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He II-He I Transition in the Presence of a Heat Current

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The He II-He I transition has been investigated by measuring the temperature at which the conductivity of He II exhibits a sharp drop. It is found that this temperature is a function of the heat current Q. A qualitative explanation in terms of the Ginzburg-Pitaevskii phenomenological theory is suggested.

I. INTRODUCTION

It is well known that superfluidity breaks down when the superfluid velocity v_s exceeds a certain critical value, say, v_c. However, there is no universal agreement regarding the processes that occur when breakdown takes place. It is possible that there is an hierarchy of critical velocities, some of which are temperature and/or geometry dependent. Presumably, in any given experimental situation one observes the smallest critical velocity.

In isothermal flow, the breakdown is marked by the onset of very large dissipative processes, and usually it is not possible to drive the superfluid at velocities greater than v_c. Several experiments have reported measurements of such critical velocities at temperatures far from T_λ.[1] Following Feynman,[2] the breakdown has been generally attributed to the appearance of vorticity. On the other hand, at temperatures close to T_λ, Clow and Reppy have studied the variation of another critical velocity.[3] It is claimed that this critical velocity represents an intrinsic property of He II in that it is determined, following Langer and Fisher,[4] by the probability of transition from the state of pure superflow to a state carrying vortex rings.

When the flow is induced by a heat current, the situation appears to be somewhat different.

A. Temperatures Far Away from T_λ

The breakdown usually manifests itself through the appearance of a temperature gradient (in addition to any gradients attributable to viscosity of the normal component) at a well-defined critical heat current Q_c.[5] For $Q > Q_c$ (henceforth referred to as supercritical flow), the additional temperature gradients usually increase according to the relation $Q \propto (\mathrm{grad}T)^{1/m}$ with $m = 3$, and this has been regarded as evidence for the existence of a mutual friction force proposed originally by Gorter and Mellink.[6] It is generally agreed that this force can at least qualitatively be understood in terms of scattering of rotons and phonons by the vortex lines which are presumed to appear in the superfluid for $Q > Q_c$.[7] Measurements on Q_c can be used to obtain v_c through the usual two-fluid hydrodynamic equations. For $T \leq 2\,°\mathrm{K}$, the agreement between the values of v_c measured using the two types of flow appears to be quite satisfactory.[1]

B. T Close to T_λ

In this case there are very few experimental results.[8] On the other hand, following the phenomenological expressions for the free energy suggested by Ginsburg and Pitaevskii,[9] several theoretical papers have discussed this regime.[10-12] The main result can be summarized by the relationship between Q and $w = v_s - v_n$, exhibited in Fig. 1. Here $[w_c]_{\mathrm{Th}}$, which allows one to define a "thermodynamic" critical heat current through the relationship $[Q_c]_{\mathrm{Th}} = \rho_s ST\,[w_c]_{\mathrm{Th}}$, marks the point where superfluidity breaks down. At w_c', the order parameter (or ρ_s) vanishes. It is clear that neither of these critical currents has anything to do with vorticity; they follow from purely thermodynamic considerations. At present, there is no description available for the state of the system between $[w_c]_{\mathrm{Th}}$ and w_c'. The existence of w_c' implies that the presence of conterflow should cause $\rho_s(Q, T)$ to go to zero at $T < T_\lambda$, thus giving a shift in the transition temperature.

From the experimental viewpoint, two situations can be envisaged.

(a) $[Q_c]_{\mathrm{Th}} < Q_c$: This condition may be realized if one chooses a geometry in which generation of vorticity is inhibited from general hydrodynamic

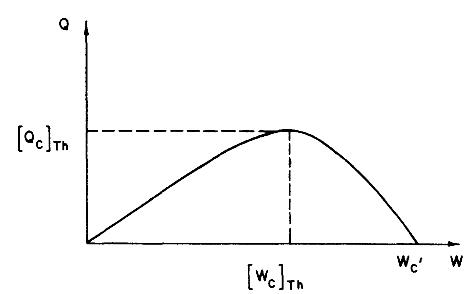

FIG. 1. Q (heat current) versus w (counterflow velocity) for a constant temperature $T < T_\lambda$. $[Q_c]_{\mathrm{Th}}$ and $[w_c]_{\mathrm{Th}}$ are the critical heat current and the critical counterflow velocity for breakdown of superfluidity. w_c' marks the point where the order parameter ρ_s vanishes.

considerations. Such an experiment was reported by us some time ago.[13] However, the interpretation of the results was not as unequivocal as we had expected. Further details regarding the results and their implications were presented in an unpublished report.[14]

(b) $[Q_c]_{Th} > Q_c$: That is, the thermodynamic critical current is larger than the critical heat current at which generation and/or growth of vorticity becomes energetically favorable. In the present paper we wish to report the results of an experiment in which we feel that this regime prevails.

