.,

$$\frac{W(\text{N}_2 + 2\text{H}_2)}{W(\text{N}_2\text{H}_4)} = 30 \quad (12)$$

which suggests $A_m \gg A_r$ as the cross section of radiative atomic capture in hydrogen should not depend on the properties of the $Z-\text{H}$ bond. Thus, $E \approx 0$, the probability of nuclear capture of the pion by the proton from the molecular orbit is negligible. This is consistent with theoretical estimations and other data: the radiative nuclear capture by a proton has a small probability even in the free $p\pi^-$ atom¹⁻³ [cf. $\beta_r \approx 0$ in Eq. (5)] and the transition from molecular to atomic state is assumed to be very fast.^{1,2,19} Our conclusion on $A_r \ll A_m$ is also supported by theory: in the involved energy region of the meson the cross section of the Auger capture is much larger than that of radiative capture.^{13-16,19-22}

The experimental data obtained on the mixtures of

methane (ethane, ethylene) with noble gases⁸ gives support to all considerations made by us earlier, as the observed effect of the noble gas added to C_mH_n could be approximated as

$$\frac{W_{\text{exp}}(C_mH_n + Z)}{W_{\text{exp}}(C_mH_n)} = \frac{1 + \bar{A}_C(m/n)}{1 + \bar{A}_C(m/n) + \bar{A}_Z C_Z} \times \frac{1 + \bar{\Lambda}_C(m/n)}{1 + \bar{\Lambda}_C(m/n) + \bar{\Lambda}_Z C_Z} \quad (13)$$

using (10a) for \bar{A}_C and \bar{A}_Z and an estimation from earlier data for $\bar{\Lambda}_C$, with only $\bar{\Lambda}_Z$ as a parameter. In the case of a competitive "internal" transfer of pions from hydrogen to carbon atoms the data would have shown an additional, density-dependent term in the denominator of Eq. (13), as in Eq. (5).

The results of the analysis of gas-mixture studies can be extrapolated to condensed molecular systems as all models work with isolated molecules (for the energies involved in mesic processes are much higher than the Van der Waals forces) and, furthermore, no condensation effect was observed in studies of pion capture in gaseous and liquid hydrocarbons,¹⁸ solid and liquid ammonia,²³ or solid and melted aquacomplexes.³ We did not include in our present

analysis the numerous data on pion capture in composite molecular (inorganic and organic) systems.¹⁻³ It is extremely difficult to use those data for a conclusive study on the basic processes for even in the simplest case (pion capture in binary hydrides $({}_Z X)_m H_n$, when comparing probabilities in various compounds, one encounters many simultaneous, mutually interrelated processes (see Fig. 1), all of which depend on the molecular composition, on atomic number Z , and on the properties of the ${}_Z X-H$ and ${}_Z X-{}_Z X$ chemical bonds¹⁻³ in a not yet understood manner. For example, we observed⁹ a much weaker pion transfer to noble-gas atoms ${}_Z X$ from $p\pi^-$ atoms formed in $CH_4 + {}_Z X$ than from those formed in $H_2 + {}_Z X$.

Thus, we have shown that, within our phenomenological, model-free description, the assumption of a considerable role of processes (i)-(iii) is inconsistent with the available experimental information. The smallness of the corresponding probabilities is in agreement with the recent theoretical picture^{13-16, 19-22} of the atomic capture of mesons.

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