We have argued in the subjunctive sense for the adoption of the term "unstable pseudohole gas" when the lifetimes of unstable particles become negative. Regarded as a gas of particles, pseudoholes have negative number densities given by Eq. (28) and from Ref. 1 tend to exert negative pressures:

$$\frac{P_I}{kT} = n^2\left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2} \frac{2^{3/2}}{\pi} \sum_l (2l+1) \times \int_0^\infty \frac{d\eta_l}{dE} (4\sin^2\eta_l - 1)\, e^{-E/kT} dE \ . \tag{29}$$

But these are effective properties. We may say that a gas of unstable particles with negative lifetimes behaves as if it were a gas of holes. But as individuals, pseudoholes lack the identity of unstable particles because it is impossible even to consider writing down localized wave functions describing them. Unlike holes or bubbles in a degenerate Fermi sea, pseudoholes only exist in the contrary sense of removed-particle pairs. As such, they are only intended as a conceptual convenience with which to extend the unstable-particle formalism to the negative-lifetime case. The formal equivalence of Eqs. (2), (3), and (4) with Eqs. (28), (26), and (27), respectively, was demonstrated without recourse to this interpretation, and should therefore indicate the permissibility of extending the formulas of Ref. 1 when $d\eta/dE < 0$.

*Work supported in part by the U.S. AEC and NASA.

[1]Michael D. Delano, Phys. Rev. A 1, 1175 (1970).

PHYSICAL REVIEW A VOLUME 3, NUMBER 2 FEBRUARY 1971

N-Fold Photoelectric Counting Statistics of Mixed Chaotic and Coherent Light*

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(Received 14 July 1970)

General formulas for the factorial cumulants of the photocounting distribution and the cumulants of the integrated intensity are obtained for chaotic light superposed with coherent light, for experiments involving N different counting intervals. The chaotic light may be partially polarized, need not be time stationary or single mode, may differ in frequency from the coherent light, and may have any spectral distribution.

I. INTRODUCTION

Chaotic light superposed with coherent light is one of a small number of cases for which an exact solution can be found for the probability distribution of photoelectric counts, or for arbitrarily high moments of this distribution. As in other areas of physics, exactly solvable problems in photoelectric counting facilitate physical understanding of problems which are less tractable theoretically, such as the counting statistics of scattered light.

The probability distribution of the number of photoelectrons produced in a counting interval T by chaotic-plus-coherent light incident on a single detector has been studied theoretically by several authors.[1-10] Closed general formulas for the generating function, or the factorial moments, or the probability distribution itself, were derived by Lachs,[1] Glauber,[3] and Magill and Soni,[4] assuming that: (i) The incident light is fully polarized; (ii) the counting time is short compared to the coherence time of the chaotic light; and (iii) the incident light is confined to a single mode of the electromagnetic field (which implies that the mean frequency of the chaotic light ω_T equals the frequency of the coherent light ω_C). Without using (ii) or (iii), Korenman calculated both the generating function and the factorial cumulants of the photocount distribution[5]; the latter have proved to be a particularly simple and useful form for summarizing the complete photocounting statistics. Assumptions (ii) and (iii) were removed in the formalism set up by Lachs, although closed formulas for the generating function and the higher-factorial moments were obtained only in the limit of short counting times.[6] Jakeman and Pike calculated the photocount generating function without (ii) or (iii) for the case of chaotic light with a Lorentzian spectrum[8]; results for an arbitrary spectral distribution were obtained by Peřina and Horâk.[9] Recently, Jaiswal and Mehta derived the factorial cumulants without any of these assumptions.[10]

Photoelectric counting experiments involving more than one counting interval and more than one detector contain information on both spatial and temporal coherence properties of the light incident on the detectors. The generalization of the generating function, cumulants, and factorial cumulants to such N-fold counting experiments has been discussed elsewhere.[11,12] In this paper, the factorial cumulants will be calculated for N-fold photocounting experiments on chaotic-plus-coherent light, for arbitrary counting times, for arbitrary values of $\omega_T-\omega_C$, and for partially polarized multimode chaotic light of arbitrary spectral distribution. The coherent light is assumed to be fully polarized, but may be multimode. A different method due to Jaiswal and Mehta yields the same results.[13]

The N-fold cumulant $K_{m_1\ldots m_N}(x_1, \ldots, x_N)$ of N variables $x_1, \ldots, x_N$ is a measure of the "true" $m_1\cdots m_N$-order correlation among the variables $x_1, \ldots, x_N$, with all lower orders of correlation subtracted away.[14,15] The N-fold cumulants $K_{m_1\cdots m_N}(I_1, \ldots, I_N)$ provide a formally convenient and physically appealing summary of the statistics of the optical intensities $I_1, \ldots, I_N$ at N different detectors. Onefold intensity cumulants have been discussed and used to analyze experimental data by Chang *et al.*[15]

