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Analysis of Scattering and Excitation Amplitudes in Polarized-Electron-Atom Collisions. I. Elastic Scattering on One-Electron Atoms and the Excitation Process $^2S_{1/2} \rightarrow {}^2P_{1/2,3/2}$†

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Scattering amplitudes are analyzed for the scattering of polarized or unpolarized electrons on polarized or unpolarized atoms (one-electron atoms). The elastic scattering of unpolarized electrons on partially polarized atoms provides the direct and exchange cross sections when the polarization of the scattered electrons and atoms are observed, respectively. The same quantities can also be obtained from the scattering of partially polarized electrons on unpolarized atoms, while the difference between the phases of the direct and the exchange amplitude can be derived from the spin analysis with polarized electrons and polarized atoms. When the degree of polarization of the colliding particles is known, the intensity of the scattered collision particles determines the interference term of the elastic cross section. The excitation process $^2S_{1/2} \rightarrow {}^2P_{1/2,3/2}$, carried out with any combination of polarized and unpolarized electrons and atoms, has been analyzed in terms of differential and integral excitation amplitudes distinguished in terms of direct, exchange, and interference processes and also in terms related to the excitation of the magnetic substates $m_l = 0$ or ± 1. The analysis of the polarization of the inelastically scattered electrons gives differential excitation amplitudes for the direct, exchange, and interference terms of the excitation cross section without distinguishing between the magnetic substates. Based upon the observations of the intensity and the circular polarization of the emitted line radiation, the analysis of the $^2P_{1/2,3/2} \rightarrow {}^2S_{1/2}$ deexcitation process provides integral excitation amplitudes. Observation of the total intensity of the $^2P_{1/2,3/2} \rightarrow {}^2S_{1/2}$ transition in an excitation process with polarized electrons and polarized atoms enables one to determine interference terms of the excitation cross section. Equal polarization for the exciting electrons and target atoms allows us to separate the interference amplitudes for the magnetic substates ($|F_1 - G_1|$ and $|F_0 - G_0|$). Analysis of the circular polarization of the $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition excited with unpolarized electrons and partially polarized atoms results in partial cross sections describing excitation processes in which the quantum number m_j of the excited state differs from that of the ground state at most in sign. Excitation with polarized electrons and unpolarized atoms should also result in the emission of circular polarized light of the $P \rightarrow S$ transitions. In this context Farago's proposal for measuring electron-spin polarization by circularly polarized line excitation is discussed as it applies to alkali atoms. Finally, it is pointed out how sets of integral excitation amplitudes determined by collisions, as discussed above, can be used for comparing single-excitation amplitudes with theory.

I. INTRODUCTION

Recent advances[1] in the technique of producing polarized electrons and atoms should allow more detailed studies of the different types of interactions occurring in electron-atom collisions. The usual experiments determining differential and total cross sections simply result in some kind of averaging over several different interactions of the scattering process. This paper deals with an analysis of scattering and excitation amplitudes when studies of the elastic and inelastic scattering are carried out by all combinations of scattering of polarized electrons on unpolarized atoms, unpolarized electrons on polarized atoms, and polarized electrons on polarized atoms. We restrict ourselves to one-electron atoms; a subsequent paper will discuss a scattering analysis with two-electron systems and also special cases for scattering of polarized electrons with ions. Based upon the density matrix formalism, the theory of elastic spin exchange of electrons by spin $-\frac{1}{2}$ atoms was developed by Burke and Schey,[2] and has been applied to the scattering of electrons by lithium atoms.[3] A recent paper by Rubin *et al.*[4] deals with the special application of a spin analysis in electron–alkali-metal inelastic recoil experiments with polarized atoms. A short survey of elastic scattering experiments with polarized electrons and atoms has also been given by Bederson.[5]

II. SCATTERING AND EXCITATION AMPLITUDES

A. Elastic Scattering

We begin by defining scattering amplitudes for the case in which the colliding electrons and atoms are completely polarized. We restrict ourselves to one-electron atoms and we exclude the following interactions: spin-orbit interactions (to be taken into account only for heavy atoms), scattering interactions producing longitudinal polarization (parity violation), and spin flips without electron exchange between the projectile and the atomic electron [e.g., the collision process with polarized electrons $e'(\uparrow)$ and polarized atoms $A(\downarrow)$: $e(\uparrow)+A(\downarrow)\to e'(\downarrow)+A(\uparrow)$]. The remaining types of scattering processes to be included are

$$e(\downarrow)+A(\uparrow)\to A(\uparrow)+e(\downarrow)\,,\quad \text{scattering amplitude } f, \tag{1}$$

$$e(\downarrow)+A(\uparrow)\to A(\downarrow)+e(\uparrow)\,,\quad \text{scattering amplitude } g, \tag{2}$$

$$e(\uparrow)+A(\uparrow)\to A(\uparrow)+e(\uparrow)\,,\quad \text{scattering amplitude } f-g\,. \tag{3}$$

The cross sections for these scattering processes are then equal to the square of the magnitude of the scattering amplitudes: We call f the direct, g the exchange, and $f-g$ the interference or triplet amplitude ($f-g={}^3F$).

The relation of these amplitudes to the differential cross section $\sigma(\theta,\phi)$ for the scattering with unpolarized colliding particles $[e(\uparrow\downarrow)+A(\uparrow\downarrow)]$ is given by

$$\sigma(\theta,\phi)=\tfrac{1}{2}|f|^2+\tfrac{1}{2}|g|^2+\tfrac{1}{2}|f-g|^2 = \tfrac{1}{4}|f+g|^2+\tfrac{3}{4}|f-g|^2=\tfrac{1}{4}|{}^1F|^2+\tfrac{3}{4}|{}^3F|^2\,, \tag{4}$$

where ${}^1F$ is the singlet scattering amplitude.

For simplicity we may write Eq. (4) as

$$\sigma(\theta,\phi)=\sigma_d(\theta,\phi)+\sigma_{ex}(\theta,\phi)+\sigma_{int}(\theta,\phi) \tag{5}$$

and call $\sigma_d(\theta,\phi)=\frac{1}{2}|f|^2=\sigma_d$ the direct cross section, $\sigma_{ex}(\theta,\phi)=\frac{1}{2}|g|^2=\sigma_{ex}$ the exchange cross section, and $\sigma_{int}(\theta,\phi)=\frac{1}{2}|f-g|^2=\sigma_{int}$ the interference cross section. It follows from Eqs. (1)–(3) and from a more detailed discussion in Sec. III that the polarized-electron–atom techniques provide a tool for determining these partial cross sections.

B. Inelastic Scattering

The details of the theory for the analysis of scattering amplitudes related to inelastic scattering processes are described extensively in the literature.[6,7]

From a practical point of view one is primarily interested in the excitation and deexcitation process $S\to P\to S$ (excitation to a P state and subsequent transition from the excited P state to an S state). We therefore restrict our discussion in detail to that particular excitation process.

