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Elastic Scattering of Beryllium Doubly Charged Ions in Helium †

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The scattering of Be^{++} ions of energy 100–800 eV by He has been measured by observing the energy-loss spectrum of the ions on traversing various path lengths of the scattering gas. The scattering data have been analyzed to determine an interaction energy for Be^{++} and He for the internuclear separations from 0.15 to 1.0 Å. While not strictly logarithmic, the results can be approximately fitted by an expression for the interaction which is $V(r) = 6.0 \times 10^2 \times e^{-5.70r}$ eV, with r in Å. A Thomas-Fermi-Dirac calculation of the interaction energy fits the experimental result well for $r > 0.3$ Å.

I. INTRODUCTION

The method previously reported[1] for the observation of the scattering of Li^+ by He, and the determination of the interaction energy, has been extended to the case of Be^{++} in He. These systems, as He-He, are of particular interest, since the simplicity of the electron configuration permits highly accurate, molecular-orbital calculations of the interaction energy. Numerous calculations[2] have been done for the He-He system, with the most elaborate that by Phillipson,[3] which agrees to about 20% with the revised experimental results from the scattering of He by He by Jordan and Amdur.[4] A similar but less extensive series of calculations have been done by Mason *et al.*,[5] Fischer,[6] Junker and Brown,[7] and Catlow *et al.*[8] for the Li^+-He combination. These compare favorably with that obtained from the scattering of Li^+ in He by Zehr and Berry, and more recently by Olson *et al.*[9] for nuclear separations from 0.3 to 0.9 Å. In this range, both the experimental and the theoretical results are within 10–20% of each other. But from 0.3 to 0.1 Å, the lower limit of the experimental results, the calculated energy is increasingly higher, becoming almost twice the experimental value at 0.1 Å. It is interesting to note that the Thomas-Fermi-Dirac (TFD) two-center calculation of Abrahamson *et al.*[10] agrees surprisingly well with the *ab initio* calculations for He-He and Li^+-He, and follows the same trend at small internuclear separations.

The scattering of Be^{++} in He was investigated with the same technique used for Li^+-He. In this method, the energy-loss spectrum of the scattered ions is observed for various scattering-path lengths. While no MO calculation seems to exist for the Be^{++}-He system, a TFD calculation shows good agreement for $0.3 < r < 0.8$ Å, but with an increasing difference below 0.3 Å.

II. EXPERIMENTAL METHOD

Except for the Be^{++} source, the apparatus was similar to that used for Li^+ in He, and the details can be found in Ref. 1. The production of a sufficiently ample Be^{++} beam proved considerably more difficult than for Li^+. A number of ion sources were tried in which various salts of Be were dissociated and ionized by electrons bombardment. These proved unsatisfactory in yield and in the short life of the source. The source finally used was like that described by Magnuson *et al.*[11] Here Be metal is vaporized in an electrically heated oven and is then ionized by electron bombardment. An axial magnetic field confined the electron paths, and thus, the ion production, at the center of the cylindrical source. Ions which diffused along the axis through the aperture in the end of the source were accelerated by a several kilovolts extraction potential through a grid to the focusing and decelerating system.

A mass spectrometer designed to produce simultaneous vertical and horizontal focusing selected the Be^{++} beam. The beam was again defined and focused to be approximately plane parallel before entering the scattering chamber. The Be^{++} current ranged from 6×10^{-9} to 5×10^{-10} A and was about 5% of the Be^+ current from the source.

As stated above, the scattering was observed through the energy-loss spectrum of the ions after passage through various lengths of the scattering gas. The detector consisted of a flat stainless-steel circular plate preceded by a fine wire grid which limited the extent of the retarding field used for the energy analysis. The diameter of these was sufficient to intercept all elastically scattered ions for all lengths of the scattering path. The assembly was mounted on a rigid way and driven by a precision screw. See Ref. 1 for a discussion of the effects of the beam divergence and the nonuniformity of the retarding field on the data.

The original method of observing the ion current as a continuous function of the path length was abandoned for Be^{++} because of the smallness of the beam. Motion of the detector produced instabilities in the electronic system sufficient to mask

the changes in the collector current. Consequently, readings of the collector current with and without the retarding field were taken at specific detector positions as step functions on a strip chart. Up to 30 such pairs were recorded for each position and each value of the retarding field. For a given beam energy and retarding potential, the ratios of these currents were averaged. The data were then subjected to a series of tests such that eventually any that differed from the average by more than ± 1% were rejected. The logarithms of the new averages were corrected for backgrounds, such as secondary electrons from the grid, and were fitted by least squares to a straight line and the slope computed (see Fig. 1). The linearity of the plot was used as a check on single scattering.

The inelastic scattering which results in charge exchange would be essentially unobservable by the method of measurement here. A charge-exchange collision would generally produce a fast Be^+ ion and a slower He^+ ion. For a retarding voltage in the range consistent with elastic scattering, these Be^+ ions would always reach the collector, and thus would not contribute to the attenuation of the detector current with path length. Since both currents, with and without retarding voltage are reduced in like amounts, the effect is negligible.

