Nonlinear generation of magnetostatic fluctuations by drift waves

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It is shown that nonlinear-mode coupling of drift waves can generate magnetostatic fluctuations in an inhomogeneous magnetized plasma. The growth rate of the parametric instability is estimated for typical tokamak parameters.

A number of experimental observations[1-3] have reported the presence of the drift waves and the magnetic fluctuations in tokamak discharges. Recently, Kaw and Chen[4] estimated the level of enhanced magnetostatic fluctuations which are created by nonlinear coupling of a given level of drift-Alfvén wave turbulence. It is found that even very weak electric fields ($e\phi/T_e \sim 1\%$) of the drift waves can drive up very high levels ($|b_\perp/B_0| \sim 10^{-4}$) of the magnetostatic mode.

In this Brief Report, we present a self-consistent investigation of nonlinear coupling of drift waves with magnetostatic modes. Specifically, we show that the latter are driven unstable because they couple back to the drift waves. This parametric interaction was considered by Shukla, Rahman, Yu, and Varma[5] who ignored the density-gradient effects on the magnetostatic modes,[6] and retained only the electron-polarization drift nonlinearity. Here, we incorporate the important $\vec{E}\times\vec{B}_0$ nonlinearity of the drift waves which is very essential for the enhanced-magnetostatic fluctuations in the presence of the density gradient.[4]

We consider a low-β plasma and treat the drift waves within the electrostatic approximation. Since the parallel (to $\vec{B}_0$) phase velocity of the drift waves lies between the electron and ion thermal velocities, the inertialess electrons establish an equilibrium along the external magnetic field $B_0\hat{z}$. Thus, the electron density perturbation is given by

$$\frac{n_e}{n_0} = \Phi + \partial_z^{-1}\frac{\hat{z}\times\vec{\nabla}\Phi\cdot\vec{\nabla}}{B_0}(1-\lambda^2\nabla^2)A + \frac{e\lambda^2}{cT_e}\nabla^2 A\,\partial_z^{-1}(\partial_t + \vec{v}_D\cdot\vec{\nabla})\Phi \;, \tag{1}$$

where n_0 is the average plasma density, $\nabla\equiv\nabla_\perp$, $\Phi = e\phi/T_e$, ϕ is the ambipolar potential of the drift wave, T_e is the electron temperature, $\lambda = c/\omega_{pe}$ is the collisionless electron skin depth, $\vec{v}_D = D\vec{\nabla}\ln n_0\times\hat{z}$, $D = cT_e/eB_0$, and A is the parallel (to $\vec{B}_0$) component of the vector potential associated with the magnetostatic mode. The second term on the right-hand side of (1) comes from the $\vec{v}^{\,d}_{e\perp}\cdot\vec{\nabla}v^m_{ez}$ and $(-e/m_e c)\vec{v}^{\,d}_{e\perp}\times\vec{b}_\perp\cdot\hat{z}$ terms, whereas the last one originates from the $v^m_{ez}\partial_z v^d_{ez}$ term. For $\partial_t \ll \Omega_i$ ($\Omega_i = eB_0/m_i c$ is the ion gyrofrequency), the perpendicular and parallel (to $\vec{B}_0$) components of the electron velocities in the drift wave fields are, respectively,

$$\vec{v}^{\,d}_{e\perp} = \frac{c}{B_0}\hat{z}\times\vec{\nabla}\phi \equiv D\hat{z}\times\vec{\nabla}\Phi \;, \tag{2}$$

$$v^d_{ez} = -\partial_z^{-1}(\partial_t + \vec{v}_D\cdot\vec{\nabla})\Phi \;, \tag{3}$$

where in (3) we have used $n_e/n_0 \approx \Phi$. The parallel velocity v^m_{ez} of the electrons in the magnetostatic fields ($E^m_z = -c^{-1}\partial_t A$, $\vec{b}_\perp = \vec{\nabla}A\times\hat{z}$) is given by Ampère's law

$$v^m_{ez} = (c/4\pi n_0 e)\nabla^2 A \;, \tag{4}$$

where the ion current is neglected because $j^m_{ez}/j^m_{iz} \approx m_i/m_e$.