It follows from Percival and Seaton[8] that the total cross section $Q(S\to P)$ of the excitation $S\to P$ can be expressed in terms of cross sections for the components of the initial angular momenta of the excited state:

$$Q(S\to P)=Q(S\to P)_{m_l=0}+2Q(S\to P)_{m_l=\pm1}\,, \tag{6}$$

with

$$Q(S\to P)_{m_l=+1}=Q(S\to P)_{m_l=-1}\ .$$

As in the case of the elastic scattering, the excitation process can also be described in terms of scattering amplitudes. General expressions of such scattering amplitudes, including the inelastic process, are, e.g., discussed in Burke's paper[6] [see in particular Eq. (I.12) of Burke's paper]. It follows that the excitation process $S\to P$ of a one-electron atom can also be described in terms of direct (F) and exchange (G) amplitudes. The partial cross sections for the orbital angular momentum components are to be expressed by the amplitudes F_0 and G_0 for $Q(S\to P)_{m_l=0}$, and F_1 and G_1 for $Q(S\to P)_{m_l=\pm1}$, respectively. In the same way as for elastic scattering[8] it can be shown that

$$Q(S\to P)_{m_l=0}=\tfrac{1}{2}|F_0|^2+\tfrac{1}{2}|G_0|^2+\tfrac{1}{2}|F_0-G_0|^2 = \tfrac{3}{4}|F_0-G_0|^2+\tfrac{1}{4}|F_0+G_0|^2\,, \tag{7}$$

$$Q(S\to P)_{m_l=\pm1}=\tfrac{1}{2}|F_1|^2+\tfrac{1}{2}|G_1|^2+\tfrac{1}{2}|F_1-G_1|^2 = \tfrac{3}{4}|F_1-G_1|^2+\tfrac{1}{4}|F_1+G_1|^2\ . \tag{8}$$

The total cross section is then

$$Q(S\to P)=\tfrac{1}{2}|F_0|^2+|F_1|^2+\tfrac{1}{2}|G_0|^2+|G_1|^2 +\tfrac{1}{2}|F_0-G_0|^2+|F_1-G_1|^2\ . \tag{9}$$

Analogous to the elastic scattering (Sec. II A) we call $\frac{1}{2}|F_0|^2+|F_1|^2=Q_d(S\to P)$ the direct excitation cross section, $\frac{1}{2}|G_0|^2+|G_1|=Q_{ex}(S\to P)$ the exchange excitation cross section, and $\frac{1}{2}|F_0-G_0|^2+|F_1-G_1|^2=Q_{int}(S\to P)$ the interference excitation cross section. Relations (6)–(8) remain valid even if the atoms show fine- and hyperfine-structure splitting. As will be seen in Sec. IV the analysis of the excitation process carried out by polarized electrons and atoms will be affected by the fine- and hyperfine-structure interaction. In our description of the excitation process $S\to P$ we will take into account the fine-structure separation (it can even be used to simplify the analysis of the excitation process), while the complication caused by the hyperfine structure can be overcome by some special experimental arrangements (e.g., by decoupling the hyperfine-structure by an external magnetic field, or preferably by observing the inelastically scattered electrons).

We can also express the total excitation cross section in terms of the cross sections for the fine-structure states:

$$Q(S\to P)=Q({}^2S_{1/2}\to{}^2P_{1/2})+Q({}^2S_{1/2}\to{}^2P_{3/2}). \tag{10}$$

The fine-structure cross sections are proportional to their statistical weights[7]:

$$Q({}^2S_{1/2}\to{}^2P_{1/2})=\tfrac{1}{3}Q(S\to P) =\tfrac{1}{3}Q(S\to P)_{m_l=0}+\tfrac{2}{3}Q(S\to P)_{m_l=\pm1}\,,$$

$$Q({}^2S_{1/2}\to{}^2P_{1/2})=\tfrac{1}{6}|F_0|^2+\tfrac{1}{3}|F_1|^2+\tfrac{1}{6}|G_0|^2+\tfrac{1}{3}|G_1|^2 +\tfrac{1}{6}|F_0-G_0|^2+\tfrac{1}{3}|F_1-G_1|^2$$

$$=\tfrac{1}{3}(Q_d+Q_{\mathrm{ex}}+Q_{\mathrm{int}})\,, \tag{11}$$

$$\begin{aligned} Q({}^2S_{1/2}\to{}^2P_{3/2}) &= \tfrac{2}{3}Q(S\to P) \\ &= \tfrac{2}{3}Q(S\to P)_{m_l=0}+\tfrac{4}{3}\,Q(S\to P)_{m_l=\pm1}\,, \end{aligned}$$

$$\begin{aligned} Q({}^2S_{1/2}\to{}^2P_{3/2}) &= \tfrac{1}{3}\,|F_0|^2+\tfrac{2}{3}\,|F_1|^2+\tfrac{1}{3}\,|G_0|^2+\tfrac{2}{3}\,|G_1|^2 \\ &\quad +\tfrac{1}{3}\,|F_0-G_0|^2+\tfrac{2}{3}\,|F_1-G_1|^2 \\ &= \tfrac{2}{3}(Q_d+Q_{\mathrm{ex}}+Q_{\mathrm{int}})\,. \end{aligned} \tag{12}$$

Describing the excitation process in terms of differential cross sections, the following relations between the differential excitation amplitudes (f_0, f_1) and (g_0, g_1) and the differential cross sections (including the partial differential cross section σ_{m_l}) are then given by

$$\begin{aligned} \sigma(S\to P) &= \sigma({}^2S_{1/2}\to{}^2P_{1/2})+\sigma({}^2S_{1/2}\to{}^2P_{3/2}) \\ &= \sigma(S\to P)_{m_l=0}+2\sigma(S\to P)_{m_l=1} \\ &= \tfrac{1}{2}|f_0|^2+|f_1|^2 \qquad (=\sigma_d) \\ &\quad +\tfrac{1}{2}|g_0|^2+|g_1|^2 \qquad (=\sigma_{\mathrm{ex}}) \\ &\quad +\tfrac{1}{2}|f_0-g_0|^2+|f_1-g_1|^2 \quad (=\sigma_{\mathrm{int}})\,, \end{aligned} \tag{13}$$

$$\sigma({}^2S_{1/2}\to{}^2P_{3/2})=\tfrac{2}{3}(\sigma_d+\sigma_{\mathrm{ex}}+\sigma_{\mathrm{int}})\,, \tag{14}$$

$$\sigma({}^2S_{1/2}\to{}^2P_{1/2})=\tfrac{1}{3}(\sigma_d+\sigma_{\mathrm{ex}}+\sigma_{\mathrm{int}})\,. \tag{15}$$

Transformation of Eqs. (13)–(15) to those of Eqs. (11) and (12) is to be carried out by means of the relations between the integral and the differential amplitudes ($i=0$ or 1, θ is the scattering angle of the inelastically scattered electron):

$$F_i=2\pi\int_0^\pi f_i(\theta)\,\sin\theta\,d\theta\;,\quad G_i=2\pi\int_0^\pi g_i(\theta)\,\sin\theta\,d\theta\;.$$