Inelastic-scattering cross sections resulting in ionization would appear to be much smaller than the elastic cross section found here. Extrapolation of the cross sections for ionization by Be^+ in He by Sherwin, [12] and by He in He by Hayden and Utterback, [13] to the energy range of this experiment yields values 2 to 4 orders of magnitude below the elastic cross sections. (See, also, the calculation for excitation cross sections for Li^+ in He by Junker and Brown. [7]) Any inelastic collision which would leave the Be^{++} charge unchanged may be observed as an energy loss greater than that possible in an elastic collision. However, for retarding voltages below the minimum for which no elastically scattered ion will be rejected, the detector current

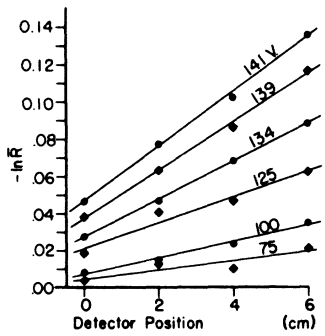

FIG. 1. Attenuation of the collector current ratio $\bar{R}$ with the detector position. The numbers on the curves are the retarding potentials in volts for an ion beam which was accelerated through a potential of 146.1 V. The zero position on the abscissa is arbitrary.

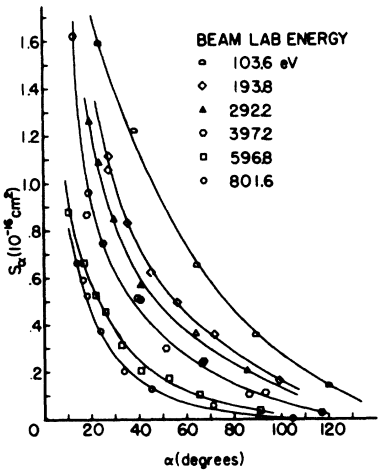

FIG. 2. Cross section S_α for scattering of Be^{++} outside the center-of-mass angle α by He. The number on each curve is the ion energy in electron volts in the laboratory system.

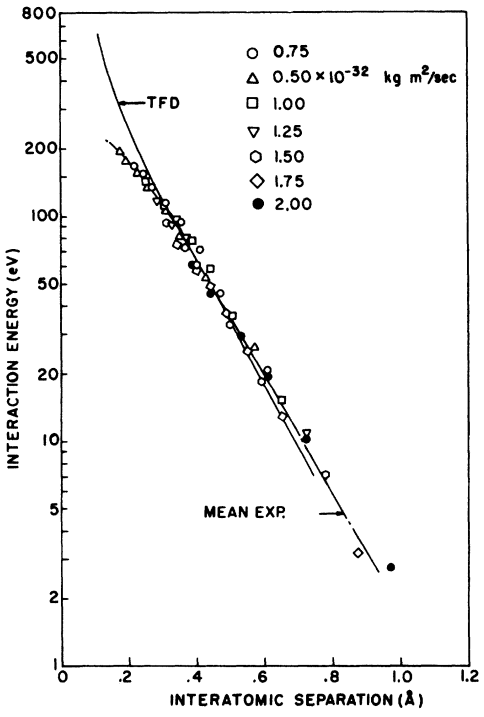

FIG. 3. Interaction energy, in electron volts of Be^{++} and He as a function of the interatomic separation in angstroms. The different symbols indicate the different values of the angular momentum used in the calculations. The curve labeled TFD is the interaction determined by the two-center TFD statistical model calculation.

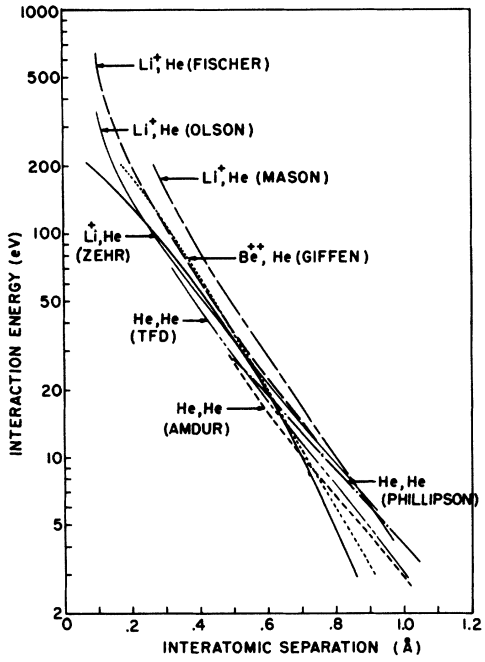

FIG. 4. Interaction energy for the isoelectronic systems He-He, Li^+-He, and Be^{++}-He as a function of the interatomic separation. Those labeled Amdur, Zehr, Olson, and Giffen are experimental, and the others are calculated.

remained essentially constant with path length, though showing the presence of the electron current from the grid.

