# Solitary surface-charge propagation along a plasma boundary

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The nonlinear properties of long-wavelength ion acoustic surface modes in a semi-infinite plasma are studied. It is shown that finite-amplitude surface-charge layers can propagate along the plasma boundary with a velocity that exceeds the ion sound speed by a small term which is proportional to the squared inverse width of the solitary layer.

Although the nonlinear behavior of ion acoustic volume waves has been analyzed in numerous works[1] for uniform as well as slightly nonuniform plasmas it seems to us that a corresponding investigation for ion acoustic surface waves does not yet exist. In the present Brief Report we shall thus extend previous linear theory[2] for such long-wavelength surface modes in order to present a comparatively very simple formalism taking into account perturbations which are so large that the nonlinear terms play a significant role.

To simplify the calculations we consider a semi-infinite ($z>0$) two-component plasma which is bounded by a dielectric medium at $z \leqslant 0$. The electron and ion densities are denoted by $n_{e,i}=n_0+\delta n_{e,i}$, where $n_0$ is constant for $z>0$ and zero for $z \leqslant 0$, and the perturbations $\delta n_{e,i}$ are much smaller than $n_0$. Considering the equation of momentum for the electrons in the long-wavelength limit, we then adopt the Boltzmann distribution[1]

$$\frac{\delta n_e}{n_0}=\exp\tilde{\phi}-1 \approx \tilde{\phi}+\frac{\tilde{\phi}^2}{2} \quad , \tag{1}$$

where $\tilde{\phi}=\tilde{\phi}(x,z,t)=q\Phi/T_e$, $q$ is the magnitude of the electron charge, $T_e$ the electron temperature, and $\Phi$ the potential of the electrostatic surface wave we intend to study. Introducing the dimensionless coordinates[1] $\tau=\omega_{pi}t$, $\tilde{\xi}=x/\lambda_D$, and $\eta=z/\lambda_D$, where $\omega_{pi}$ is the ion plasma frequency and $\lambda_D$ the electron Debye length, we then note that, for $z>0$, the Poisson equation with the symbols $N\equiv\delta n_i/(n_0)_{z>0}$ and $\nabla^2=\partial^2/\partial\tilde{\xi}^2+\partial^2/\partial\eta^2$, can be written as

$$\nabla^2\tilde{\phi}=\tilde{\phi}+\frac{\tilde{\phi}^2}{2}-N \quad , \tag{2}$$

whereas for $z<0$ the potential satisfies the Laplace equation

$$\nabla^2\tilde{\phi}=0 \quad . \tag{3}$$

Similar equations have previously been studied for a cold semi-infinite plasma interacting with a high-frequency external field.[3] The boundary condition, which follows from an integration of Eqs. (2) and (3) across the boundary, is

$$S=\left.\frac{\partial\tilde{\phi}}{\partial\eta}\right|_{\eta=0-} \quad , \tag{4}$$

where $S=\int_{0-}^{0+}N\,d\eta$. In deriving (4) we have neglected $\partial\tilde{\phi}/\partial\eta|_{0+}$ in comparison to $\partial\tilde{\phi}/\partial\eta|_{0-}$. The validity of this approximation, which is well known within linear theory,[2] can subsequently be verified also for the present slightly nonlinear equations.

In addition to the equations above we shall also for $z>0$ make use of the equation of continuity for the ions

$$\frac{\partial N}{\partial\tau}+\vec{\nabla}\cdot(\vec{u}+N\vec{u})=0 \tag{5}$$

as well as the ion momentum equation

$$\frac{\partial\vec{u}}{\partial\tau}+\vec{u}\cdot\vec{\nabla}\vec{u}=-\vec{\nabla}\tilde{\phi} \quad . \tag{6}$$