Using completely polarized electrons and atoms, the excitation processes ${}^2S_{1/2}\to{}^2P_{1/2,3/2}$ or ${}^2S_{1/2}\to{}^2P_{1/2}$ or ${}^2S_{1/2}\to{}^2P_{3/2}$ can then be described by means of the amplitudes defined above:

$$e(\uparrow)+A({}^2S_{1/2},\uparrow)\to A({}^2P_{1/2,3/2},\uparrow)+e(\uparrow)\,,$$
$$|f_0-g_0|^2+2|f_1-g_1|^2\,, \tag{16}$$

$$e(\uparrow)+A({}^2S_{1/2},\downarrow)\to A({}^2P_{1/2,3/2},\downarrow)+e(\uparrow)\,,$$
$$|f_0|^2+2|f_1|^2\,, \tag{17}$$

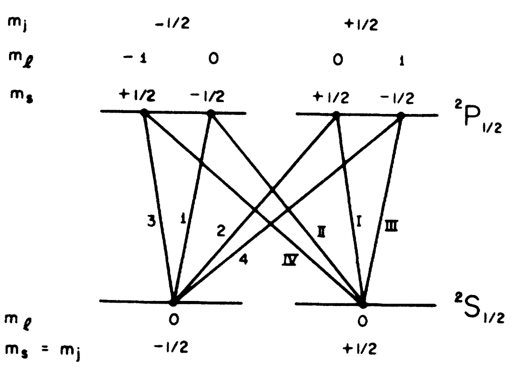
$$e(\uparrow)+A({}^2S_{1/2},\downarrow)\to A({}^2P_{1/2,3/2},\uparrow)+e(\downarrow)\,,$$
$$|g_0|^2+2|g_1|^2\,. \tag{18}$$

The corresponding excitation cross sections for the separated excitation of the fine-structure levels ${}^2P_{3/2}$ and ${}^2P_{1/2}$ are the same as in Eqs. (16)–(18) multiplied, however, by the factors $\frac{2}{3}$ and $\frac{1}{3}$, respectively.

Figures 1 and 2 show how the excitation amplitudes are related to the excitation processes for transitions from a given magnetic sublevel of the ground state to a magnetic sublevel m_l of the excited states ${}^2P_{1/2}$ or ${}^2P_{3/2}$. It should be noted that the coefficients of these amplitudes are correct for excitation of unpolarized atoms by unpolarized electrons. These coefficients can be derived from the expression relating the cross section for fine-structure states to those for the excitation of the components of the orbital angular momentum.[7]

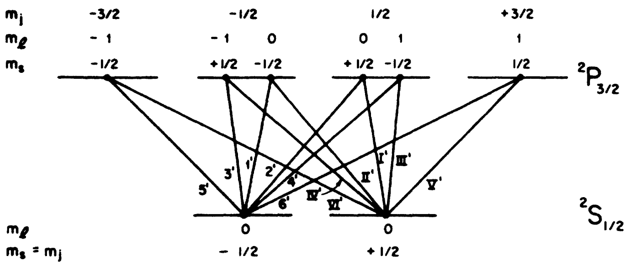
III. ELASTIC SCATTERING WITH POLARIZED ELECTRONS AND ATOMS

Figure 3 illustrates a crossed-beam experiment with partially polarized electrons and atoms intersecting the interacting volume. The quantization axis is assumed parallel to the Z axis; accordingly the direction of the spin polarization of the colliding particles before the collision is transverse. We restrict the observation of the scattered particles to the scattering plane (azimuthal angles $\phi=0$ or $\chi=0$), which has the consequence that the polarization of the scattered particles remains transverse. The density of the atoms in the interaction volume may be divided into two parts, one part n_p which will be completely polarized and the other part n_u, completely unpolarized. If the total atomic density is $n=n_p+n_u$, and the degree of polarization of the atoms is $n_p'=n_p/n=P_A$, then $n_u'=n_u/n=1-P_A$. The flux density j of the incoming electrons can similarly be divided into one completely polarized part j_p and one unpolarized part j_u : $j=j_p+j_u$, $j_p'=j_p/j=P_e$



1 or I	$\frac{1}{12}\,\lvert f_0\rvert^2+\frac{1}{12}\,\lvert f_0-g_0\rvert^2$
2 or II	$\frac{1}{12}\,\lvert g_0\rvert^2$
3 or III	$\frac{1}{6}\,\lvert g_1\rvert^2$
4 or IV	$\frac{1}{6}\,\lvert f_1\rvert^2+\frac{1}{6}\,\lvert f_1-g_1\rvert^2$

FIG. 1. Two magnetic sublevels of the ${}^2S_{1/2}$ ground state and the excited fine-structure state ${}^2P_{1/2}$ labeled by their quantum numbers m_j, m_l, and m_s. The table on the right gives the excitation amplitudes squared times the intensity coefficients of the different transitions when the excitation is carried out with unpolarized electrons and atoms.



1′ or I′	$\frac{1}{6}\lvert f_0\rvert^2 + \frac{1}{6}\lvert f_0 - g_0\rvert^2$
2′ or II′	$\frac{1}{6}\lvert g_0\rvert^2$
3′ or III′	$\frac{1}{12}\lvert g_1\rvert^2$
4′ or IV′	$\frac{1}{12}\lvert f_1\rvert^2 + \frac{1}{12}\lvert f_1 - g_1\rvert^2$
5′ or V′	$\frac{1}{4}\lvert f_1\rvert^2 + \frac{1}{4}\lvert f_1 - g_1\rvert^2$
6′ or VI′	$\frac{1}{4}\lvert g_1\rvert^2$

FIG. 2. Magnetic sublevels of $^2S_{1/2}$ and $^2P_{3/2}$. The table contains the excitation amplitudes squared (with intensity coefficients) of the different transitions when the excitation is carried out with unpolarized electrons and atoms.

(the degree of polarization of the incoming electrons) and $j_u' = j_u/j = 1 - P_e$.

We study the collision process by either observing the scattered electrons (we measure the scattered intensity and the polarization P_e' after the collision) or the scattered atoms (intensity and polarization P_A'), or by detecting both of the collisional particles. We denote the differential cross section determined from the observation of the scattered electrons by $\sigma_e(\theta, \phi) = \sigma_e$, and the differential cross section obtained from the scattered atoms is denoted by $\sigma_A(\psi, \chi) = \sigma_A$. For a given single-collision process both these cross sections are certainly identical. The general relation between these two different collision processes expressed in terms of σ_e and σ_A follows from a momentum-transfer analysis.[4]

Table I shows the different types of scattering processes to be discussed in connection with the scattering amplitudes in Secs. III A–III C.

A. Scattering of Unpolarized Electrons on Polarized Atoms

In order to describe this collision process we introduce partial differential cross sections which are specified by the spin direction; thus $\sigma_e^\uparrow, \sigma_A^\uparrow$ or $\sigma_e^\downarrow, \sigma_A^\downarrow$ are defined as differential cross sections for the elastic collision process in which the spin of either the scattered electron or atom is up (↑) or down (↓), respectively, after the scattering. Table I includes such cross sections for the case in which the atomic target is completely polarized. Obviously, it appears that the measurement of the cross sections $\sigma_e^\downarrow$ and $\sigma_A^\downarrow$ supplement each other in the way that they give the direct and the exchange cross sections, $\sigma_e^\downarrow = \frac{1}{2}|f|^2 = \sigma_d$ and $\sigma_A^\downarrow = \frac{1}{2}|g|^2 = \sigma_{ex}$. $\sigma_A^\downarrow$ can be measured by selecting the atoms with spin down by a Stern-Gerlach magnet. $\sigma_e^\downarrow$ can be obtained by means of a Mott analysis of the scattered electrons and by the measurement of the differential cross section as noted in the following discussion on the scattering of unpolarized electrons on partially polarized atoms.