Nonlinear interaction of the drift waves with the magnetostatic modes in a nonuniform plasma is governed by[5]

$$\begin{aligned}\partial_t[(1-\rho_s^2\nabla^2)\partial_t^2 + \vec{v}_D\cdot\vec{\nabla}\partial_t - c_s^2\partial_z^2]\Phi &= \frac{c_s^2}{B_0}[\partial_t^2(\vec{\nabla}A\times\hat{z}\cdot\vec{\nabla}\partial_t^{-1}\partial_z\Phi) - \Omega_i^{-1}\partial_t^2\vec{\nabla}\cdot(A\,\partial_z\vec{\nabla}\Phi] \\ &+ \frac{\Omega_i}{B_0}\partial_z[\partial_t^2(\Phi A) + c_s^2 A\,\partial_z^2\Phi] \\ &- \frac{1}{B_0}\partial_t^3[\partial_z^{-1}\hat{z}\times\vec{\nabla}\Phi\cdot\vec{\nabla}(1-\lambda^2\nabla^2)A \\ &\quad + \frac{\Omega_i}{c_s^2}\lambda^2\nabla^2 A\,\partial_z^{-1}(\partial_t + \vec{v}_D\cdot\vec{\nabla})\Phi] \;,\end{aligned} \tag{5}$$

where $\rho_s = c_s/\Omega_i$, and c_s is the ion-acoustic velocity. Note that Eq. (5) is derived by means of the ion continuity and momentum equations under the quasineutrality ($n_e = n_i$) and the drift ($\partial_t \ll \Omega_i$) approximations. The ions are assumed to be cold, and it is noted that the magnetostatic modes have zero-density perturbations.[6]

The beating of the drift waves gives rise to the nonlinear two-dimensional magnetic fluctuations which are described by the parallel electron momentum equation[4]

$$(\partial_t + \nu)v^m_{ez} + \frac{e}{m_e}E^m_z - \frac{e}{m_e c}\vec{v}_D\cdot\vec{\nabla}A = -\langle\vec{v}^{\,d}_{e\perp}\cdot\vec{\nabla}v^d_{ez}\rangle \;, \tag{6}$$

where ν ($\nu > \Delta k_z v_{te} \sim v_{te}/qR$; v_{te} is the electron thermal velocity, q is the safety factor, and R is the major radius of the torus) is the electron-ion collision frequency. The right-hand side of (6) represents nonlinear driving term associated with the drift waves.

Combining (2), (3), (4), and (6), one readily finds

$$(\partial_t+\nu)\lambda^2\nabla^2 A - (\partial_t + \vec{v}_D\cdot\vec{\nabla})A = \frac{cT_e}{e\Omega_e}\hat{z}\times\vec{\nabla}\Phi\cdot\vec{\nabla}[\partial_z^{-1}(\partial_t+\vec{v}_D\cdot\vec{\nabla})\Phi] \ , \quad (7)$$

where $\Omega_e = eB_0/m_e c$ is the electron gyrofrequency. Equations (5) and (7) are the coupled system describing nonlinear interaction of the drift waves with the magnetostatic modes. Note that we have neglected temperature gradients, which can lead to linear instability[7] of the magnetic drift waves.