The quantities which similarly measure the "true" correlations in the distribution of the numbers $n_1, \ldots, n_N$ of photoelectric counts received at N detectors in the time intervals $[t_1,\ t_1+T_1], \ldots, [t_N,\ t_N+T_N]$ are the factorial cumulants $k_{m_1\cdots m_N}(n_1, \ldots, n_N)$, which are equal to the cumulants (of the same order) of the integrated intensities[12,15]

$$k_{m_1\cdots m_N}(n_1, \ldots, n_N) = K_{m_1\cdots m_N}(W_1, \ldots, W_N) .$$

In this equation, we have

$$W_j = \alpha_j \int_{t_j}^{t_j+T_j} I_j(t)\, dt ,$$

where $I_j(t)$ is the intensity of light incident on the jth detector, and the mean number of photoelectrons emitted is

$$\langle n_j\rangle = \alpha_j T_j \langle I_j\rangle = \langle W_j\rangle .$$

All the factorial cumulants with $\sum_j m_j > 1$ vanish when the intensities at the N detectors are statistically independent (in which case all correlations in the photocounts are accidental).

To derive $K_{m_1\ldots m_N}(W_1, \ldots, W_N)$ it is convenient to calculate $K_{m_1\ldots m_N}(I_1, \ldots, I_N)$ first, since the latter cumulant can be found by purely algebraic methods using the Gaussian moment theorem, Eq. (6), and the relation between cumulants and moments, Eq. (3). This is done in Sec. II. The results from combinatorial analysis which are needed for this purpose have recently been discussed in the context of photocounting experiments,[12] and hence are summarized very briefly. The factorial cumulants can be found from the intensity cumulants by integration,

$$k_{m_1\ldots m_N}(n_1, \ldots, n_N) = \alpha_1 \cdots \alpha_N \int_{t_1}^{t_1+T_1} dt_1' \cdots \int_{t_N}^{t_N+T_N} dt_N' \times K_{m_1\ldots m_N}(I_1(t_1'), \ldots, I_N(t_N')) . \tag{1}$$

The results for coherent light plus fully polarized chaotic light are discussed in Sec. III, and are generalized to include partially polarized chaotic light in Sec. IV.

II. INTENSITY CUMULANTS OF MIXED CHAOTIC AND COHERENT LIGHT

The values of the optical field at the N detectors, and at times $t_1, \ldots, t_N$, are

$$V_j'(t_j) = V_j + U_j , \tag{2}$$

where $V_j (j = 1, 2, \ldots, N)$, the values of the chaotic field, are Gaussian random variables with zero mean and covariance matrix Γ,

$$\Gamma_{jk} = \Gamma_{jk}(t_j, t_k) = \langle V_j^* V_k\rangle ,$$

and $\{U_j\}$ are the values of the coherent field. Here and in Sec. III the optical fields are assumed to be fully polarized.

It is convenient to define the intensities

$$I_j^T = |V_j|^2, \quad I_j^C = |U_j|^2$$

of the chaotic and coherent light alone.

The cumulants are related to the joint multivariable moments by the formula[12]

$$K_{m_1\ldots m_N}(x_1, \ldots, x_n) = \sum_{\mathcal{P}} (-1)^{n(\mathcal{P})-1}(n(\mathcal{P})-1)! \prod_{S\in\mathcal{P}} \left\langle \prod_{j\in S} x_j^{m_j} \right\rangle , \tag{3}$$

where $\mathcal{P}$ is any partition[12] of the set of numbers $1, 1, \ldots, 1(m_1 \text{ times}); \ldots; N, N, \ldots, N(m_N \text{ times})$ into $n(\mathcal{P})$ disjoint subsets S. The N-fold factorial cumulants $k_{m_1\ldots m_N}(n_1, \ldots, n_N)$ are related to the N-fold factorial moments

$$F_{m_1\ldots m_N}(n_1, \ldots, n_N) = \left\langle \prod_{j=1}^{N} n_j(n_j-1)\cdots(n_j-m_j+1)\right\rangle$$

in the same way as the ordinary cumulants are related to the ordinary joint moments, Eq. (3).