Given now the degree of polarization P_A of the target atom, the cross sections specified by the direction of the electron spin or the spin of the atom are

$$\sigma_e^\uparrow = P_A(\tfrac{1}{2}|f-g|^2 + \tfrac{1}{2}|g|^2) + \tfrac{1}{2}(1-P_A)\sigma\ ,$$

$$\sigma_e^\downarrow = P_A\tfrac{1}{2}|f|^2 + \tfrac{1}{2}(1-P_A)\sigma\ ,$$

$$\sigma_A^\uparrow = P_A(\tfrac{1}{2}|f-g|^2 + \tfrac{1}{2}|f|^2) + \tfrac{1}{2}(1-P_A)\sigma\ ,$$

$$\sigma_A^\downarrow = P_A\tfrac{1}{2}|g|^2 + \tfrac{1}{2}(1-P_A)\sigma\ .$$

It follows that

$$\sigma_e^\uparrow + \sigma_e^\downarrow = \sigma, \quad \sigma_A^\uparrow + \sigma_A^\downarrow = \sigma\ ,$$

$$M_e = \sigma_e^\uparrow - \sigma_e^\downarrow = P_A(\sigma - |f|^2), \tag{19}$$

$$M_A = \sigma_A^\uparrow - \sigma_A^\downarrow = P_A(\sigma - |g|^2)\ ,$$

and with $P_e' = M_e/\sigma$ and $P_A' = M_A/\sigma$ we have

TABLE I. Partial differential cross sections specified by the spin directions of the scattered electrons ($\sigma_e^\uparrow$ or $\sigma_e^\downarrow$) and the atoms ($\sigma_A^\uparrow$ or $\sigma_A^\downarrow$) for the elastic scattering processes.

Unpolarized electrons scattered on completely polarized atoms[a]	Completely polarized electrons scattered on unpolarized atoms[a]	Completely polarized electrons scattered on completely polarized atoms[a]
$e(\uparrow\downarrow)+A(\uparrow)$	$e(\uparrow)+A(\uparrow\downarrow)$	$e(\uparrow)+A(\uparrow)$
$\sigma_e^\uparrow = \frac{1}{2}\lvert f-g\rvert^2 + \frac{1}{2}\lvert g\rvert^2$, $\sigma_e^\downarrow = \frac{1}{2}\lvert f\rvert^2$	$\sigma_e^\uparrow = \frac{1}{2}\lvert f-g\rvert^2 + \frac{1}{2}\lvert f\rvert^2$, $\sigma_e^\downarrow = \frac{1}{2}\lvert g\rvert^2$	$\sigma_e^\uparrow = \lvert f-g\rvert^2$, $\sigma_e^\downarrow = 0$
$\sigma_A^\uparrow = \frac{1}{2}\lvert f-g\rvert^2 + \frac{1}{2}\lvert f\rvert^2$, $\sigma_A^\downarrow = \frac{1}{2}\lvert g\rvert^2$	$\sigma_A^\uparrow = \frac{1}{2}\lvert f-g\rvert^2 + \frac{1}{2}\lvert g\rvert^2$, $\sigma_A^\downarrow = \frac{1}{2}\lvert f\rvert^2$	$\sigma_A^\uparrow = \lvert f-g\rvert^2$, $\sigma_A^\downarrow = 0$
		$e(\uparrow)+A(\downarrow)$
		$\sigma_e^\uparrow = \lvert f\rvert^2$, $\sigma_e^\downarrow = \lvert g\rvert^2$
		$\sigma_A^\uparrow = \lvert g\rvert^2$, $\sigma_A^\downarrow = \lvert f\rvert^2$

[a] One–electron atoms.

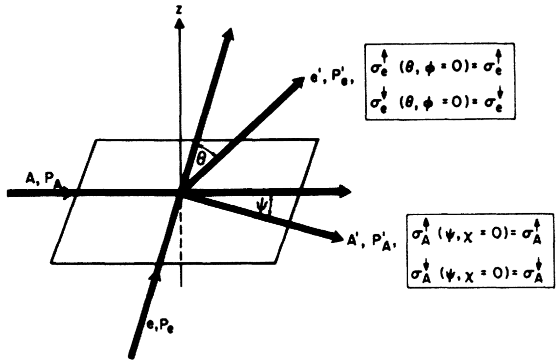


FIG. 3. Scheme of the crossed-beam experiment to be analyzed with partially polarized incoming electrons e (degree of polarization P_e) and partially polarized target atoms A (degree of polarization P_A). The quantization axis is parallel to the Z axis. P'_e and P'_A represent the polarization of scattered electrons or atoms, respectively. $\sigma_e^{\uparrow}$ and $\sigma_e^{\downarrow}$ or $\sigma_A^{\uparrow}$ and $\sigma_A^{\downarrow}$ denote the partial differential cross sections for electrons or atoms with final spin up or down, respectively.

$$|f|^2=\sigma(1-P'_e/P_A), \quad |g|^2=\sigma(1-P'_A/P_A) . \tag{20}$$

With the knowledge of σ and P_A, the measurement of the spin polarization of the scattered electrons and atoms (P'_e and P'_A, respectively) gives the direct and the exchange cross sections.

B. Scattering of Polarized Electrons on Unpolarized Atoms

The basic scattering processes for this case are described in Table I. The situation is similar to that in Sec. III A, with the quantities $|g|$ and $|f|$ interchanged in the expressions for $\sigma_e^{\downarrow}$ and $\sigma_A^{\downarrow}$. Again of practical interest is the case of the scattering of partially polarized electrons on unpolarized atoms.

The cross sections specified by the spin directions of the scattered electrons and atoms are now given as follows (P_e is the degree of polarization of the incoming electrons):

$$\sigma_e^{\uparrow}=P_e(\tfrac12|f-g|^2+\tfrac12|f|^2)+\tfrac12(1-P_e)\sigma ,$$

$$\sigma_e^{\downarrow}=P_e\tfrac12|g|^2+\tfrac12(1-P_e)\sigma ,$$

$$\sigma_A^{\uparrow}=P_e(\tfrac12|f-g|^2+\tfrac12|g|^2)+\tfrac12(1-P_e)\sigma ,$$

$$\sigma_A^{\downarrow}=P_e\tfrac12|f|^2+\tfrac12(1-P_e)\sigma .$$

It follows that $\sigma=\sigma_e^{\uparrow}+\sigma_e^{\downarrow}$, $\sigma=\sigma_A^{\uparrow}+\sigma_A^{\downarrow}$, and

$$M_e=\sigma_e^{\uparrow}-\sigma_e^{\downarrow}=P_e(\sigma-|g|^2),$$
$$M_A=\sigma_A^{\uparrow}-\sigma_A^{\downarrow}=P_e(\sigma-|f|^2) , \tag{21}$$

and with $M_e/\sigma=P'_e$ and $M_A/\sigma=P'_A$ we have

$$|g|^2=\sigma(1-P'_e/P_e), \quad |f|^2=\sigma(1-P'_A/P_e). \tag{22}$$

The measurement of the spin polarization of the scattered electrons and atoms determines, with the knowledge of σ and P_e, the direct and the exchange cross section.