We have studied heat flow in a wide channel containing liquid He II at T close to T_λ. The main results are (i.) Even at the smallest heat currents used there exists a finite temperature gradient in the liquid. This is taken as evidence to suggest that for the range of heat currents used by us $Q > Q_c$. (ii) We have observed a sharp drop in the conductivity of the liquid at a well-defined temperature. This "transition" temperature is a fairly strong function of the heat current (Sec. III B). A somewhat similar effect was noticed by Johnson and Crooks.[15] However, they did not study the phenomena in any detail.

II. EXPERIMENTAL METHOD AND OBSERVATIONS

The measurements were made using a conventional thermal conductivity apparatus.[16] The liquid He II was contained in a 20-cm-long thin-walled stainless-steel tube which was mounted inside a high vacuum (better than 10^{-5} Torr) enclosure (Fig. 2). Two tubes have been used, one of diameter 0.6 cm and one of 0.4 cm. Several carbon resistance thermometers were mounted on the tube. Thin copper wires (No. 24 AWG) were soldered to pass diametrically through the tube and were cemented securely to the body and leads of the thermometer to give improved thermal contact. The heater was made of high-resistance Karma wire. A conventional ac bridge, followed by a lock-in detector and recorder, was used to monitor the thermometer resistance. The voltage across the heater could also be displayed continuously on a chart recorder. The thermometer had a sensitivity of about 30 Ω/mK in the neighborhood of T_λ. Resistances could be measured to ±1 Ω or better, thus allowing one to measure relative temperatures to better than 30 μdeg. The resistances were calibrated against the He^4 vapor pressure. Over the present range of measurements $(T_\lambda - 0.0003) > T > (T_\lambda - 0.01)$ °K, dR/dT was essentially constant. Typical values of Q were between 1 and 100 mW/cm^2.

The observations were made as follows: A given heat current Q was switched on. The pumping rate was controlled so as to give a slow rate of warm-up and the temperature at one thermometer was

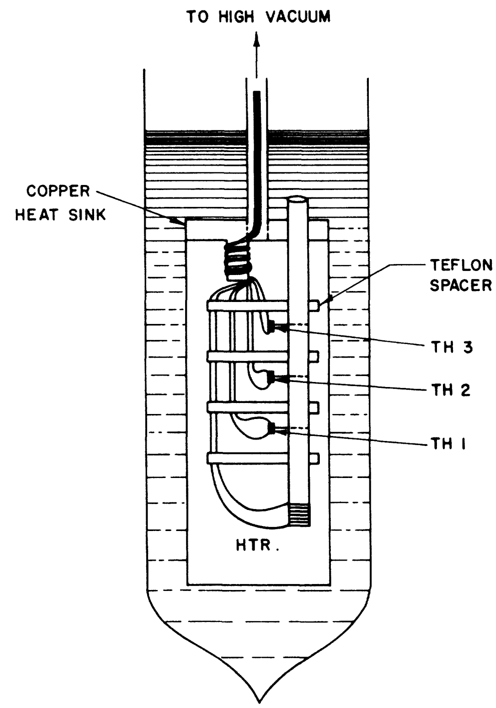

FIG. 2. Experimental schematic. Th1, Th2, Th3 are thermometers (1/8 W Allen-Bradley resistors). The heater at the bottom of tube is made of Karma wire (nominal resistance 700 Ω). The top of the stainless-steel tube is open directly to the helium bath.

recorded as a function of time. At frequent intervals the thermogram (i.e., T-versus-time curve) was obtained for no heat input. Such a thermogram has two parts, AB and BC [Fig. 3(a)]. Following the practice of several authors we identify the quiescent λ temperature from the flat part of this curve.[17] The value of R_λ (for $Q = 0$) drifts with time. To minimize the effects of the drift the thermometer bridges were left "on" for 12 to 14 h prior to taking any data points. All temperatures were deduced by reference to these values of R_λ. For a finite Q the thermogram has a completely different shape. Figure 3(b) shows the observations for two thermometers situated, respectively, 5 and 10 cm away from the heater. An initial linear increase DE, changes abruptly to a slower one, EF, and after a few minutes the temperature shows a sharp increase FG. Several features are worthy of note. (i) The duration of EF depends on the thermometer heater distance, l, being larger for larger l, and the heat input Q being smaller for larger Q. (ii) As Q is increased, considerable rounding appears at F and the change in slope at E is not so marked. (iii) *For all thermometers the points E are simultaneous*. (iv) Within limits of error of our experiment *T_F is the same for all thermometers*. The "rounding" of the curve at F makes the precision