As in Ref. 12, we need to consider only the cumulants for which $m_1 = m_2 = \cdots m_N = 1$, since the cumulants for which some m's are greater than unity can be obtained from an N'-fold cumulant $K_{11\cdots 1}(x_1, \ldots\ (x_1, \cdots, x_{N'})$ with $N' = \sum_{j=1}^{N} m_j$. Substituting (2) into (3), we find

$$K_{11\cdots 1}(I_1, \ldots, I_N) = K_{11\cdots 1}(|V_1|^2 + V_1^* U_1 + V_1 U_1^* + |U_1|^2, \cdots) .$$

It can be shown from the cumulant generating function[14] that when

$\sum m_j > 1$,

$$K_{11\cdots 1}(x_1, \ldots, x_N) = K_{11\cdots 1}(x_1 + c_1, \ldots, x_N + c_N) \ , \quad (4)$$

where $c_1, \ldots, c_N$ are constants (i.e., not subject to the implied statistical average). Therefore, we have

$$K_{11\cdots 1}(I_1, \ldots, I_N) = K_{11\cdots 1}(I_1^T + V_1^* U_1 + V_1 U_1^*, \cdots) \ .$$

Equation (3) implies that the latter cumulant can be expressed as a sum of terms involving the multinomials U_j, $U_j U_k^*(j \neq k)$, $U_j^* U_k U_m^*(j \neq k \neq m)$, etc., up to Nth order in the coherent field

$$K_{11\cdots 1}(I_1, \ldots, I_N) = K_{11\cdots 1}(I_1^T, \ldots, I_N^T) + \sum_{j=1}^{N} U_j K_{11\cdots 1}(I_1^T, \ldots, I_{j-1}^T, V_j^*, I_{j+1}^T, \ldots, I_N^T) + \sum_{j,k=1, j\neq k}^{N} U_j^* U_k K_{11\cdots 1}(I_1^T, \ldots, V_j, \ldots, V_k^*, \ldots, I_N^T) + \cdots \ . \quad (5)$$

The mixed N-fold cumulants of intensities and amplitudes which appear in (5) can be evaluated from (3), using the Gaussian moment theorem for complex Gaussian variables $z_j (j = 1, 2, \ldots, N)$,[16,17]

$$\langle z_{j_1}^* \cdots z_{j_M}^* z_{k_1} \cdots z_{k_N} \rangle = \delta_{MN} \sum_P \prod_{m=1}^{N} \langle z_{j_m}^* z_{Pk_m} \rangle \ . \quad (6)$$

In (6), $P: k_m \to Pk_m$ is any permutation of the numbers $k_1, \ldots, k_N$.

Equation (6) implies that only those terms with an equal number of U's and U^*'s will be nonzero in (5). Consider the cumulant

$$K_{11\cdots 1}(V_1^*, \ldots, V_L^*, V_{L+1}, \ldots, V_{2L}, I_{2L+1}^T, \ldots, I_N^T) \ . \quad (7)$$

(The order of variables is irrelevant in a cumulant in which all m_j's are equal.) By (3) and (6), (7) will be equal to a sum of products of averages $\langle V_j^* V_k \rangle = \Gamma_{jk}$, where $j = 1, 2, \ldots, L, 2L+1, \ldots, N$, and $k = L+1, \ldots, N$. We can identify any given product

$$\Gamma_{j_1 k_1} \cdots \Gamma_{j_M k_M}$$

by the symbol

$$R = (j_1 k_1) \cdots (j_M k_M) = \begin{pmatrix} j_1 & \cdots & j_M \\ k_1 & \cdots & k_M \end{pmatrix} , \quad (8)$$

where $M = N - L$. Expression (8) can be taken to represent an operation in which $V_{j_1}^*$ is "replaced" by $V_{k_1}; \ldots; V_{j_M}^*$ by V_{k_M}. [In the calculation of the cumulants of the intensities alone, the indices j_m and k_m run through the same set of integers, and (8) represents a permutation of this set factored into transpositions $(j_m k_m)$.] Thus, Eq. (7) equals

$$\sum_R c_R \Gamma_{j_1 k_1} \cdots \Gamma_{j_M k_M} \ , \quad (9)$$

where the sum runs over all replacements of the set $V_1^*, \ldots, V_L^*, V_{2L+1}^*, \ldots, V_N^*$ by the set $V_{L+1}, \ldots, V_N$. The coefficient c_R is the sum of integers $(-1)^{n(\mathcal{P})-1}(n(\mathcal{P}) - 1)!$ over all partitions $\mathcal{P}$ in (3), which can give rise to a term of type R when the Gaussian moment theorem is used to evaluate the moments appearing in (3). If there are no $\mathcal{P}$'s which can give rise to R, then we define $c_R = 0$.

A *cyclic* replacement is one in which the parentheses can be ordered so that the replacement has the form

$$(ab)(bc)(cd)(de) \cdots (xy)(yz) = \begin{pmatrix} a\,b\,c\,d & \cdots & x\,y \\ b\,c\,d\,e & \cdots & y\,z \end{pmatrix}$$

[the order of parentheses on the left-hand side of (8) is immaterial]. As with permutations, a replacement can be expressed uniquely as a product of cyclic replacements. In a cyclic replacement, all but two of the symbols a, b, c, $\cdots$ occur twice; however, not all replacements of which this is true are cyclic.