C. Scattering of Polarized Electrons on Polarized Atoms

It follows from the discussion of the scattering processes with completely polarized electrons and atoms (Sec. II A) that measurement of the intensity of either the scattered electrons or atoms gives the cross sections σ_d, σ_{ex}, and σ_{int}. These quantities can also be determined from the scattering of partially polarized electrons on partially polarized atoms.

We denote the degree of polarization of the incoming electrons and the target atoms, as in Sec. I, by $P_e=j'_P$, $P_A=n'_P$. The partial cross sections specified by the spin directions of the scattered electrons and atoms are now given as follows:

$$\sigma_e^{\uparrow}=[j'_Pn'_P+\tfrac12(j'_Pn'_u+j'_un'_P)]\,|f-g|^2 + \tfrac12 j'_Pn'_u|g|^2+\tfrac12 j'_un'_u\cdot\sigma ,$$

$$\sigma_e^{\downarrow}=\tfrac12 j'_Pn'_u|g|^2+\tfrac12 j'_un'_P|f|^2+\tfrac12 j'_un'_u\cdot\sigma ,$$

$$\sigma_A^{\uparrow}=[j'_Pn'_P+\tfrac12(j'_Pn'_u+j'_un'_P)]\,|f-g|^2+\tfrac12 j'_Pn'_u|g|^2 + \tfrac12 j'_un'_P|f|^2+\tfrac12 j'_un'_u\sigma ,$$

$$\sigma_A^{\downarrow}=\tfrac12 j'_Pn'_u|f|^2+\tfrac12 j'_un'_P|g|^2+\tfrac12 j'_un'_u\cdot\sigma .$$

From these equations we obtain the up-down asymmetry relations

$$M_e=\sigma_e^{\uparrow}-\sigma_e^{\downarrow}=P_e(\sigma-|g|^2)+P_A(\sigma-|f|^2) , \tag{23}$$

$$M_A=\sigma_A^{\uparrow}-\sigma_A^{\downarrow}=P_A(\sigma-|g|^2)+P_e(\sigma-|f|^2) . \tag{24}$$

The sums of the partial cross sections are

$$\begin{aligned} S_e=\sigma_e^{\uparrow}+\sigma_e^{\downarrow}&=\sigma-P_eP_A(\sigma-|f-g|^2)\\ &=\sigma-P_eP_A(|f|^2+|g|^2-\sigma)\\ &=\sigma-P_eP_A\,\mathrm{Re}(f^*g) . \end{aligned} \tag{25}$$

One can verify that

$$S_e\equiv S_A=\sigma_A^{\uparrow}+\sigma_A^{\downarrow}=S ,$$

which is valid for a given collision process observed via the scattered electrons or atoms.

The sum of the partial cross sections does not coincide with the total differential cross section for the case of the scattering of polarized electrons on polarized atoms (contrary to the two previous cases).

It should also be noted that as a result, the total intensity S of either the scattered electrons or atoms determines the magnitude of the interference amplitude (with knowledge of σ, P_e, and P_A),

$$|f-g|^2=\sigma+(1/P_eP_A)(S-\sigma) . \tag{26}$$

The quantities $|f|$ and $|g|$ cannot be obtained directly from the up-down asymmetries [Eqs. (23) and (24)] alone, as is possible with the scattering of polarized electrons on unpolarized atoms or with the scattering of unpolarized electrons on polarized atoms. $|f|$ and $|g|$ can, however, be determined if one applies either of the following combinations of measurements: For a given single-collision process (assuming that the relation between the electron and atom scattering is known) one combines the measurement of M_e with that of M_A and applies Eqs. (24) and (25). It then follows that

$$|g|^2=\sigma-\frac{M_AP_A-M_eP_e}{P_A^2-P_e^2}\,,\quad \frac{|g|^2}{S}=\frac{\sigma}{S}-\frac{P_A'P_A-P_e'P_e}{P_A^2-P_e^2}\,, \tag{27}$$

$$|f|^2=\sigma-\frac{M_AP_e-M_eP_A}{P_e^2-P_A^2}\,,\quad \frac{|g|^2}{S}=\frac{\sigma}{S}-\frac{P_A'P_e-P_e'P_A}{P_e^2-P_A^2}\,. \tag{28}$$

The amplitudes $|f|$ and $|g|$ can also be obtained from the combination of the measurement M_e and S_e or M_A and S_A. It then follows from Eqs. (23) and (25) that

$$|f|^2=\frac{1}{P_A-P_e}\left[\frac{S}{P_A}-M_e-\sigma\left(\frac{1}{P_A}-P_A\right)\right]\,,$$
$$\frac{|f|^2}{S}=\frac{1}{P_A-P_e}\left[\frac{1}{P_A}-P_e'-\frac{\sigma}{S}\left(\frac{1}{P_A}-P_A\right)\right]\,, \tag{29}$$

$$|g|^2=\frac{1}{P_e-P_A}\left[\frac{S}{P_e}-M_e-\sigma\left(\frac{1}{P_e}-P_e\right)\right]\,,$$
$$\frac{|g|^2}{S}=\frac{1}{P_e-P_A}\left[\frac{1}{P_e}-P_e'-\frac{\sigma}{S}\left(\frac{1}{P_e}-P_e\right)\right]\,. \tag{30}$$

Combining Eqs. (24) and (25), one obtains

$$|f|^2=\frac{1}{P_e-P_A}\left[\frac{S}{P_e}-M_A-\sigma\left(\frac{1}{P_e}-P_e\right)\right]\,,$$
$$\frac{|f|^2}{S}=\frac{1}{P_e-P_A}\left[\frac{1}{P_e}-P_A'-\frac{\sigma}{S}\left(\frac{1}{P_e}-P_e\right)\right]\,, \tag{31}$$

$$|g|^2=\frac{1}{P_A-P_e}\left[\frac{S}{P_A}-M_A-\sigma\left(\frac{1}{P_A}-P_A\right)\right]\,,$$
$$\frac{|g|^2}{S}=\frac{1}{P_A-P_e}\left[\frac{1}{P_A}-P_A'-\sigma\left(\frac{1}{P_A}-P_A\right)\right]\,. \tag{32}$$

With the knowledge of $|f|$ and $|g|$ the phase difference $\delta=\psi-\gamma$ between f and g ($f=|f|e^{i\psi}$ and $g=|g|e^{i\gamma}$) can be determined from Eq. (25):

$$\cos\delta=(\sigma-S)/P_eP_A|f|\,|g|\,. \tag{33}$$

Although it is possible to determine all three cross sections σ_d, σ_{ex}, and σ_{int} and the phase difference between f and g by means of scattering of partially polarized electrons by partially polarized atoms, the analysis is only relatively simple for the measurement of σ_{int}. For the measurement of σ_d and σ_{ex}, the cases discussed in Secs. II A (unpolarized electrons on polarized atoms) and II B (polarized electrons on unpolarized atoms) appear to be simpler in the analysis and also more suitable and more promising from the point of view of experimental techniques.

