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New route to optical turbulence in detuned lasers with a compound cavity

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A successive subharmonic modulation cascade of self-sustained relaxation oscillations in an inverse order leading to optical turbulence in detuned lasers with compound-cavity configurations is predicted. A brief experimental result which supports the predicted subharmonic modulation phenomenon is shown. The suppression effect of optical turbulence by external light injection, which is important for practical applications, is demonstrated theoretically.

A light-injection-induced dissipation structure, exhibiting period-doubling cascades to optical turbulence, in gain-linked dispersive optical systems has been predicted and demonstrated experimentally for detuned lasers with injected signals, where the detuned lasers have an anomalous dispersion effect at the lasing wavelength.[1,2] The basic idea has been derived from the dependence of the active region refractive index on population inversion density.

The present Rapid Communication discusses a new route to turbulence in detuned-laser systems having compound-cavity configurations and the suppression of optical turbulence by external light injection. Figure 1(a) conceptually illustrates a compound-cavity model, where the laser resonator consists of two coupled cavities (No. 1, No. 2) that include the detuned-laser gain medium with anomalous dispersion. Figure 1(b) shows the relationship between the two cavity modes.

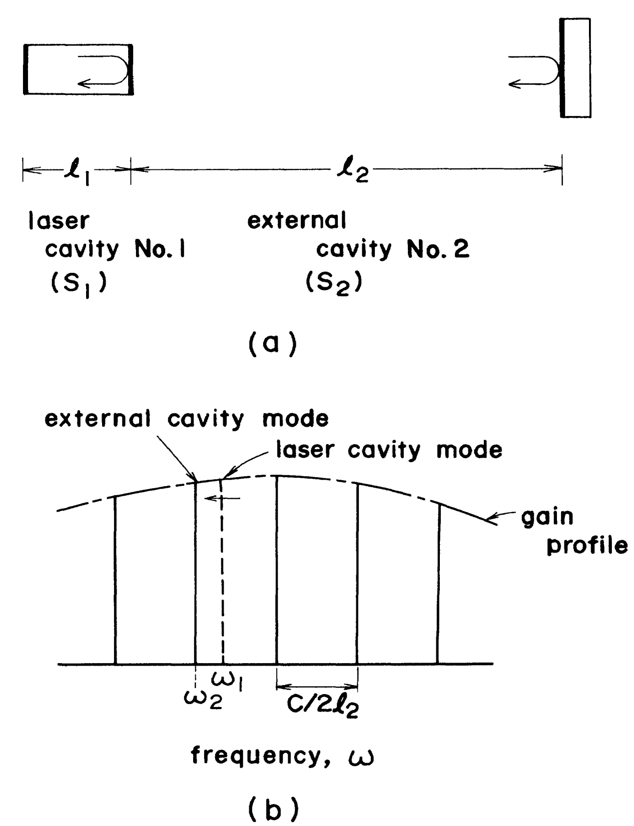

FIG. 1. (a) Conceptual model of a detuned laser with a compound-cavity configuration. (b) Relationship between the laser cavity mode S_1 and the external cavity mode S_2.

Taking into account the anomalous dispersion effect in the detuned-laser medium, and assuming the No. 2 cavity length is much longer than that of the No. 1 cavity (i.e., $l_2 >> l_1$), the system can be regarded as being governed by the following coupled generalized van der Pol equations:[3]

$$\frac{dN}{dt} = P - N/\tau - G(N)[S_1 + S_2] \quad , \tag{1}$$

$$\frac{dE_1}{dt} = [G(N) - 1/\tau_{p1}](E_1/2) + qE_2 \cos\psi \quad , \tag{2}$$

$$\frac{d\psi}{dt} = \omega_1(N) - \omega_2 - q(E_2/E_1)\sin\psi \quad , \tag{3}$$

$$\frac{dS_2}{dt} = [G(N) - 1/\tau_{p2}]S_2 + \beta N/\tau, \quad S_1 \equiv E_1^2, \quad S_2 \equiv E_2^2 \quad . \tag{4}$$

Here, P is the pump rate, G the gain function, N the population inversion density, E_1, E_2 the electric fields for the No. 1 and No. 2 cavity modes, τ_{p1}, τ_{p2} the photon lifetimes, τ the upper state lifetime, q the coupling coefficient, β is the spontaneous-emission coefficient, and ψ the external phase angle. For brevity, only a single axial mode is assumed to exist in the gain bandwidth for the No. 1 cavity mode.