Expressing R as a product of cyclic replacements separates the set of indices $1, 2, \ldots, N$ in (9) into subsets consisting of the indices which occur in one cycle. These subsets have no indices in common. Thus, each R which actually occurs corresponds uniquely to a partition $\mathcal{Q}(R)$ of the set $1, 2, \ldots, N$. No partition $\mathcal{P}$ which is a refinement[12] of $\mathcal{Q}(R)$ can give rise to a term of type R, but every $\mathcal{P}$ of which $\mathcal{Q}(R)$ is a refinement gives rise to exactly one term of type R when (3) and (6) are used to evaluate (7). Hence, Eq. (7) equals

$$\sum_R \left[\sum_{\mathcal{P} \leq \mathcal{Q}(R)} (-1)^{n(\mathcal{P})-1}(n(\mathcal{P}) - 1)! \right] \Gamma_{j_1 k_1} \cdots \Gamma_{j_M k_M} \ . \quad (10)$$

It was shown in Ref. 12, Appendix B, that the sum in brackets is the Kronecker delta $\delta_{1, n[\mathcal{Q}(R)]}$, where $n(\mathcal{Q})$ is the number of subsets in the partition $\mathcal{Q}$. Thus, the sum over R in (9) reduces to a sum (with all coefficients equal to 1) over those replacements which consist of exactly one cycle.

For $L > 1$, there are no cyclic replacements of the set $V_1^*, \ldots, V_L^*, V_{2L+1}^*, \ldots, V_N^*$ by the set $V_{L+1}, \ldots, V_N$, since the indices $1, 2, \ldots, 2L$ must occur singly in any replacement of these sets. Consequently, only the terms of zero order and second order in the coherent field survive in (5). The cyclic replacements of the above sets with $L = 1$ are in one-to-one correspondence with the cyclic permutations of the $N-1$ integers $1, 3, \ldots, N$, the difference between a permutation C' and a replacement being in this case that in the latter, where 1 occurs in the permuted set $C'1, C'3, \ldots, C'N$, it must be replaced by 2.

The result is

$$K_{11\cdots 1}(I_1, \ldots, I_N) = \sum_C \prod_{j=1}^{N} \Gamma_{j, Cj}$$

$$+\sum_{j,k=1,j\neq k}^{N} U_j U_k^* \sum_{C'(k)} \prod_{m=1}^{N}{}'^{(k)} \Gamma_{m,C'(k)m} , \tag{11}$$

where C is any cyclic permutation of 1, 2, ..., N, and $C'(k)$ is any cyclic permutation of 1, 2,..., $k-1$, $k+1$, ..., N; where j occurs in the permuted set of integers $\{C'(k)m\}$, it is to be replaced by k. The notation

$$\prod_{m=1}^{N}{}'^{(k)}$$

is intended to indicate that the product runs from $m=1$ to $m=N$, omitting $m=k$.

As an example of these formulas, the cyclic rearrangements of two and three objects are

$$\begin{pmatrix}1&3\\3&2\end{pmatrix} \text{ and } \begin{pmatrix}1&3&4\\3&4&2\end{pmatrix}, \begin{pmatrix}1&3&4\\4&2&3\end{pmatrix},$$

respectively, and the intensity cumulants for $N=2-4$ are

$$K_{11}(I_1, I_2) = |\Gamma_{12}|^2 + 2\,\mathrm{Re}(U_1 U_2^* \Gamma_{12}) , \tag{12}$$

$$K_{111}(I_1, I_2, I_3) = 2\,\mathrm{Re}(\Gamma_{12}\Gamma_{23}\Gamma_{31} + U_1U_2^*\Gamma_{13}\Gamma_{32} + U_1U_3^*\Gamma_{12}\Gamma_{23} + U_2U_3^*\Gamma_{21}\Gamma_{13}) , \tag{13}$$

$$\begin{aligned} K_{1111}(I_1, I_2, I_3, I_4) = 2\,\mathrm{Re}[&\Gamma_{12}\Gamma_{23}\Gamma_{34}\Gamma_{41} + \Gamma_{12}\Gamma_{24}\Gamma_{43}\Gamma_{31} + \Gamma_{13}\Gamma_{32}\Gamma_{24}\Gamma_{41} + U_1U_2^*(\Gamma_{13}\Gamma_{34}\Gamma_{42} + \Gamma_{14}\Gamma_{43}\Gamma_{32}) \\ &+ U_1U_3^*(\Gamma_{12}\Gamma_{24}\Gamma_{43} + \Gamma_{14}\Gamma_{42}\Gamma_{23}) + U_1U_4^*(\Gamma_{13}\Gamma_{32}\Gamma_{24} + \Gamma_{12}\Gamma_{23}\Gamma_{34}) \\ &+ U_2U_3^*(\Gamma_{24}\Gamma_{41}\Gamma_{13} + \Gamma_{21}\Gamma_{14}\Gamma_{43}) + U_2U_4^*(\Gamma_{23}\Gamma_{31}\Gamma_{14} + \Gamma_{21}\Gamma_{13}\Gamma_{34}) \\ &+ U_3U_4^*(\Gamma_{31}\Gamma_{12}\Gamma_{24} + \Gamma_{32}\Gamma_{21}\Gamma_{14})] . \end{aligned} \tag{14}$$