Here, we have introduced the notation $\vec{u}=\vec{v}_i/c_s$, where $\vec{v}_i$ is the ion fluid velocity and $c_s$ the ion sound velocity. In the nonlinear terms in (5) and (6) we will then neglect $u_z$ in comparison to $u_x$ as corrections of third and higher order in the wave amplitude are to be omitted.[2] Furthermore, considering the long-wavelength limit we appropriate Eq. (2) by

$$\tilde{\phi}=N-\frac{N^2}{2}+\nabla^2N \quad . \tag{7}$$

We shall now look for solutions of Eqs. (5)–(7) which are functions of the new coordinate $\xi=\tilde{\xi}-\tau$ as well as of $\eta$ and an additional weak $\tau$ dependence. This means that it is convenient to introduce the normalized potential $\phi(\xi,\eta,\tau)\equiv\tilde{\phi}(\tilde{\xi},\eta,\tau)$ together with corresponding symbols for $N$ and $u_x$. Following previously developed techniques[1,3] to reduce similar equations, i.e., eliminating $\vec{u}$ from the system of Eqs. (5)–(7), and neglecting third-order terms, we then obtain

$$\frac{2\partial N}{\partial\tau}+\frac{\partial\nabla^2N}{\partial\xi}+\frac{\partial(N\phi)}{\partial\xi}=0 \quad , \tag{8}$$

which, when integrated from $\eta=0-$ to $\eta=0+$, yields

$$2\frac{\partial S}{\partial\tau}+\frac{\partial^3S}{\partial\xi^3}+\frac{\partial[S\phi(\eta=0)]}{\partial\xi}=0 \quad . \tag{9}$$

We have accordingly now at our disposal a system of two coupled equations, namely, (3) and (9) which are connected by (4). Although it is impossible for us to present a general analysis of these equations, we are able to propose the solitary wave solution

$$\phi=\frac{4\phi_0}{\pi^2}\int_0^\infty dt\frac{t\cos(\theta t)}{\sinh t}\exp\left[\pi^{-1}\left(\frac{\phi_0}{3}\right)^{1/2}\eta t\right] \tag{10}$$

 

and

$$S = \left(\frac{16}{\pi^3}\right)\phi_0\left(\frac{\phi_0}{3}\right)^{1/2} S_g(\theta) \;, \tag{11}$$

where the constant $\phi_0$ represents the amplitude of the potential at the surface, $\theta = \pi^{-1}(\phi_0/3)^{1/2}(\tilde{\xi} - \tau - \phi_0\tau/6)$, and the function $S_g$ is

$$S_g(\theta) = \frac{1}{4}\int_0^\infty dt \frac{t^2 \cos(\theta t)}{\sinh t} \;. \tag{12}$$

It is easy to check that (10) and (11) actually satisfy the system (3), (4), and (9). We have thus found an expression for the nonlinear surface-charge density, which has the form of a solitary triple layer that is described by the function $S_g(\theta)$ [note that $S_g(\theta)$ is positive for $|\theta| < \theta_0$ and negative for $|\theta| > \theta_0$, where $\theta_0 \approx 0.6$, that $S_g$ has a maximum at $\theta = 0$ and minima at $\theta \approx \pm 1.0$ and that $S_g(\theta) \approx -1/4\theta^2$ for $\theta \rightarrow \pm\infty$]. The argument $\theta$, which already was defined above, can here be rewritten in the alternative form

$$\theta = \frac{x - (1 + 2\lambda_D^2/x_0^2)c_s t}{x_0} \;, \tag{13}$$

where $x_0[= \lambda_D(12/\phi_0)^{1/2} >> \lambda_D]$ is the characteristic width of the solitary layer. The maximum surface-charge density is determined by the initial perturbation.

In conclusion, we have, thus, by means of a simple choice of model, supplemented the nonlinear theory for ion acoustic solitary volume waves in uniform or slightly nonuniform plasmas with a corresponding theory for surface waves on a sharply bounded plasma. An exact solution of (3), (4), and (9) has been found. An additional experiment[4] intending to study ion acoustic solitary surface waves would be of much interest.

[1]For example, see R. C. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972), Chap. 2.

[2]Yu. A. Romanov, Fiz. Tverd. Tela **7**, 970 (1965) [Sov. Phys. Solid State **7**, 782 (1965)].

[3]O. M. Gradov and L. Stenflo, IEEE Trans. Plasma Sci. **PS-11**, 249 (1983).

[4]For example, see M. Moisan, A. Shivarova, and A. W. Trivelpiece, Plasma Phys. **24**, 1331 (1982); K. E. Lonngren, *ibid.* **25**, 943 (1983).