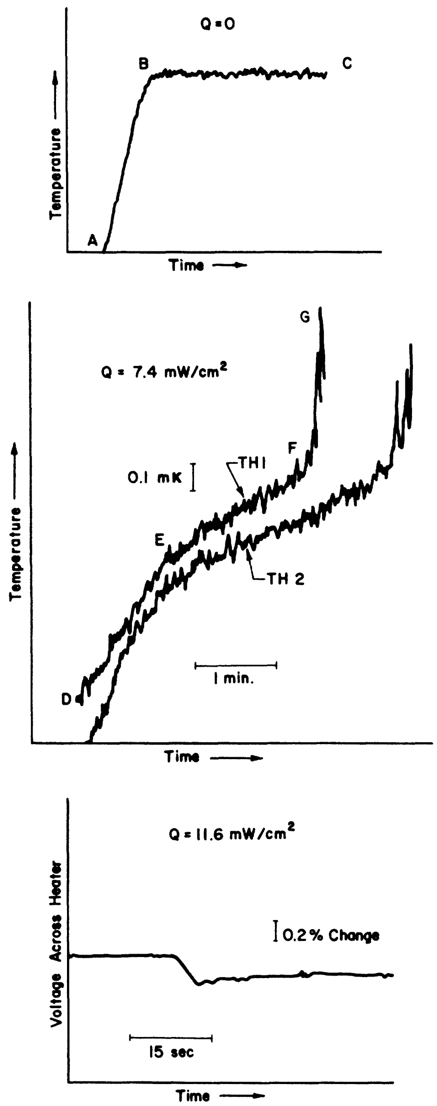

FIG. 3. (a) Thermogram for zero heat current ($Q=0$), shows the time variation of temperature at a typical thermometer as the helium bath is warmed up slowly through λ point. The flat portion of the curve *BC* gives quiescent T_λ. (b) Thermograms of two thermometers, Th1 and Th2, placed 5 and 10 cm away from heater for $Q=7.4$ mW/cm^2. Note that points *E* are simultaneous and that points *F* occur at the same temperature. (c) Change in heater voltage as a function of time. Note that the "jog" in voltage occurs simultaneously with point *E* in (b).

of these measurements somewhat poorer. T_F values are good to about 0.05 °K. (v) As a consequence of (i) and (ii) above, data taken for very small l (< 2 cm) and large Q are highly suspect and were rejected. In addition, several checks were made to ensure that none of the measurements depended in any way upon (a) the measuring current in the thermometer and (b) the exact rate of warm-up of the bath.

Simultaneous with the above, the voltage across the heater, (for a given current) was monitored as a function of time [Fig. 3(c)]. It is interesting to note that the sharp "jog" occurs *exactly at the same time* as the point *E* in Fig. 3(b). It is believed that this jog reflects a rapid change in the heater temperature. In hindsight, it was fortunate that the heater was made of Karma wire. Karma contains magnetic impurities and it turns out that for $T \sim T_\lambda$, the material has a sizable dR/dT. This was checked by making an independent set of dc resistance measurements.

III. RESULTS AND DISCUSSION

A. Q_c

As mentioned earlier, even for the smallest heat currents used by us ($\simeq 1$ mW/cm^2), a finite temperature gradient is observed. For a typical value of $Q = 25$ mW/cm^2 this temperature gradient is about 0.1 mK/cm. If the viscosity of the normal component was the only source of dissipation the expected gradients would be about three orders of magnitude smaller than the measured value.[18] Thus it is claimed that all the data described here pertain to the supercritical flow regime ($Q > Q_c$).

B. Thermogram

It is suggested that the thermograms in Fig. 3(b) be interpreted as follows: The point *E* marks the instant at which a drastic reduction in conductivity occurs in the immediate vicinity of the heater. This interpretation is confirmed by the type of observation shown in Fig. 3(c), i.e., simultaneous with *E* the heater exhibits a sudden rise in temperature. Admittedly, at the heater itself the total change in temperature will reflect (i) changes in the Kapitza resistance (R_K) at the transition point and (ii) rise of temperature associated with the possible appearance of a gas film. Thus one cannot use the change in the heater temperature to get a measure for the sudden drop in conductivity.