IV. EXCITATION PROCESS $^2S_{1/2}\to{}^2P_{1/2,3/2}$ WITH POLARIZED ELECTRONS AND POLARIZED ATOMS

We will now analyze the excitation process $^2S_{1/2}\to{}^2P_{1/2,3/2}$ in terms of excitation amplitudes when all kinds of collision processes occur, with polarized or unpolarized electrons and also with polarized or unpolarized ground-state atoms. We also assume that the excited atoms return to the ground state under the emission of resonance light.

In principle we could follow the same scheme of analysis as discussed for the elastic scattering. There is, however, one important difference: The excited ^2P states have a short lifetime which prevents direct observation of the scattered atoms in the excited states unless one carries out experiments with very fast atoms. In analyzing the spin of the ground-state atoms that have returned from the excited state to the ground state by an optical transition, all the complications resulting from the fine- and hyperfine-structure splittings, from the finite width of the excited states, and from the rules governing the transitions have to be taken into account. The electron scattering, however, is unaffected by these complications because the collision process takes place in times too short to have set up magnetic and spin-orbit interactions in the atom or between the atom and the colliding electron. Accordingly, the observation of electron scattering gives results in terms of scattering amplitudes which appear simpler than those obtained from an analysis carried out with the scattering of atoms (e.g., the partial cross sections σ_d, σ_{ex}, and σ_{int} follow directly from the electron scattering analysis, but not from the atom scattering analysis).

Also of interest are the total intensity and the analysis of circularly and linearly polarized components of the $P\to S$ resonance-line radiation which provide some details about the integral excitation amplitudes. Accordingly, our analysis for excitation amplitudes is restricted to electron scattering and the observation of the intensity and polarization of the resonance radiation excited by electron bombardment. We discuss the analysis of the electron scattering without distinguishing between the fine-structure states, and we deal also with the analysis of a few cases in which either the $^2P_{1/2}$ or the $^2P_{3/2}$ state is excited.

A. Unpolarized Electrons Exciting Polarized Ground-State Atoms to $^2P_{1/2,3/2}$ States

Given the degree P_A of the polarization of the target atoms, the partial differential excitation cross sections of $\sigma(^2S_{1/2}\to{}^2P_{1/2,3/2})$ specified by the spin direction of the inelastically scattered electrons can be expressed in terms of the excitation amplitudes (see Figs. 1 and 2 and Table II which display the amplitudes for the excitational transitions between the magnetic sublevels for the case discussed in this section):

$$\sigma_e^{\uparrow}(^2S_{1/2}\to{}^2P_{1/2,3/2}) = P_A\left(\tfrac{1}{2}|f_0-g_0|^2+|f_1-g_1|^2+\tfrac{1}{2}|g_0|^2+|g_1|^2\right) + \tfrac{1}{2}(1-P_A)\,\sigma(^2S_{1/2}\to{}^2P_{1/2,3/2})\,,$$

$$\sigma_e^{\downarrow}(^2S_{1/2}\to{}^2P_{1/2,3/2}) = P_A\left(\tfrac{1}{2}|f_0|+|f_1|^2\right)+\tfrac{1}{2}(1-P_A)\,\sigma(^2S_{1/2}\to{}^2P_{1/2,3/2})\,.$$

The up-down asymmetry and the sum of these cross section are then

$$\begin{aligned}
&\sigma_e^{\uparrow}(^2S_{1/2}\to{}^2P_{1/2,3/2})-\sigma_e^{\downarrow}(^2S_{1/2}\to{}^2P_{1/2,3/2})=M_e\,,\\
&M_e=P_A[\sigma(^2S_{1/2}\to{}^2P_{1/2,3/2})-2\left(\tfrac{1}{2}|f_0|^2+|f_1|^2\right)]\,,\\
&S_e=\sigma_e^{\uparrow}(^2S_{1/2}\to{}^2P_{1/2,3/2})+\sigma_e^{\downarrow}(^2S_{1/2}\to{}^2P_{1/2,3/2})\\
&\quad=\sigma(^2S_{1/2}\to{}^2P_{1/2,3/2})\,.
\end{aligned}\tag{34}$$

With these last equations and with

$$M_e/\sigma(^2S_{1/2}\to{}^2P_{1/2,3/2})=P_e',$$

we obtain for the direct excitation cross section

TABLE II. Various characteristics of the excitation process $^2S_{1/2}-{}^2P_{1/2,3/2}$ by excitation with unpolarized electrons and completely polarized ground-state atoms: $e(\uparrow\downarrow)+A\ (^2S_{1/2},\uparrow)$. [a]

Transition[b]	Excitation amplitude	Spin direction of scattered electrons	Circular polarization of $\Delta m_j=\pm 1$ deexcitation transition
	$^2S_{1/2}\to{}^2P_{1/2}$ transitions		
I	$\frac{1}{6}\lvert f_0\rvert^2+\frac{1}{6}\lvert f_0-g_0\rvert^2$	$\downarrow$ or $\uparrow$	$I^{\sigma+}$
II	$\frac{1}{6}\lvert g_0\rvert^2$	$\uparrow$	$I^{\sigma-}$
III	$\frac{1}{3}\lvert g_1\rvert^2$	$\uparrow$	$I^{\sigma+}$
IV	$\frac{1}{3}\lvert f_1\rvert^2+\frac{1}{3}\lvert f_1-g\rvert^2$	$\downarrow$ or $\uparrow$	$I^{\sigma-}$
	$^2S_{1/2}\to{}^2P_{3/2}$ transitions		
I	$\frac{1}{3}\lvert f_0\rvert^2+\frac{1}{3}\lvert f_0-g\rvert^2$	$\downarrow$ or $\uparrow$	$I^{\sigma+}$
II′	$\frac{1}{3}\lvert g_0\rvert^2$	$\uparrow$	$I^{\sigma-}$
III′	$\frac{1}{6}\lvert g_1\rvert^2$	$\uparrow$	$I^{\sigma+}$
IV′	$\frac{1}{6}\lvert f_1\rvert^2+\frac{1}{6}\lvert f_1-g_1\rvert^2$	$\downarrow$ or $\uparrow$	$I^{\sigma-}$
V′	$\frac{1}{2}\lvert f_1\rvert^2+\frac{1}{2}\lvert f_1-g_1\rvert^2$	$\downarrow$ or $\uparrow$	$I^{\sigma+}$
VI′	$\frac{1}{2}\lvert g_1\rvert^2$	$\uparrow$	$I^{\sigma-}$

[a] One-electron atoms.