The dependence of G and ω_1 on N due to the anomalous dispersion effect can be approximated using Taylor's series as

$$G(N) = 1/N_{th}\tau_{p1} + \left|\frac{\partial G}{\partial N}\right|(N - N_{th}) \quad , \tag{5}$$

$$\omega_1(N, \Omega) = \omega_1(N_{th}) + \left|\frac{\partial \omega}{\partial N}\right|(N - N_{th}) - (n_e/n - 1)[\Omega - \omega_1(N_{th})] \quad , \tag{6}$$

where N_{th} is the threshold population inversion density, Ω the oscillation frequency, and $n_e = n + \Omega(\partial n/\partial\Omega)$ the effective refractive index. Here, we introduce the parameter $R = -2(\partial\omega/\partial N)/(\partial G/\partial N)$ and employ the dimensionless parameters given in Ref. 2.

Numerical simulations were carried out using Eqs. (1)–(4) based on the Runge-Kutta-Gill algorithm for various normalized detunings defined as $\Delta \equiv (n_e/n)[\omega_2 - \omega_1(N_{th})]\tau_{p1}$, and assuming $w = P/P_{th} = 1.5$, $q\sqrt{\tau_{p1}/P_{th}} = 0.01$, $R = -2$, $\beta = 10^{-5}$, and $\tau_{p1} = 0.2\tau_{p2}$. It has been found that the system undergoes successive inverse subharmonic modulations of sustained relaxation oscillations leading to intermittent turbulence with increasing the detuning. The simulated waveforms are shown in Figs. 2(a)–2(e). For Fig. 2(a), two cavity-resonant frequencies were locked together and axial modes, except the mode at frequency ω_2, were completely suppressed. Consequently, the system produced a stable output. When Δ was increased toward the negative side (negative detuning), self-sustained pulsations appeared at frequency f_{R0}, whose envelope was modulated at frequency $\sim\Delta/2\pi\tau_{p1}(=f_{R0}/m, m$: integer), and m decreased with successive detunings [see Figs. 2(b)–2(c)] until self-sustained relaxation oscillations finally developed at f_{R0} [see Fig. 2(d)]. f_{R0} is given by $(1/2\pi)\sqrt{(w-1)/\tau_{p1}\tau}$, where

$$\tau_{p1}=\tau_{p0}\{1+q\tau_d(n/n_e)[R\sin(\delta\Omega\tau_d)+\cos(\delta\Omega\tau_d)]\} \ ,$$

$\tau_d=2l_2/c$ (c: velocity of light); $\delta\Omega=\Omega-\omega_1$ and τ_{p0} is the photon lifetime for laser without an external mirror, assuming $q\tau_d>>1$.[5] Increasing Δ further led to intermittent turbulence in the present system as shown in Fig. 2(e).

Such a new route to turbulence, that is a successive subharmonic modulation cascade of self-sustained relaxation oscillations in an inverse order, is significantly different from the period-doubling bifurcation route for the injection locking system described in Ref. 2. In the present system, two coupled resonators share the same nonlinear gain-linked dispersive laser medium (symmetrical coupling scheme), while in the injection locking system, the self-excited laser is optically coupled to the external injection light through an optical isolator where the coupling is asymmetrical.