At later instants of time, i.e., during *EF*, the low-conductivity regime spreads through the fluid and the sharp rise observed at *F* happens when the "boundary" passes the thermometer being monitored.

Admittedly, the thermal conductance of the stainless-steel tube becomes comparable to that of He I (if convection is neglected) and thus over that length of the tube which is filled with He I the heat flow is not entirely through the fluid. However, during most of the interval *EF* the temperature is being monitored at a point which is still surrounded entirely by He II. Another source of difficulty may be the Kapitza resistance since this will make the tube wall warmer than the fluid. However, it is easy to show that this effect becomes important only when the boundary arrives within $\sim 10^{-2}$ cm of the thermometer. The two thermal resistances to be compared are the surface (solid-liquid helium II) resistance $R_K/2\pi al$ and the thermal resistance of the stainless wall $l/K_s 2\pi a\delta$ where δ is the wall thickness and a is the tube radius. Near T_λ the Kapitza resistance, $R_K \approx 1\ \text{cm}^2\,\text{K/W}$[19] and $K_s = 2\times 10^{-3}$ W.[20] For a 10-mil-thick stainless tube, the two will become comparable for $l \sim 10^{-2}$ cm. For longer lengths the Kapitza term is so small that no heat flows through the stainless. The sharp rise at F therefore must be related to the presence of a boundary in the fluid across which the sharp change of thermal conductivity causes a sizable temperature "jump."

The slowdown in the warming rate may be attributed to the fact that part of the applied heat is being used up to heat the He I locally and, perhaps, to cause bubbling. It is very difficult to make precise statements about this regime because it involves a number of unknown factors: degree of bubbling, convection, etc. However, appeal to experiment shows that for the following discussion it is not important to be able to catalog all these facts. The experiments show that whereas for a given Q the change in slope depends on the bath warm-up rate, thermometer position, and/or the channel diameter, the value of T_F does not. In fact, as shown in Fig. 4, T_F is the same for all the thermometers, i.e., the temperature at the boundary appears to be a unique function of the applied heat current. Admittedly, this is a very surprising

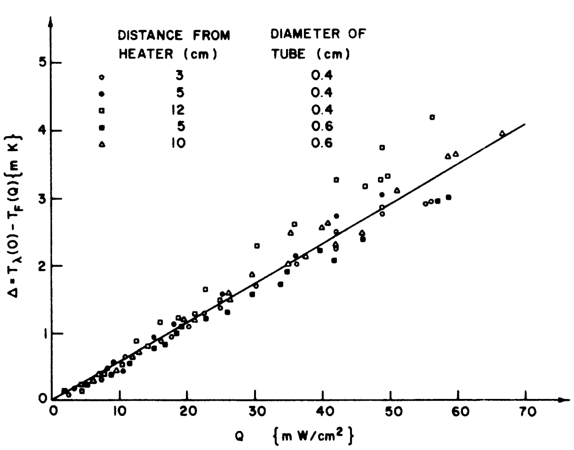

FIG. 4. Heat current Q versus temperature $T_F(Q)$ of the He II-He I transition. Each data point corresponds to point F, $T_F(Q) \equiv T_\lambda(Q)$, of Fig. 3(b) for a given Q. The various symbols represent data on different thermometers. Several data points have been left out for the sake of clarity. As discussed in the text, the full line is a least-squares fit to the data points.

result. We therefore feel justified in calling T_F a transition temperature. A least-squares fit to the data shows that

$$\Delta = T_\lambda(0) - T_F(Q) = 0.059Q, \tag{1}$$

for $0.1 < \Delta < 10$ mK. Here Q is in mW/cm^2. Several sets of data are shown in Fig. 4, the full line being given by Eq. (1). A somewhat similar set of measurements was reported by Erben and Pobell.[21] However, they did not describe their thermogram sufficiently and therefore a comparison of Eq. (1) with their data is not likely to be meaningful.

Next, one would like to elucidate the nature of T_F. We suggest that T_F represents the solution of the equation

$$\rho_s(Q, T_F) = 0, \tag{2}$$

that is, for a given Q, T_F represents the temperature at which the order parameter vanishes or the He II-He I transition takes place. That this should happen first of all in the warmest part of the fluid (i.e., near the heater) is obvious enough. In this sense $T_F(Q) \equiv T_\lambda(Q)$, and it is claimed that Eq. (1) represents the manner in which the transition temperature is depressed by a heat current in the fluid.

