shed additional light onto the phenomena associated with the ^{3}He and the ^{4}He liquid-solid transition. We hope to revert to this problem in subsequent work.

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Tunable Far-Infrared Radiation Generated from the Difference Frequency between Two Ruby Lasers

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Far-infrared radiation generated from the difference frequency between two temperature-tuned ruby lasers operated on the R_1 and R_2 transitions has been observed. This radiation is continuously tunable over the frequency range 20–38 cm^{-1}. Lithium niobate was used as a mixing crystal. The expected frequencies of the far-infrared radiation were measured using a Fabry-Perot interferometer. The phase-matching conditions were also verified.

In a previous paper, we described the generation of tunable far-infrared radiation over the frequency range 1.5–8.1 cm^{-1} by beating two temperature-tuned ruby laser beams in a nonlinear crystal.[1] By operating one laser on the R_1 transition and the other on the R_2 transition, we have now obtained tunable radiation in the range 20–38 cm^{-1}. In order to produce laser action on the R_2 transition,[2–4] we used essentially a Lyot-Ohman[5,6] filter in the laser cavity to discriminate against the R_1 transition.[7] The power output of the R_2 laser was about $\frac{1}{2}$ MW, compared with the 1-MW output of the R_1 laser. Lithium niobate was used for the nonlinear crystal. The additional experimental apparatus and the measurement technique were essentially the same as described earlier.[1]

If both lasers are operated at room temperature, the difference-frequency radiation generated should be at 29 cm^{-1}. In order to phase-match this difference-frequency generation process, a 1.5-mm slice of $LiNbO_3$ was cut with the c axis tilted approximately 18° away from the normal to the surface. The frequency of the far-infrared output was measured using a Fabry-Perot interferometer with electroformed metal mesh mirrors. The measured transmission curve[8] of the Fabry-Perot interferometer is compared in Fig. 1(a) with a theoretical curve calculated for a frequency of 28.8 cm^{-1}. Since the interferometer has a finesse of about 4, the spectral purity of the far-infrared radiation could not be measured. In Fig. 1(b) the Fabry-Perot transmission is shown for radiation generated with the R_1 laser at room temperature and the R_2 laser at − 23 °C. The calculated transmission for 35.8 cm^{-1}, the frequency expected from the known temperature dependence of the R_2 transition, is also shown.[9] The same $LiNbO_3$ crystal was used, but it was oriented for phase matching at 35.8 cm^{-1}.

The phase-matching curve for production of the difference frequency at 29 cm^{-1} is shown in Fig. 2. The absorption coefficient for $LiNbO_3$ at 29 cm^{-1} is 18 cm^{-1}.[10] Normalized theoretical curves obtained by solving Maxwell's equations in the plane-wave approximation, with and without absorption,[1,11] are shown for comparison. The effect of absorption changes the width at half-maximum very little, but shows a definite difference at the wings of the curves. The shapes of the curves would not be changed appreciably by including diffraction and boundary conditions. The observed peak power in this case was about 5 mW, compared with a previously observed value of ~1 mW at 8.1 cm^{-1} from a 0.47-mm-thick crystal.[1]

From the simple theory with plane-wave approximation, we expect an ω^2 dependence of the far-infrared output power on frequency. Since the extinction length at 29 cm^{-1} is 0.055 cm, as compared to crystal length of 0.47 mm in the 8.1-cm^{-1} case, we estimate that the power at 29 cm^{-1} should be about 15 times the power at 8.1 cm^{-1} for the same laser power. Computer calculations[11] including the effects of absorption, diffraction, boundary conditions, the spatial distribution and mode structure of the laser light, the birefringent property of the $LiNbO_3$ crystal, and the collection angle of the detector give a factor of 12. Since the power of the R_2 transition laser is a factor of about two less than that of the R_1 transition, the complete theory predicts a factor of 6 compared to the factor 5 observed. This result seems satisfactory considering the uncertainties in our experiment.[11] The absolute value of the power obtained at 8.1 cm^{-1} was shown to agree with theory in the previous paper.

The tunable range of the far-infrared radiation produced by the beating between R_1 and R_1 lasers and between R_1 and R_2 lasers could be easily extended from 0 to 50 cm^{-1} by using liquid nitrogen as a coolant. Other systems might enlarge the range of tunability. Many tunable laser sources exist which can be used to produce difference frequency radiation. Among the more promising are the dye lasers,[12] the spin-flip Raman lasers,[13] stimulated polariton scattering,[14] and parametric oscillators.[15] A system using two dye lasers could