First, the self-sustained relaxation oscillations will be explained. The field intensity increase due to light injection reduces the population inversion density and increases the refractive index because $R<0$ in this case. The refractive index increase results in a decrease of $\omega_1(N)$. As a result,

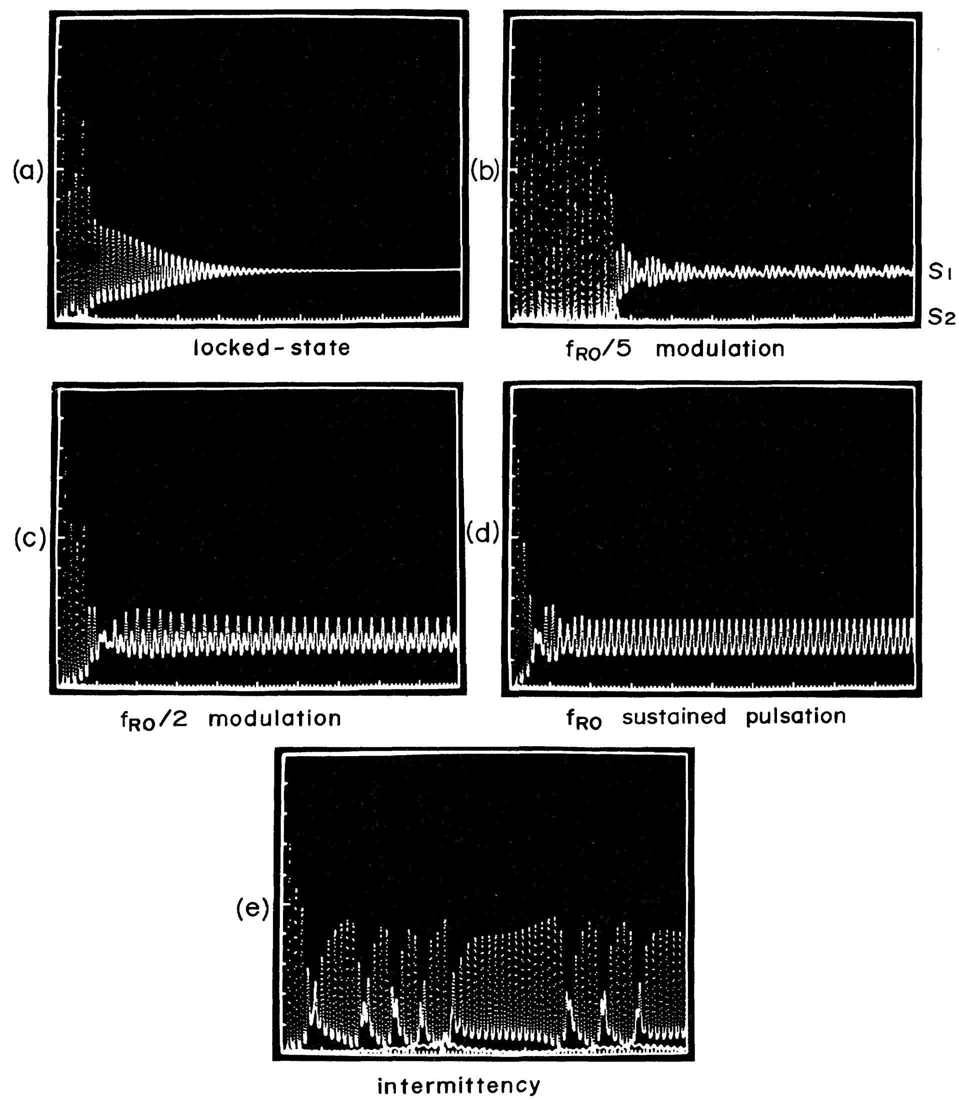

FIG. 2. Simulated responses of a detuned laser for various effective detunings, $\Delta=(n_e/n)(\omega_2-\omega_1)\tau_{p1}$. Vertical axis (normalized photon number): $S_1/\tau_{p1}P_{th}$, $S_2/\tau_{p1}P_{th}$, 0.3/division. Horizontal axis (normalized time): T/τ_{p1}, 200/division, $n_e/n=1.25$. (a) $\Delta=-0.001\times1.25$ (stable), (b) -0.0042×1.25 ($f_{R0}/5$ modulation), (c) -0.011×1.25 ($f_{R0}/2$ modulation), (d) -0.015×1.25 (f_{R0} modulation), (e) -0.018×1.25 (intermittency).