As noted in the Introduction all of the phenomenological theories of superfluidity at T close to T_λ predict a depression in T_λ in the presence of a counterflow velocity. However, before one can expect to compare Eq. (1) with any of the theoretical expressions, one has to ask oneself the question: How are the predictions of the theory modified if one has vorticity in the superfluid? Can one still talk of a thermodynamic critical current and a w_c' (cf. Fig. 1)? It is suggested that even when vorticity is present the general structure of the thermodynamic solution should be the same. This can be seen as follows. Following Ginzburg and Pitaevskii[9] the free energy near T_λ may be written as

$$F_s - F_n = \Delta F = \int -\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\hbar^2}{2m}|\vec{\nabla}\psi|^2 d\tau. \tag{3}$$

As before, the macroscopic variables ρ_s and v_s are to be defined through the relations $\rho_s = m|\psi|^2$, $v_s = (\hbar/m)\nabla\Phi$ where $\psi = \chi e^{i\Phi}$. As a starting point one wishes to consider the case of uniform superfluid velocity, $v_s\hat{z}$ in a cylindrical tube of radius a and ask the question: If a vortex line of circulation $\kappa\hat{z}$ centered on the tube axis is introduced, what is the change in the free energy of the system? The superfluid velocity components will be

$$[\kappa\sin\theta/r,\ \kappa\cos\theta/r,\ v_s]\ .$$

Further, the presence of the vortex line should alter the order parameter locally and this is taken into account by writing

$$\psi = \chi(r)e^{i\Phi(r,\theta)}\ ,$$

with

$$\chi(r)=\chi_\infty\,[1-e^{-(r-a_0)/\xi}],\quad r>a_0$$
$$\chi(r)=0,\qquad r\le a_0, \tag{4}$$

where a_0 is the so-called core radius of the vortex, and ξ has the meaning of a correlation length. In the present case, ρ_s becomes a function of position with the following form:

$$\rho_s=m\,\chi_\infty^2,\quad r\gg a_0$$
$$\rho_s=0,\qquad r\le a_0. \tag{5}$$

Substituting in Eq. (3) and integrating over the volume one gets

$$\Delta F_1=-\,\alpha\chi_\infty^2\,[(v-v_0)-\pi l a_0\xi(3+7\xi/2a_0)]$$
$$+\tfrac{1}{2}\beta\chi_\infty^4\,[(v-v_0)-0(\xi^2 l)]$$
$$+\rho_s\frac{v_s^2}{2}\,[v-v_0-0(\xi^2 l)]+\rho_s\pi l\frac{\hbar^2}{m^2}\left[\frac{a_0}{2\xi}+\frac{1}{4}\right]$$
$$+\rho_s\kappa^2\pi l\left[\ln\frac{a}{a_0}-2P\left(\frac{a_0}{\xi}\right)+P\left(\frac{2a_0}{\xi}\right)\right], \tag{6}$$

where $v_0=\pi a^2 l$ (l being the length of the tube) and

$$P(\lambda)=e^{\lambda}\left(-0.58-\ln\lambda-\sum_{n=1}\frac{(-)^n\lambda^n}{n\,n!}\right).$$

Generalization of the above to the case when an arbitrary distribution of vortex lines is present in the fluid is algebraically a very difficult problem. A possible simplification is to consider N lines, each of strength $\kappa\hat{z}$, with their axes parallel to the tube axis. In this case (6) can be generalized in a straightforward way to read

$$\Delta F_N=\alpha\chi_\infty^2\left[(v-Nv_0)-N\pi l\,a_0\xi\;\;3+\frac{7}{2}\frac{\xi}{a_0}\right]$$
$$+\frac{\beta}{2}\chi_\infty^4\,[(v-Nv_0)-N\,0(\xi^2 l)]$$
$$+\frac{\rho_s v_s^2}{2}\,[(v-Nv_0)-N0(\xi^2 l)]+\rho_s\pi l\frac{\hbar^2}{m^2}N\left[\frac{a_0}{2\xi}+\frac{1}{4}\right]$$
$$+\rho_s\kappa^2 N\pi l\left[\ln\frac{a}{N^{1/2}a_0}-2P\left(\frac{a_0}{\xi}\right)+P\left(\frac{2a_0}{\xi}\right)\right]. \tag{7}$$

Measurements on ρ_s near T_λ imply[22]

$$\xi=0.45\,(T_\lambda-T)^{-2/3}\ \text{Å}.$$

Usually, the core radius $a_0\approx\xi$ and $\kappa=\hbar/m\sim10^{-4}$ cm^2/sec. For typical heat currents used in the experiments $v_s\sim1$ cm/sec. Using these values, preliminary calculations[23] suggest that if $N\sim10^{13}$ $(T_\lambda-T)^{4/3}$ the calculated Δ is very close to that given by Eq. (1).