[b] Labels of the transitions as in Figs. 1 and 2.

$$\sigma_d(^2S_{1/2}\to{}^2P_{1/2,3/2})=\tfrac{1}{2}|f_0|^2+|f_1|^2 =\tfrac{1}{2}\sigma(^2S_{1/2}\to{}^2P_{1/2,3/2})\,(1-P_e'/P_A)\,. \tag{35}$$

In exciting the fine-structure states separately we obtain the following expressions which relate $\sigma_d(^2S_{1/2}\to{}^2P_{1/2,3/2})$ to the cross sections of the fine-structure states:

$$\sigma_d(^2S_{1/2}\to{}^2P_{1/2,3/2})=\tfrac{3}{4}\sigma(^2S_{1/2}\to{}^2P_{3/2})\,(1-P_e'/P_A)\,, \tag{36}$$

$$\sigma_d(^2S_{1/2}\to{}^2P_{1/2,3/2})=\tfrac{3}{2}\sigma(^2S_{1/2}\to{}^2P_{1/2})\,(1-P_e'/P_A)\,. \tag{37}$$

We now discuss how the observation of circularly polarized light emitted by transitions from the $^2P_{1/2}$ or $^2P_{3/2}$ state can be used to obtain detailed information on the excitation process. Figures 1 and 2 can also be used to illustrate the excitation and line-emission processes involved; obviously the integral excitation amplitudes have to be applied for the analysis of the polarized light components.

We first discuss the excitation of the $^2P_{1/2}$ state in detail. It follows from Sommerfeld's intensity rules[9] that the total intensity of the circularly polarized component is twice as large as the linearly polarized component of the $^2P_{1/2}\to{}^2S_{1/2}$ transition. Accordingly, the total number of transitions with linearly (I^{π}) and circularly polarized light emission ($I^{\sigma+}$ and $I^{\sigma-}$) can be expressed in terms of the integral excitation amplitudes (see Table II, $^2P_{1/2}\to{}^2S_{1/2}$ transitions):

$$\begin{aligned}
I^{\pi}(^2P_{1/2}\to{}^2S_{1/2})&=\tfrac{1}{18}|F_0|^2+\tfrac{1}{9}|F_1|^2+\tfrac{1}{18}|G_0|^2 +\tfrac{1}{9}|G_1|^2+\tfrac{1}{18}|F_0-G_0|^2+\tfrac{1}{9}|F_1-G_1|^2\,,\\
I^{\sigma+}(^2P_{1/2}\to{}^2S_{1/2})&=\tfrac{1}{9}|F_1|^2+\tfrac{2}{9}|G_1|^2+\tfrac{1}{9}|F_0-G_0|^2\,,\\
I^{\sigma-}(^2P_{1/2}\to{}^2S_{1/2})&=\tfrac{2}{9}|F_1|^2+\tfrac{1}{9}|G_0|^2+\tfrac{2}{9}|F_1-G_1|^2\,.
\end{aligned}$$

With

$$I^{\pi}(^2P_{1/2}\to{}^2S_{1/2})=\tfrac{1}{3}Q(^2S_{1/2}\to{}^2P_{1/2})$$

and

$$I^{\sigma+}(^2P_{1/2}\to{}^2S_{1/2})+I^{\sigma-}(^2P_{1/2}\to{}^2S_{1/2})=\tfrac{2}{3}Q(^2S_{1/2}\to{}^2P_{1/2})$$

we can relate $I^{\sigma+}$ and $I^{\sigma-}$ to the following partial cross sections:

$$I^{\sigma+}(^2P_{1/2}\to{}^2S_{1/2})=\tfrac{2}{3}Q(^2S_{1/2}\to{}^2P_{1/2},\ m_j\to m_j) =\tfrac{2}{3}\left(\tfrac{1}{3}|G_1|^2+\tfrac{1}{6}|F_0|^2+\tfrac{1}{6}|F_0-G_0|^2\right)\,, \tag{38}$$

$$I^{\sigma-}(^2P_{1/2}\to{}^2S_{1/2})=\tfrac{2}{3}Q(^2S_{1/2}\to{}^2P_{1/2},\ m_j\to m_j) =\tfrac{2}{3}\left(\tfrac{1}{6}|G_0|^2+\tfrac{1}{3}|F_1|^2+\tfrac{1}{3}|F_1-G_1|^2\right) \tag{39}$$

These two cross sections

$$[Q(^2S_{1/2} \to {}^2P_{1/2}, m_j \to \pm m_j)]$$

describe the excitation process by which either the magnetic quantum number in the $^2P_{1/2}$ state is equal to that in the ground state [Eq. (38)] or is changed to the opposite sign [Eq. (39)] of m_j.

Assuming that the degree of the polarization of the atoms is P_A, the number of transitions with circular polarization is then

$$I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) = P_A(\tfrac{2}{9}|G_1|^2 + \tfrac{1}{9}|F_0| + \tfrac{1}{12}|F_0 - G_0|^2) + \tfrac{1}{3}(1-P_A)\,Q(^2S_{1/2} \to {}^2P_{1/2}),$$

$$I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2}) = P_A(\tfrac{1}{9}|G_0|^2 + \tfrac{2}{9}|F_1|^2 + \tfrac{2}{9}|F_1 - G_1|^2) + \tfrac{1}{3}(1-P_A)\,Q(^2S_{1/2} \to {}^2P_{1/2}).$$

Applying Eqs. (38) and (39) it follows that

$$I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) = P_A[\tfrac{2}{3}Q(^2S_{1/2} \to {}^2P_{1/2}) - \tfrac{2}{3}Q(^2S_{1/2} \to {}^2P_{1/2},\ m_j \to -m_j)] + \tfrac{1}{3}(1-P_A)Q(^2S_{1/2} \to {}^2P_{1/2}),$$

$$I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2}) = P_A[\tfrac{2}{3}Q(^2S_{1/2} \to {}^2P_{1/2},\ m_j \to -m_j)] + \tfrac{1}{3}(1-P_A)Q(^2S_{1/2} \to {}^2P_{1/2}),$$

$$I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) - I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2}) = P_A[\tfrac{2}{3}Q(^2S_{1/2} \to {}^2P_{1/2}) - \tfrac{4}{3}Q(^2S_{1/2} \to {}^2P_{1/2},\ m_j \to -mj)].$$

With the degree of circular polarization $P^{\sigma\pm} = (I^{\sigma+} - I^{\sigma-})/(I^{\sigma+} + I^{\sigma-})$, it follows that

$$Q(^2S_{1/2} \to {}^2P_{1/2},\ m_j \to -m_j) = \tfrac{1}{2}Q(^2S_{1/2} \to {}^2P_{1/2})(1 - P^{\sigma\pm}/P_A) = \tfrac{1}{3}|F_1|^2 + \tfrac{1}{6}|G_0|^2 + \tfrac{1}{3}|F_1 - G_1|^2, \quad (40)$$

$$Q(^2S_{1/2} \to {}^2P_{1/2},\ m_j \to m_j) = Q(^2S_{1/2} \to {}^2P_{1/2}) - Q(^2S_{1/2} \to {}^2P_{1/2},\ m_j \to -m_j) = \tfrac{1}{6}|F_0| + \tfrac{1}{3}|G_1|^2 + \tfrac{1}{6}|F_0 - G_0|^2. \quad (41)$$