the locking condition is improved because of the decrease in effective detuning. Therefore, a field increase above its stationary value is induced, giving rise to an enhanced relaxation oscillation. Such self-sustained relaxation oscillations corresponding to Fig. 2(d) have been theoretically predicted and demonstrated experimentally in a laser diode coupled to an external reflector placed 1–2 cm apart from the diode.[4,5] When the detuning $\Delta/2\pi\tau_{p1}$ coincides with f_{R0}/m, such self-sustained relaxation oscillations are thought to be modulated at frequency f_{R0}/m. A similar subharmonic modulation of the relaxation oscillation was reported for the laser loss and/or phase modulated at f_{R0}/m.[6,7] It should be pointed out that the unlocked field component is quite small, though not zero in the above regime [Figs. 2(b)–2(d)]. This means that these states may be considered as "quasilocked states," although their output is not stationary.

Intermittent turbulence occurs when the detuning reaches a point just outside the "lock-in" range. At this point, the unlocked component builds up and competes with the locked one, resulting in rather complex output waveforms, as shown in Fig. 2(e). Moreover, as $\Delta/2\pi\tau_{p1}$ is close to f_{R0}, the self-sustained relaxations are modulated at frequencies slightly different from f_{R0}, resulting in intermittency.

The numerical results indicate that when the detuning increases further than in Fig. 2(e), the system exhibits chaotic self-pulsations leading to self-modulations at frequency $\Delta/2\pi\tau_{p1}$, which is similar to the result for the injection locking system.[2]

In Fig. 3 the phase-space trajectories of complex-field and intensity spectra are shown for f_{R0}/m modulation ($m=3$) [Figs. 3(a) and 3(b)] and intermittent turbulence [Figs. 3(c) and 3(d)]. It is interesting to note that the complex field rotates by 2π in the phase space at frequency f_{R0}/m in the case of the subharmonic modulation regime. In the intermittency regime, the intensity spectrum shows f^{-2} dependence, which is similar to the intermittent turbulence which can be observed in the driven Josephson junction.[8]

Experiments were carried out using a GaAlAs/GaAs semiconductor laser coupled to a bandwidth-limited external cavity. The intracavity etalon was used to maintain the single-axial-mode oscillation of laser cavity mode S_1. Figure 4 shows the observed noise spectrum for $w=1.59$, $l_1=300$ μm, and $l_2=42$ cm. The sharp noise peak near f_{R0} is due to the beat noise between external cavity modes S_2 and its frequency coincides with $1/\tau_d$. Since $f_{R0}\cong 1/\tau_d$, the condition of $E(t)-E(t-\tau_d) << 1$ is satisfied in this experiment.[3] The observed spectrum clearly corresponds to Fig. 3(a) and supports the existence of subharmonic modulation phenomenon predicted above. Details will be published