The terms containing only products of Γ's are the intensity cumulants for chaotic light alone; the remaining terms containing products of Γ's and U's result from the interference of the chaotic light with the coherent light.

III. FACTORIAL CUMULANTS OF PHOTOELECTRIC COUNTING DISTRIBUTION

The factorial cumulants of $n_1, \cdots, n_N$ are, from (1) and (11),

$$\begin{aligned} k_{11\cdots1}(n_1, \ldots, n_N) &= K_{11\cdots1}(W_1, \ldots, W_N) = \alpha_1 \cdots \alpha_N \int_{t_1}^{t_1+T_1} dt_1' \cdots \int_{t_N}^{t_N+T_N} dt_N' \Big(\sum_C \prod_{j=1}^{N} \Gamma_{j,Cj}(t_j', t_{Cj}') \\ &+ \sum_{j,k=1,j\neq k}^{N} U_j(t_j')U_k(t_k')^* \sum_{C'(k)} \prod_{m=1}^{N}{}'^{(k)} \Gamma_{m,C'(k)m}(t_m', t_{C'(k)m}') \Big) . \end{aligned} \tag{15}$$

The notation does not adequately indicate the structure of the second term of (15), but it can be described as follows: The mutual coherence functions $\Gamma_{mm'}$ form the cyclic product $\Gamma_{ja}\Gamma_{ab}\Gamma_{bc}\Gamma_{cd}\cdots\Gamma_{hi}\Gamma_{ik}$, and the product $U_jU_k^*$ completes the cycle.

The factorial cumulant of the photocounting distribution is seen to be the sum of two sets of terms, the first representing the effect of the chaotic light alone, and the second representing the interference of the chaotic and the coherent light. The coherent light alone would give rise to a Poisson distribution of photocounts, and hence to factorial moments

$$F_{m_1\cdots m_N}(n_1, \ldots, n_N) = \langle n_1\rangle^{m_1} \cdots \langle n_N\rangle^{m_N} .$$

This expression has the same form as the joint ordinary moment of a set of constants; for this case, the cumulants beyond lowest order all vanish. Consequently, the factorial cumulants for coherent light are all 0 except for first order, and we can say that the factorial cumulants for superposed chaotic and coherent light are the sum of the factorial cumulants for the two alone, plus an interference term which is linear in the intensity of the coherent light.

The coherent field $U(\vec{r}, t)$ in (15) need not be monochromatic, but it cannot be subject to statistical fluctuations. Thus, (15) applies in cases where the coherent light is multimode, but perfectly mode locked.

All that has been assumed about the chaotic field in deriving (15) is that the amplitudes of the field at different space-time points obey the multivariable Gaussian distribution. It has not been assumed that the random process represented by the chaotic field is stationary, i.e., that

$$\Gamma_{jk}(t_j, t_k) = \Gamma_{jk}(t_k - t_j) .$$

Thus, (15) is valid for nonstationary Gaussian fields such as light derived from a thermal source by chopping or amplitude modulation. Many different mode structures are compatible with these assumptions, as long as the mode amplitudes also obey the multivariable Gaussian distribution. It is not necessary that the mode amplitudes be statistically independent.

For detectors of finite size, the integrated intensities are

$$W_j = \alpha_j \int_{A_j} dS(\vec{r}_j) \int_{t_j}^{t_j+T_j} dt\, I(\vec{r}_j, t) ,$$

where the surface integral extends over the area A_j of the jth detector. Thus, the factorial cumulants can be obtained from the point-detector relation (15) by integrating with respect to $\vec{r}_j (j=1, 2, \ldots, N)$. If each A_j is small compared to a coherence area of the chaotic light, then the integrations with respect to the space variables which occur purely in the mutual coherence functions $\Gamma(\vec{r}_m, \vec{r}_{m'}; t_m, t_{m'})$ simply multiply (15) by the areas of the respective detectors. However, the factors

$$U_j \Gamma_{jm} = U(\vec{r}_j, t_j')\Gamma(\vec{r}_j, \vec{r}_m; t_j', t_m') ,$$

$$U_k^* \Gamma_{m'k} = U(\vec{r}_k, t_k')^* \Gamma(\vec{r}_{m'}, \vec{r}_k; t_{m'}', t_k')$$

will cause the integral of the second term of (15) with respect to $\vec{r}_j$ and $\vec{r}_k$ to vanish unless the wavefronts of the chaotic and coherent light are parallel to within less than half a wavelength over the areas of the jth and kth detectors. [This can be seen by expanding $U(\vec{r}, t)$ and $V(\vec{r}, t)$ in plane waves.] This effect is familiar in optical heterodyning experiments.