The above discussion will break down for $(T_\lambda-T)$ $\lesssim10^{-8}$ °K where ξ will become large in comparison with atomic dimensions. In that temperature regime, the behavior of the superfluid will, in many ways, be quite analogous to that of a type-II superconductor.

One question remains to be answered: Why do we suggest that T_F is a solution of Eq. (2) rather than claiming that T_F should be given by the equation $\partial Q/\partial w=0$? A closer look at the experimental situation helps to clarify this point. It can be shown quite easily that once the system is driven past the point $\partial Q/\partial w=0$, ρ_s is likely to fall to zero precipitously (cf. p. 73 of Ref. 14). From the experimental point of view, therefore, there is no way of distinguishing between the position of the maximum and the point where $\rho_s(Q,T)=0$. Thus T_F must be very close to, if not exactly, the solution of Eq. (2).

Closely allied to the above discussion is the question of the kinds of processes that control thermal conduction in the superflow regime. We are at present attempting to analyze this aspect of the work and expect to publish it shortly.

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Ground State of a One-Dimensional Many-Boson System

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The ground state of a one-dimensional system of many bosons interacting through a repulsive δ-function potential is studied when the coupling parameter γ of the system is small. The procedure is based on the variational analysis in the Bijl-Dingle-Jastrow wave-function space. The ground-state energy, determined in a series form in powers of $\gamma^{1/2}$ and truncated after the third leading term, is found to be in fair agreement with exact results of Lieb and Liniger over a range of the coupling parameter extending up to $\gamma = 6$. Two slightly different approaches are considered, and they are shown to be equivalent.

I. INTRODUCTION

The one-dimensional problem of many bosons with repulsive δ-function interactions has been investigated extensively by Lieb and Liniger[1] and by Lieb[2] throughout the entire range of the coupling parameter γ, which is proportional to the ratio of the interaction strength to the number density. The same problem in the limit of infinite coupling ($\gamma = \infty$), i.e., one-dimensional system of impenetrable bosons, was studied earlier by Girardeau,[3] who pointed out that the energy spectrum in this limit is identical to that of an ideal fermion system. Although no real physical system is known to resemble this model, the problem is interesting in that it is one of only a very few many-body model problems whose solutions have been found in the entire domain of the coupling parameter. Thus, it provides theorists with a testing ground of many-body theories at arbitrary values of the coupling parameter.

In this paper, we present a variational study on the ground state of the one-dimensional many-boson system in the weak-coupling limit, using the method of series expansion based on a Bijl-Dingle-Jastrow (BDJ) correlated wave function.[4-6] The essence of the procedure consists of (i) development of a cluster-type expansion for the expectation value of the Hamiltonian $\langle H\rangle$ in terms of the liquid-structure function $S(k)$, and (ii) minimization of the expectation value $\langle H\rangle$ with respect to the indirect variational function $S(k)$. In Sec. II it is shown that the resulting ground-state energies, with truncation of third- and higher-order terms, agree quite closely with exact numerical results of Lieb and Liniger[1] over a wider range of γ values than the range where the Bogoliubov approximation[7] is valid. In Sec. III the problem is reexamined in a double-variational formalism, which proves to be equivalent but particularly helpful in understanding the physical basis of the relation $T_1/U_1 = -\frac{1}{3}$, where T_1 and U_1 are the first-order terms of the kinetic and potential energies, respectively.

II. GROUND-STATE ENERGY

We consider a system of N bosons interacting in a one-dimensional space of length L through a repulsive δ-function potential

$$v(x) = 2c\delta(x), \quad c > 0. \tag{1}$$

The end effects may be neglected by considering the ordinary many-body limit $N \to \infty$, $L \to \infty$ while the number density $\rho = N/L$ remains finite. The Hamiltonian of the system is

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + 2c\sum_{i<j}^{N}\delta(x_{ij}), \tag{2}$$

where

$$x_{ij} = x_i - x_j.$$