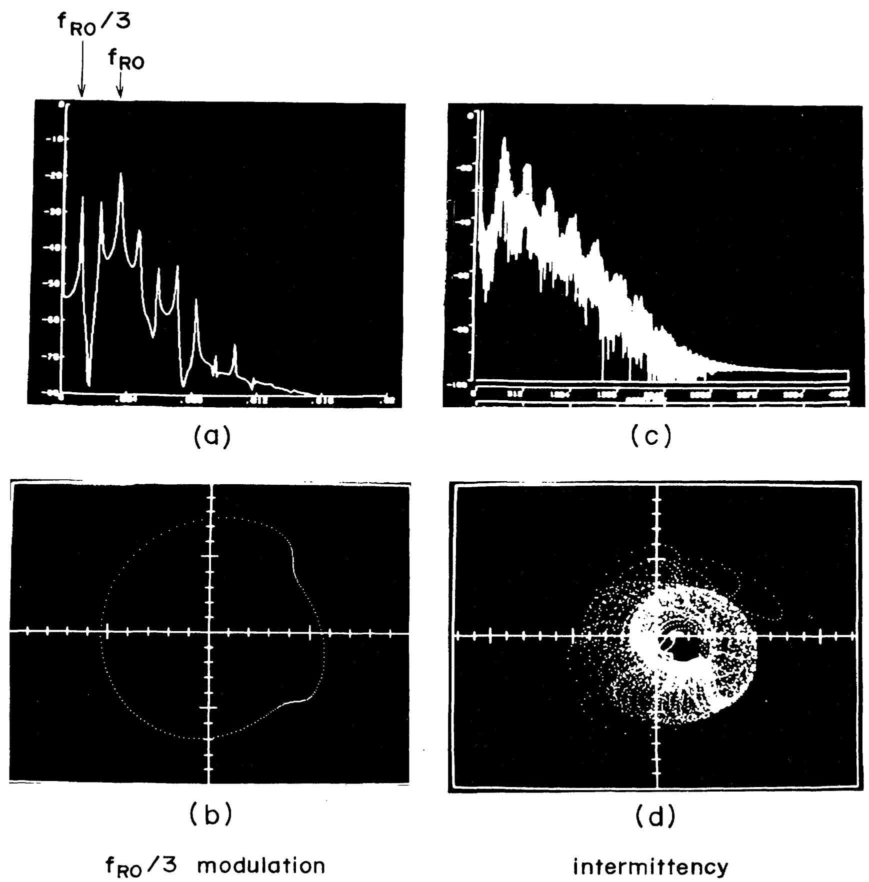

FIG. 3. Phase-space field trajectories $[\mathrm{Im}(E_1/\sqrt{\tau_{p1}P_{th}})-\mathrm{Re}(E_1/\sqrt{\tau_{p1}P_{th}})]$ and noise spectra for subharmonic modulation ($m=3$) and for intermittent turbulence. The adopted parameter values are the same as those for Fig. 2 (see the text). (a), (b): $f_{R0}/3$ modulation ($\Delta=-0.0067\times 1.25$); (c),(d): intermittency ($\Delta=-0.018\times 1.25$). (a) 0.1/division; (c) 0.2/division origin [0,0]; (b) vertical axis: 10 dB/division, horizontal axis: $f\tau_{p1}$, 0.004/division; (d) vertical axis: 10 dB/division, horizontal axis: 0.006 25/division.

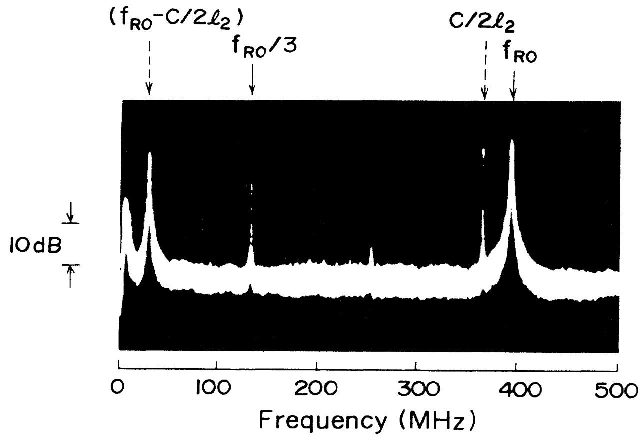

FIG. 4. Observed noise spectrum in a GaAlAs/GaAs semiconductor laser coupled to an external cavity, demonstrating the subharmonic modulation phenomenon. $w=1.59$, $l_1=300$ μm, and $l_2=42$ cm.

elsewhere.[9]

In the following, we will show that such autonomous turbulence brought about by the nonlinear interaction, which gives rise to serious problems in practical applications, can be completely suppressed by the external light injection. Coherent light having the frequency $\omega_1(=\omega_2)$ was assumed to be injected into the compound-cavity detuned-laser system operating in the intermittent regime. The governing equations can be expressed by adding an external injection term to Eqs. (1)–(4) (see Ref. 2). Figure 5 shows a calculated example that demonstrates the suppression effect of different external light-injection levels, assuming the same parameters as Fig. 2(e). Light injection is assumed to take place at $T=t/\tau_{p1}=10^4$. In the case of Fig. 5(a) (injected field amplitude $\epsilon_i \equiv q_i E_i \sqrt{\tau_{p1}/P_{\rm th}} = 5\times 10^{-3}$), injection synchronization is poor and some turbulence remains. However, the turbulence is found to be completely suppressed for $\epsilon_i \geqslant 7.5\times 10^{-3}$, as shown in Fig. 5(b).