The vacuum system was made of stainless steel and was separately pumped at the source and at the manifold to the mass spectrometer and the scattering chamber. Pressures before admission of the scattering gas were about 10^{-6} Torr. Admission of the He raised the pressure to about 8×10^{-3} Torr. Considerable effort was spent in a careful determination of the absolute He pressure. Several times during each run, a Schulz-Phelps gauge[14] on the scattering chamber was calibrated against a precision McLeod gauge. Precautions on trapping and use of the McLeod gauge were followed.[15] For a given gas sample, a number of readings were taken for different open and closed column heights. These were averaged by a least-squares fit of the column difference against the inverse of the length of gas sample in the closed column. A statistical analysis of goodness of fit was incorporated into the computer program of data analysis. Estimated error of the number of scattering centers per unit volume is about 3%.

III. INTERPRETATION OF DATA

The data yield directly the cross sections S_α, for scattering outside a given angle α, where α is found from the masses of the colliding particles and the ratio of the retarding voltage to the accelerating voltage for the ion. A representative set is shown in Fig. 2. A mean curve has been drawn through these points since data sets taken under other circumstances show no consistent trend. As described in Ref. 1, a series of numerical integrations is done which eventually yields the interatomic potential as a function of the internuclear separation. The data obtained contain overlapping results since there is a range of impact parameters present for each energy, and conversely, a range of energies available for each impact parameter yielding a given scattering angle. Arbitrarily choosing an angular momentum L, the calculation yields a set of values of r, the nuclear separation, against the effective potential $U = V(r) + L^2/2\mu r^2$, where μ is the reduced mass.

The values for the interaction energy $V(r)$ so found are shown in Fig. 3. Some indication of the internal consistency of the data is indicated by the spread of the points which, because of the method of analysis, arise from different regions of the data. The $V(r)$ values arising from a single L, or in turn a single U, are curves which are slightly concave downward such that the overlap of the end values produces most of the spread.

While the mean curve for $V(r)$ is not strictly logarithmic, the interaction energy can be very nearly represented by the Born-Mayer form, as

$$V(r) = 6.0\times10^2 e^{-5.70r}\ \text{eV}$$

for the range $0.15 < r < 1.0$ Å. Also shown in Fig. 3 is the interaction energy calculated by the TFD two-center model of Abrahamson *et al.*[10] The calculation followed the outline in Ref. 1 and used the same computer program. The agreement of the TFD with the more exact MO calculations for He-He and Li^+-He would suggest that, for this case too, it can be used as a substitute for a more exact calculation. Again, the agreement is good from 0.3 Å out, but increasingly poorer from 0.3 to 0.1 Å.

In Fig. 4, there are shown the combined results of these investigations of the interaction energy of these isoelectronic systems. Included are the experimental results for He-He by Amdur, the Li^+-He by Olson and by Zehr, and Be^{++}-He as well as the calculated $V(r)$ by Phillipson for He-He, Li^+-He by Fischer, and by Mason. The results of the calculation by Catlow[8] are not shown since these are nearly the same as those of Fischer for $r < 0.7$ Å but somewhat below for $0.7 < r < 1.0$. The essentially equal values near 0.6 Å are quite striking as is the increasing mean slope of the $\ln V(r)$ vs r with the progression from He-He to Be^{++}-He. This one might expect from the decreasing radii and increasing "hardness" of the projectile component. The discrepancy between the experimental and the

calculated values below 0.1 Å warrants further experimental study in scattering experiments with sufficiently energetic collisions to obtain the interaction energy at these close distances.

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Scattering Cross Sections for Multispecies Quantum Fluids

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The various possible ground-state structure factors are defined for a multispecies quantum fluid. These structure factors are then used to construct the Born approximation to the scattering cross sections for the multispecies fluid. The cross sections are used in conjuction with the results of an earlier paper to discuss the scattering of (i) charged particles, (ii) neutrons, and (iii) x rays from a two-species system composed of electrons and nuclei at high densities.

I. INTRODUCTION

In a recent paper[1] (henceforth referred to as I) Allen and Dunn developed a variational method for the treatment of the ground state of multispecies quantum fluids. The method was used to generate an approximation to the ground-state energy of a two-species system composed of N nuclei of atomic number Z, and ZN electrons. The approximation was found to be valid in the high-density limit where the Bruckner-Gell-Mann result[2] for the quantum electron gas is valid.

In this paper, the various possible ground-state structure factors[3] for the multispecies system are defined. The relationship between these various structure factors and the Born approximation to the cross section for the scattering of particles from the fluid is discussed for (i) the scattering of charged particles by the Coulomb interaction; (ii) the scattering of neutrons; and (iii) the scattering of x rays or photons.

The approximations developed in I are used to calculate the various ground-state structure factors for a two-species system composed of N nuclei and ZN electrons. These structure factors are then used to discuss the three types of scattering mentioned above. A comparison is made between the scattering of charged particles and x rays from the two-species system and from the quantum electron gas.[4]

It is found that the scattering of charged particles from the two-species system and from the quantum electron gas is almost identical for all values of the scattering angle. For the scattering of x rays it is found that at large scattering angles the two-species