If the chaotic light is cross-spectrally pure (and therefore time stationary),[18] we have

$$\Gamma_{jk}(t) = \gamma_{jj}(t)\Gamma_{jk}(0) ,$$

where

$$\gamma_{jj}(t) = \langle I_j \rangle^{-1} \langle V(\vec{r}_j, 0)^* V(\vec{r}_j, t)\rangle$$
$$\equiv e^{-i\omega_T t}\gamma(t) ,$$

ω_T being the mean frequency of the thermal light. For quasimonochromatic light, $\gamma(t)$ varies slowly on a time scale of $(\omega_T)^{-1}$. We shall also assume that the coherent light is monochromatic,

$$U_j(t) = U_{j0} e^{-i\omega_C t} .$$

In this case, we can separate the terms of (15) into factors which depend on the spatial coherence properties of the light, and factors which depend on the temporal properties (the spectral density of the chaotic light, and the frequency difference $\Delta\omega = \omega_C - \omega_T$)

$$k_{11\cdots 1}(n_1, \ldots, n_N) = \alpha_1 \cdots \alpha_N \Big(\sum_C \prod_{j=1}^N \Gamma_{j,C_j}(0) \int_{t_1}^{t_1+T_1} dt_1' \cdots \int_{t_N}^{t_N+T_N} dt_N' \times \prod_{m=1}^N \gamma(t'_{C_m} - t'_m) + \sum_{j\neq k} U_{j0}U_{k0}^* \sum_{C'(k)} \prod_{m=1}^{N}{}'^{(k)} \Gamma_{m,C'(k)m}(0) \times \int_{t_1}^{t_1+T_1} dt_1' \cdots \int_{t_N}^{t_N+T_N} dt_N' e^{i(\omega_C-\omega_T)(t_k'-t_j')} \prod_{r=1}^N{}'^{(k)} \gamma(t'_{C'(k)r} - t'_r) \Big) . \tag{16}$$

When the counting times are short compared to a coherence time, then $\gamma(t) \approx 1$ in all the integrals, and

$$k_{11\cdots 1}(n_1, \ldots, n_N) = \alpha_1 \cdots \alpha_N T_1 \cdots T_N \Big(\sum_C \prod_{j=1}^N \Gamma_{j,C_j}(0) + 8 \sum_{j<k} U_{j0}U_{k0}^* \cos\{\Delta\omega[t_j + \tfrac{1}{2}T_j - (t_k + \tfrac{1}{2}T_k)]\} \times \frac{\sin(\frac{1}{2}\Delta\omega T_j)}{\Delta\omega T_j} \frac{\sin(\frac{1}{2}\Delta\omega T_k)}{\Delta\omega T_k} \sum_{C'(k)} \prod_{m=1}^N{}'^{(k)} \Gamma_{m,C'(k)m}(0) \Big) . \tag{17}$$

The effect of the beat frequency $\Delta\omega$ on the photoelectron statistics is that the term representing the interference of the chaotic with the coherent light are modulated by a harmonic factor varying at frequency $\Delta\omega$, and are also multiplied by $(\sin x)/x$ factors which show the effect of the finite counting time.

For the onefold case, in the limit of short counting times and when $\omega_T = \omega_C$, Eq. (17) becomes

$$k_N(n) = (N-1)!(\alpha T)^N \langle I^T \rangle^N + N!(\alpha T)^N I^C \langle I^T \rangle^{N-1}$$

(where $I^T = |V|^2$, $I^C = |U|^2$), from which it follows that the cumulant generating function for the integrated intensities is

$$\mathcal{K}(W;s)=\sum_{N=1}^{\infty}K_N(W)\frac{s^N}{N!}$$
$$=\sum_{N=1}^{\infty}\frac{(\alpha T\langle I^T\rangle)^N s^N}{N}+(\alpha T I^C)s\sum_{M=0}^{\infty}(\alpha T\langle I^T\rangle)^M s^M$$
$$=-\ln[1-(\alpha T\langle I^T\rangle)s]+\frac{(\alpha T I^C)s}{1-(\alpha T\langle I^T\rangle)s}\ .$$

Thus, the photocount generating function for this case is

$$\mathcal{G}(n;s)=e^{\mathcal{K}(W;-s)}$$
$$=\frac{1}{1+\langle n^T\rangle s}\exp\left[-\frac{\langle n^C\rangle s}{1+\langle n^T\rangle s}\right],$$

where $\langle n^T\rangle=\alpha T\langle I^T\rangle$, $\langle n^C\rangle=\alpha T I^C$. This agrees with the calculations of Refs. 1, 3, and 4. The more general results of Korenman[5] and Jaiswal and Mehta[10] for the onefold case follow in a straightforward manner from (15).