In order to investigate the parametric instability, we now carry out a normal-mode analysis of Eqs. (5) and (7). We shall use the usual local approximation[6] and take a constant density gradient. In a shearless situation, the local approximation dictates that the wavelengths of the perturbations are much shorter than the density gradient scale length and the wave numbers of the modes do not depend on the eigenfunction. We divide Φ into three parts: the pump Φ_0 and the two sidebands $\Phi_\pm$,

$$\Phi = [\Phi_0\exp(i\vec{k}_0\cdot\vec{x} - i\omega_0 t) + \text{c.c.}] + \Phi_+\exp(i\vec{k}_+\cdot\vec{x} - i\omega_+ t) + \Phi_-\exp(i\vec{k}_-\cdot\vec{x} - i\omega_- t) \ , \quad (8)$$

where $\vec{k}_\pm = \vec{k}\pm\vec{k}_0$ and $\omega_\pm = \omega\pm\omega_0$ are the wave vector and the frequency of the drift wave satellites $\Phi_\pm$. Assuming $A = \tilde{A}\exp(-i\omega t + i\vec{k}\cdot\vec{x})$ and making use of (8) into (5) and (7), one obtains after Fourier transformation

$$\epsilon_\pm\Phi_\pm = \alpha_\pm\tilde{A}\begin{pmatrix}\Phi_0\\ \Phi_0^*\end{pmatrix} \ , \quad (9)$$

$$[(1+k_\perp^2\lambda^2)\omega - \vec{k}\cdot\vec{v}_D + i\nu k_\perp^2\lambda^2]\tilde{A} = \frac{icT_e}{k_{z0}e\Omega_e}(\hat{z}\times\vec{k}_{\perp 0}\cdot\vec{k}_\perp)\times(\omega-\vec{k}\cdot\vec{v}_D)(\Phi_0\Phi_- + \Phi_0^*\Phi_+) \ , \quad (10)$$

where

$$\epsilon_\pm = i\omega_\pm[(1+\rho_s^2 k_{\perp\pm}^2)\omega_\pm^2 + \omega_\pm^*\omega_\pm - c_s^2 k_{z0}^2] \ ,$$

$$\alpha_\pm = \mp\frac{c_s^2 k_{z0}}{B_0}\left(\omega_0\vec{k}_{\perp 0}\cdot\vec{k}\times\hat{z} \pm i\frac{\omega_0^2}{\Omega_i}\vec{k}_{\perp 0}\cdot\vec{k}_{\perp\pm} - F_1 + iF_2 + i\frac{\Omega_i}{c_s^2}(\omega_0^2 + c_s^2 k_{z0}^2)\right) \ ,$$

with

$$k_{z+} = |k_{z-}| = k_{z0}, \quad \omega_\pm^* = \vec{k}_{\perp\pm}\cdot\vec{v}_D, \quad F_1 = \omega_0^3(1+k_\perp^2\lambda^2)\vec{k}_{\perp 0}\cdot\vec{k}\times\hat{z}/k_{z0}^2 c_s^2 \ ,$$

and

$$F_2 = \omega_0^3\Omega_i c^2 k_\perp^2(\omega_0 - \vec{k}_0\cdot\vec{v}_D)/\omega_{pe}^2 k_{z0}^2 c_s^4 \ .$$

We now consider nonresonant forced magnetic fluctuations[4] satisfying $\omega \ll \vec{k}\cdot\vec{v}_D$, and $|\vec{k}\cdot\vec{v}_D/\omega + i\nu| \gg k_\perp^2\lambda^2$. Hence, in these limits, Eqs. (9) and (10) yield

$$1 = \frac{icT_e}{e\Omega_e}\frac{\hat{z}\times\vec{k}_{\perp 0}\cdot\vec{k}_\perp}{k_{z0}}\left(\frac{\alpha_-}{\epsilon_-} + \frac{\alpha_+}{\epsilon_+}\right)|\Phi_0|^2 \ . \quad (11)$$