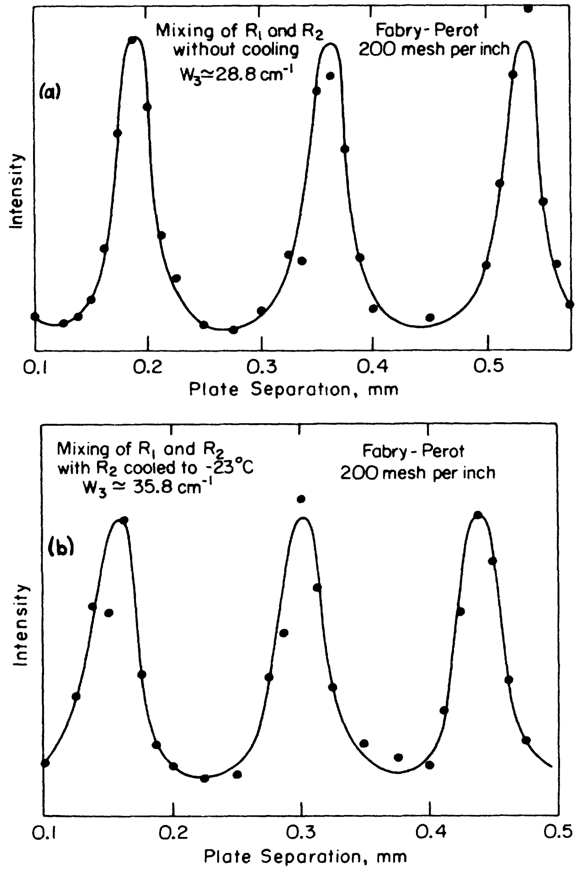

FIG. 1. Fabry-Perot scan of difference-frequency output using R_1 and R_2 lines. The first scan (a) shows a difference frequency of 28.8 cm^{-1}; the second scan (b) shows 35.8 cm^{-1}. The solid curve is calculated from the dimensions of the wire-grid interferometer and from the difference frequency expected from the laser temperature.

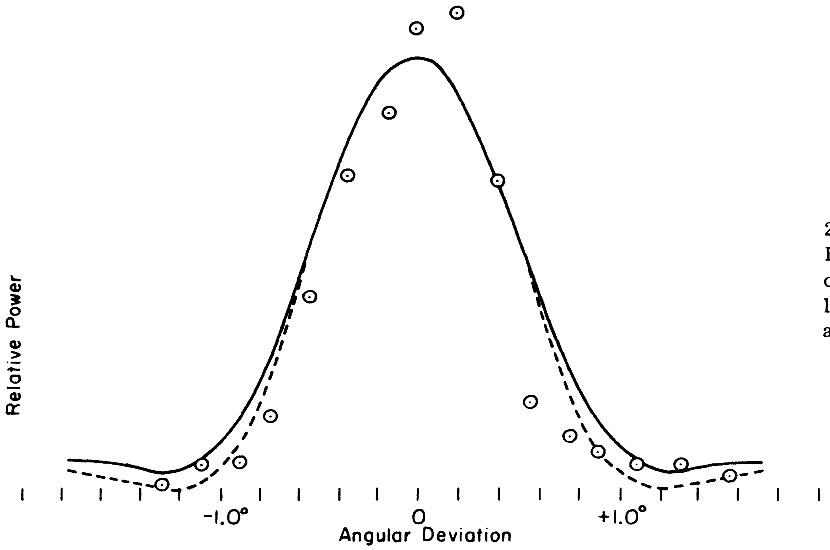

FIG. 2. Phase-matching curve for 29 cm^{-1} in 0.15-cm $LiNbO_3$ crystal. Dashed line is $[(\sin\eta)/\eta]^2$, the theoretical curve for no absorption. Solid line is the theoretical curve including an absorption coefficient of $\alpha = 18\ \text{cm}^{-1}$.

be tuned over the entire far-infrared region. The power, linewidth, and divergence will perhaps never be as good as the ruby laser, but they may be adequate for some applications. Improvements in the ruby-laser sources, particularly the divergence, should produce a significant increase in far-infrared power. We can also increase the power by choosing the optimum focusing of the lasers into the crystal, provided that damage can be avoided. It seems clear that this tunable far-infrared radiation source could be used for spectroscopy in the 1–50-cm^{-1} region, especially for saturation and other nonlinear phenomena which require large peak power.

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ERRATA

Modulated Exponential Decay in a Two-Level Quantum System, O. A. Keller and R. T. Robiscoe [Phys. Rev. 188, 82 (1969)]. In Eq. (7), the expressions on the right-hand side should have the coefficient $(4\mu^2/N^4)$ rather than $(4\mu^2/N^2)$. This error does not propagate through the rest of the paper; in particular, the results in Eq. (43) are correct, to order μ^2. Also, in Eq. (34), the left-hand side should read $\chi(t)$ rather than $x(t)$. This typographical error has no consequence.

Correlation Effects on Hyperfine-Structure Expectation Values for the Boron ^2P Ground State, R. E. Brown, S. Larsson, and V. H. Smith, Jr. [Phys. Rev. A 2, 593 (1970)]. $B''^{1,2}(q=2.0023)$ on the line above Eq. (5a) should be replaced by $\mathrm{B}^{11}(g=2.0023)^{1,2}$. q in Eqs. (5a) and (5b) should be replaced by g.

The expression for $A_{1/2}$ is the right-hand column of p. 597 and the two sentences that follow ("The results are ... above procedure.") should be replaced by the sentence: The result is $f'=0.096a_0^{-3}$. In the expression for $A_{3/2}$, q should be replaced by g.

The number 0.037 just before "CONCLUSIONS" should be replaced by 0.042.

In the expression for d' in the right-hand column of p. 597, l should be replaced by d.

Large-Scale Self-Focusing of Optical Beams in the Paraxial Ray Approximation, W. G. Wagner, H. A. Haus, and J. H. Marburger [Phys. Rev. 175, 256 (1968)]. The axes of Figs. 3–5 are labeled incorrectly. The correct labelings are

FIG. 3: $(\sqrt{\tfrac{4}{15}})\eta_m$ vs $(\sqrt{\tfrac{4}{15}})\eta_0$,

FIG. 4: ξ_f/η_0^2 vs $\tfrac{4}{15}\eta_0^2$,

FIG. 5: $\tfrac{4}{15}\eta^2$ vs ξ/η_0^2.

Classical Approximation for Ionization by Proton Impact, J. D. Garcia, E. Gerjuoy, and J. E. Welker [Phys. Rev. 165, 66 (1968)]. Equation (2) in our paper contains a typographical error: The