IV. PARTIALLY POLARIZED LIGHT

Let the light incident on the N detectors be partially polarized, so that the values of the optical field are

$$\vec{V}_j'=\vec{V}_j+\vec{U}_j$$

(where $\vec{V}_j$ is chaotic and $\vec{U}_j$ is coherent), and let the field be resolved into components with respect to an orthonormal polarization basis $\hat{e}_j^{(1)}$, $\hat{e}_j^{(2)}$ at each detector, so that

$$\vec{V}_j=V_j^{(1)}\hat{e}_j^{(1)}+V_j^{(2)}\hat{e}_j^{(2)}\ ,$$
$$\vec{U}_j=U_j^{(1)}\hat{e}_j^{(1)}+U_j^{(2)}\hat{e}_j^{(2)}\ .$$

The total intensity is

$$I_j=\vec{V}_j'^{*}\cdot\vec{V}_j'=I_j^{(1)}+I_j^{(2)}\ ,$$

where

$$I_j^{(x)}=|V_j^{(x)}|^2+V_j^{(x)*}U_j^{(x)}+V_j^{(x)}U_j^{(x)*}+|U_j^{(x)}|^2\ ,$$

where $x=1,2$. Then (3) implies that the intensity cumulants are

$$K_{11\cdots1}(I_1,\ldots,I_N)$$
$$=\sum_{x_1=1}^{2}\cdots\sum_{x_N=1}^{2}K_{11\cdots1}(I_1^{(x_1)},\ldots,I_N^{(x_N)})\ .$$

Evaluating the cumulants on the right-hand side by the techniques of Sec. II and then integrating with respect to time, we find that the factorial cumulants of the photocounting distribution are

$$k_{11\cdots1}(n_1,\ldots,n_N)=\sum_{x_1=1}^{2}\cdots\sum_{x_N=1}^{2}\int_{t_1}^{t_1+T_1}dt_1'\cdots\int_{t_N}^{t_N+T_N}dt_N'\Big(\sum_{C}\prod_{j=k}^{N}\Gamma_{j,Cj}^{(x_j,x_{Cj})}(t_j',t_{Cj}')$$
$$+\sum_{j\neq k}\sum_{C'(k)}\prod_{m=1}^{N}{}'^{(k)}\,\Gamma_{m,C'(k)m}^{(x_m,x_{C'(k)m})}(t_m',t_{C'(k)m}')\,U_j^{(x_j)}(t_j')\,U_k^{(x_k)}(t_k')^*\Big)\ . \tag{18}$$

For onefold counting, (18) agrees with the results of Jaiswal and Mehta.[10,19]

If the light is cross-spectrally pure, then the single-detector factorial cumulant $k_N(n)$ can be substantially simplified by choosing a polarization basis such that $V^{(1)}$ and $V^{(2)}$ are statistically independent.[19] This implies that only two terms in the sum over $x_1,\ldots,x_N$ can survive, since all the x's must have the same value for a term to be nonzero. Then, for monochromatic $U(t)$,

$$k_N(n)=(N-1)!\,\alpha^N(\langle I^{(1)T}\rangle^N+\langle I^{(2)T}\rangle^N)\int_0^T dt_1'\cdots\int_0^T dt_N'$$
$$\times\gamma(t_1'-t_2')\gamma(t_2'-t_3')\cdots\gamma(t_N'-t_1')\quad+N!\,\alpha^N(I^{(1)C}\langle I^{(1)T}\rangle^{N-1}+I^{(2)C}\langle I^{(2)T}\rangle^{N-1})$$
$$\times\int_0^t dt_1'\cdots\int_0^T dt_N'\,e^{i(\omega_C-\omega_T)(t_1'-t_2')}\,\gamma(t_2'-t_3')\cdots\gamma(t_N'-t_1')\ ,$$

where

$$\Gamma_{11}^{(x,x)}(t_1,t_2)=\gamma(t_2-t_1)\langle I^{(x)T}\rangle\ ,\quad I^{(x)T}=|V^{(x)}|^2,\quad I^{(x)C}=|U^{(x)}|^2\ .$$

ACKNOWLEDGMENTS

The author wishes to thank Dr. A. K. Jaiswal and Dr. C. L. Mehta for sending copies of Refs. 10 and 13 in advance of publication.