For the three-wave decay interaction, we can take $\epsilon_- = 0$ and $\epsilon_+ \neq 0$. Thus, for $\omega \ll \omega_0$ and $k_\perp^2 \ll k_{\perp 0}^2$, Eq. (11) takes the form

$$\omega - \delta\omega = P(S+iU)/Q \ , \quad (12)$$

where

$$\delta\omega = \omega_0 - \omega_{k0}^*, \quad \omega_{k0}^* = k_{y0}v_D(1+k_{\perp 0}^2\rho_s^2)^{-1} + k_{z0}^2 c_s^2/k_{y0}v_D \ ,$$

$$P = \hat{z}\times\vec{k}_{\perp 0}\cdot\vec{k}_\perp\rho_s^2/\Omega_e, \quad S = [\omega_0\vec{k}_{\perp 0}\cdot\vec{k}\times\hat{z} - F_1]\Omega_i c_s^2 \ ,$$

$$Q = \omega_0^2(1 + c_s^2 k_{z0}^2/\omega_0 k_{y0}|v_D|) \ ,$$

$$U = [\omega_0^2 G + c_s^2 k_{z0}^2]\Omega_i^2 + c_s^2\Omega_i F_2, \quad G = 1 + k_{\perp 0}^2\rho_s^2 \ .$$

The growth rate is obtained by letting $\omega = \omega_r + i\gamma$ in Eq. (12). The result is

$$\gamma \approx \frac{m_e|[G + (k_{z0}^2 c_s^2/\omega_0^2) + F_2 c_s^2/\omega_0^3\Omega_i](\hat{z}\times\vec{k}_{\perp 0}\cdot\vec{k}_\perp)|\Omega_i\rho_s^2|\Phi_0|^2}{m_i(1 + k_{z0}^2 c_s^2/\omega_0 k_{y0}|v_D|)} \ . \quad (13)$$

As an illustration, we apply our result to cold-plasma toroidal devices. Near the plasma edge, we assume that the density gradient has a constant value which is equivalent to an exponential density profile. Furthermore, we assume that the electron-temperature gradient is much smaller than the density gradient which is supported by the experimental evidence.[2,3] For

$$k_{\perp 0}\sim\rho_s^{-1}, \quad k_\perp\sim 0.1k_{\perp 0}, \quad k_{z0}\sim(qR)^{-1}, \quad T_e\sim 100\ \text{eV}, \quad n_0 = 10^{13}\ \text{cm}^{-3}, \quad B_0 = 40\ \text{kG} \ ,$$

$$R\sim 4\times 10^3\rho_s\sim 100\ \text{cm}, \quad |L_n|\sim 100\rho_s\ (L_n^{-1} = n_0^{-1}dn_0/dx) \ ,$$

and $\Phi_0 = 1\%$, the growth time (γ^{-1}) turns out to be roughly 100 msec for a hydrogen plasma.

In summary, we have given a self-consistent treatment for the nonlinear generation of nonresonant magnetostatic fluctuations by electrostatic drift waves. For typical tokamak discharge parameters, the growth time is found to be of the order of 100 ms. Since the latter is longer than the plasma confinement time (generally ~ 5 ms), it appears that the present instability would not presumably arise in the present day cold plasma devices which have low-level of drift wave fluctuations. However, since the growth rate depends quadratically on $|\Phi_0|$, a shorter growth time is expected for high-level drift-wave fluctuations (e.g., for $\Phi_0 \sim 10\%$, the growth time is reduced by two-orders of magnitude), which cannot be ruled out, in reality.

In closing, we mention that the present investigation does not account for the magnetic perturbation of the drift waves; its inclusion merely gives rise to the coupling of the drift wave with the Alfvén wave (instead of the sound branch). Furthermore, the coupling coefficients would contain some additional nonlinear terms which are generally neglected for low-β $[\equiv 8\pi n_0 T_e/B_0^2 << L_n(m_e/m_i)^{1/2}/qR]$ plasmas.[4] Finally, it is worth pointing out that in the presence of the magnetic shear the modes can be localized in the neighborhood of the $\vec{k}\cdot\vec{B}$ surfaces. Accordingly, our local treatment becomes invalid because the calculations of the parametric theory would involve an eigenvalue problem. Equations (5) and (7) should then take a form of the second-order differential equations in x, and our calculations have to be revised in a systematic manner.[8] This, however, would lead us far beyond the scope of this Brief Report.

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