In summary, self-sustained pulsations with inverse subharmonic amplitude modulations leading to chaos in detuned-laser systems with compound-cavity configurations have been predicted. A possible candidate for observing such optical turbulence is a semiconductor laser coupled to an external reflecting mirror, which has a large anomalous dispersion effect, i.e., a large R value, where the conditions $l_1 \ll l_2$ and $\tau_d \ll 1/f_{R0}$ are easily satisfied. In fact, the noise spectrum suggesting the predicted subharmonic $(f_{R0}/3)$ modulation of self-sustained relaxation oscillations has been observed in a GaAs laser diode coupled to an external cavity. The feasibility of suppressing such instabilities by injecting external light has also been demonstrated theoretically.

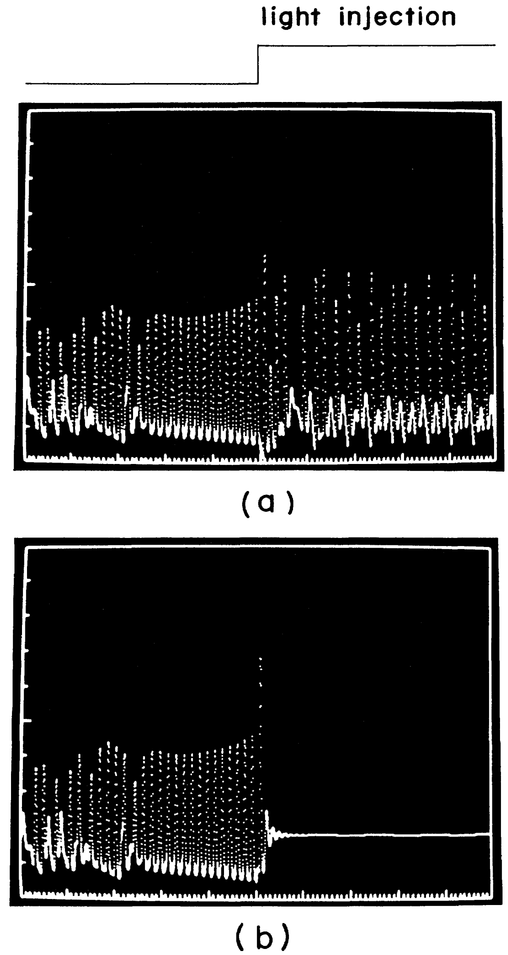

FIG. 5. Effect of light injection on suppression of optical turbulence. $(S_1+S_2)/\tau_{p1}P_{\rm th}$ vs t/τ_{p1}. The adopted parameter values are the same as those for Fig. 2(e) (see the text). (a) Insufficient suppression $\epsilon_i=5\times 10^{-3}$, (b) complete suppression $\epsilon_i=7.5\times 10^{-3}$.

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$$E_1(t)-E_1(t-\tau_d)=\tau_d[dE_1(t)/dt] \ll 1 \ ,$$

where $\tau_d=2l_2/c$ is the delay time. If $\tau_d \gg 1/f_{R0}=O(\tau)(f_{R0}$: relaxation oscillation frequency which will be discussed later), a delayed feedback effect should be added to Eqs. (2) and (3). In such an experimental situation, i.e., $\tau_d \gg \tau$, self-sustained pulsations at frequency $\sim 1/\tau_d$ leading to irregular emission properties have been observed experimentally in a diode laser coupled to an external grating. See P. Glas, R. Muller, and A. Klehr, Opt. Commun. **47**, 297 (1983).
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