*Research supported by an institutional grant from the Alfred P. Sloan Foundation.

[1]G. Lachs, Phys. Rev. 138, B1012 (1965).
[2]H. Morawitz, Phys. Rev. 139, A1072 (1965).
[3]R. J. Glauber, in *The Physics of Quantum Electronics*, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill, New York, 1966), p. 788.
[4]P. J. Magill and R. P. Soni, Phys. Rev. Letters 16, 911 (1966).
[5]V. Korenman, Phys. Rev. 154, 1233 (1967).
[6]G. Lachs, J. Appl. Phys. 38, 3439 (1967).
[7]M. D. Aldridge, J. Appl. Phys. 40, 1720 (1969).
[8]E. Jakeman and E. R. Pike, J. Phys. A 2, 115 (1969).
[9]J. Peřina and R. Horák, J. Phys. A 2, 702 (1969).
[10]A. K. Jaiswal and C. L. Mehta, Phys. Rev. A 2, 168 (1970).
[11]G. Bédard, Phys. Rev. 161, 1304 (1967).
[12]C. D. Cantrell, Phys. Rev. A 1, 672 (1970).
[13]A. K. Jaiswal and C. L. Mehta (unpublished).
[14]R. Kubo, J. Phys. Soc. Japan 17, 1100 (1962).
[15]R. F. Chang, V. Korenman, C. O. Alley, and R. W. Detenbeck, Phys. Rev. 178, 612 (1969).
[16]C. L. Mehta, in *Lectures in Theoretical Physics*, edited by W. E. Brittin (Colorado U. P., Boulder, Colorado, 1965), Vol. VII, p. 345. See especially p. 398.
[17]R. J. Glauber, in *Quantum Optics and Electronics*, edited by C. DeWitt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach, New York, 1965), p. 65. See especially Eq. (14.36).
[18]L. Mandel, J. Opt. Soc. Am. 51, 1342 (1961); L. Mandel and E. Wolf, Rev. Mod. Phys. 37, 231 (1965), Sec. 5.5.
[19]A. K. Jaiswal and C. L. Mehta, Phys. Rev. 186, 1355 (1969).

PHYSICAL REVIEW A VOLUME 3, NUMBER 2 FEBRUARY 1971

Zero-Field Mobility of an Excess Electron in Fluid Argon†

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(Received 19 August 1970)

Drift velocities of electrons in fluid argon have been measured at temperatures from 90 to 160 °K and at pressures from 10 to 100 atm for applied electric fields in the range − 25 to − 200 V/cm. The electron drift velocity is found to be linear with respect to electric field strength only to − 100 V/cm at temperatures from 90 to 125 °K and to become increasingly nonlinear at temperatures greater than 125 °K. Mobilities can be obtained from these data by extrapolation to zero field; maxima are found in the zero-field mobilities as a function of density, in the region of 0.81 g/cm^3. Using the model proposed by Lekner for electron scattering by a system of fluctuating potentials and assuming that the scattering length for electrons in fluid argon approaches zero at some density, it is possible to obtain a semi-empirical relation for the zero-field mobility as a function of density. Excellent agreement between calculated and observed mobilities is found in the high-density range studied 1.0–1.4 g/cm^3. At densities less than 1.0 g/cm^3, several qualitative aspects of the experimental data are accounted for by the theory, but quantitative agreement is lacking. It is possible that at these lower densities, gas-like scattering is of dominant importance.

I. INTRODUCTION

The injection of excess electrons into liquids provides a means of studying the electronic states of disordered systems. By adding an electron to a liquid, a conducting state becomes populated; to interpret the resultant electronic behavior it is necessary to develop physically realistic models correlating the electron-atom interactions with properties of the fluid. As might be expected, one of the most useful guides in such a development is provided by measurement of the electron drift velocity v_D (due to the application of an electric field E to the liquid). Now, the liquid systems which in principle are the simplest to understand are He, Ne, Ar, Kr, and Xe, primarily because of the spherical symmetry of the single-atom scattering potential, the absence of inelastic collisions, and the weak and short-range nature of the electron-atom interaction. This paper reports new measurements of the electron drift velocity in Ar and their interpretation.

In a dielectric liquid, such as liquid argon, where the resistivity is high (approximately 10^{19} Ω cm), the number of electrons that can be introduced into the liquid without producing space-charge effects is very small, on the order of 10^5–10^8 cm^{-3}.[1] The Coulomb energy between a pair of electrons is, then, less than the thermal energy k_BT (where k_B is the Boltzmann constant), and it becomes possible to measure directly the resistance to the electron motion arising from scattering of the electron by the fluid. The parameter of interest is the zero-field mobility, defined as $\mu_0 = \lim(v_D/E)$, as the