The analysis of the circular polarization from the excitation process $^2S_{1/2} \to {}^2P_{3/2}$ is similar to that for the excitation $^2S_{1/2} \to {}^2P_{1/2}$. One has, however, to take into account that the linearly polarized component of the $^2P_{3/2} \to {}^2S_{1/2}$ transition from the magnetic substates $m_j = \pm\frac{1}{2}$ is twice as large as the circularly polarized component (just the reverse of the previous case). The measurement of the circular polarization $P^{\sigma\pm}$ of the $^2P_{3/2} \to {}^2S_{1/2}$ transition provides the following terms for the amplitudes:

$$\tfrac{1}{6}|F_1|^2 + \tfrac{1}{3}|G_0|^2 + \tfrac{1}{2}|G_1|^2 + \tfrac{1}{6}|F_1 - G_1|^2 = \tfrac{1}{2}Q(^2S_{1/2} \to {}^2P_{3/2})(1 - P^{\sigma\pm}/P_A), \quad (42)$$

$$\tfrac{1}{3}|F_0|^2 + \tfrac{1}{2}|F_1|^2 + \tfrac{1}{6}|G_1|^2 + \tfrac{1}{3}|F_0 - G_0|^2 + \tfrac{1}{2}|F_1 - G_1|^2 = \tfrac{1}{2}Q(^2S_{1/2} \to {}^2P_{3/2})(1 + P^{\sigma\pm}/P_A). \quad (43)$$

Also straightforward is the analysis of the circular polarization of the $P \to S$ transition when both the line components ($^2P_{1/2} \to {}^2S_{1/2}$ and $^2P_{3/2} \to {}^2S_{1/2}$) are observed simultaneously:

$$(1 - P^{\sigma\pm}/P_A)Q(^2S_{1/2} \to {}^2P_{1/2,3/2}) = \tfrac{5}{4}|F_1|^2 + |G_0|^2 + \tfrac{3}{4}|G_1|^2 + \tfrac{5}{4}|F_1 - G_1|^2, \quad (44)$$

$$Q(^2S_{1/2} \to {}^2P_{1/2,3/2})(1 + P^{\sigma\pm}/P_A) = |F_0|^2 + \tfrac{3}{4}|F_1|^2 + \tfrac{5}{4}|G_1|^2 + |F_0 - G_0|^2 + \tfrac{3}{4}|F_1 - G_1|^2, \quad (45)$$

where $P^{\sigma\pm}$ is the total circular polarization of both fine-structure transitions.

Of interest are the limiting cases of threshold excitation ($E_{\rm thr}$) and infinite energy (E_∞). It follows from orbital angular momentum conservation that for threshold excitation only transitions with $\Delta_{m_l} = 0$ are possible and for excitation with infinite energy only transitions with $\Delta_{m_l} = \pm 1$ are possible. Accordingly, at threshold energy all the amplitudes with $m_l = \pm 1$ should vanish and so should the amplitudes with $m_l = 0$ at infinite energy. That is, e.g.,

$$\frac{\frac{1}{2}|G_0|^2}{Q(^2S_{1/2} \to {}^2P_{1/2}, E_{\rm thr})} = \frac{3}{2}\left(1 - \frac{P^{\sigma\pm}}{P_A}\right), \quad (46)$$

$$\frac{\frac{1}{2}|F_1|^2 + \frac{1}{6}|G_1|^2 + \frac{1}{2}|F_1 - G_1|^2}{Q(^2S_{1/2} \to {}^2P_{3/2}, E_\infty)} = \frac{1}{2}\left(1 + \frac{P^{\sigma\pm}}{P_A}\right). \quad (47)$$

B. Polarized Electrons Exciting Unpolarized Ground-State Atoms to $^2P_{1/2,3/2}$ States

Given the degree P_e of polarization of the incoming electrons, the partial differential excitation cross sections specified by the spin direction of the inelastically scattered electrons are (Table III shows the amplitudes involved)

$$\sigma_e^{\uparrow}(^2S_{1/2} \to {}^2P_{1/2,3/2}) = P_e(\tfrac{1}{2}|f_0 - g_0|^2 + |f_1 - g_1|^2 + \tfrac{1}{2}|f_0|^2 + |f_1|^2) + \tfrac{1}{2}(1 - P_e)\sigma(^2S_{1/2} \to {}^2P_{1/2,3/2}),$$

$$\sigma_e^{\downarrow}(^2S_{1/2} \to {}^2P_{1/2,3/2}) = P_e(\tfrac{1}{2}|g_0|^2 + |g_1|^2) + \tfrac{1}{2}(1 - P_e)\sigma(^2S_{1/2} \to {}^2P_{1/2,3/2}).$$

It follows that

$$M_e = \sigma_e^{\uparrow}(^2S_{1/2} \to {}^2P_{1/2,3/2}) - \sigma_e^{\downarrow}(^2S_{1/2} \to {}^2P_{1/2,3/2}) = P_e[\sigma(^2S_{1/2} \to {}^2P_{1/2,3/2}) - 2(\tfrac{1}{2}|g_0|^2 + |g_1|^2)], \quad (48)$$

$$S_e = \sigma_e^{\uparrow}(^2S_{1/2} \to {}^2P_{1/2,3/2}) + \sigma_e^{\downarrow}(^2S_{1/2} \to {}^2P_{1/2,3/2}) = \sigma(^2S_{1/2} \to {}^2P_{1/2,3/2});$$

and from the last two equations and Eq. (16) with $P_e' = M_e/\sigma(^2S_{1/2} \to {}^2P_{1/2,3/2})$ we have

$$\sigma_{\rm ex}(^2S_{1/2} \to {}^2P_{1/2,3/2}) = \tfrac{1}{2}|g_0|^2 + |g_1|^2$$

TABLE III. Various characteristics of the excitation process ${}^2S_{1/2} \to {}^2P_{1/2,3/2}$ by excitation with completely polarized electrons and unpolarized atoms: $e(\uparrow) + A({}^2S_{1/2}, \uparrow\downarrow)$.[a]

Transition[b]	Excitation amplitude	Spin direction of scattered electrons	Circular polarization of $\Delta m_j = \pm 1$ deexcitation transition
	${}^2S_{1/2} \to {}^2P_{1/2}$ transitions		
1	$\frac{1}{6}\lvert f_0\rvert^2$	$\uparrow$	$I^{\sigma-}$
2	$\frac{1}{6}\lvert g_0\rvert^2$	$\downarrow$	$I^{\sigma+}$
3	$\frac{1}{3}\lvert g_1\rvert^2$	$\downarrow$	$I^{\sigma-}$
4	$\frac{1}{3}\lvert f_1\rvert^2$	$\uparrow$	$I^{\sigma+}$
I	$\frac{1}{6}\lvert f_0 - g_0\rvert^2$	$\uparrow$	$I^{\sigma+}$
II	0		
III	0		
IV	$\frac{1}{3}\lvert f_1 - g_1\rvert^2$	$\uparrow$	$I^{\sigma-}$
	${}^2S_{1/2} \to {}^2P_{3/2}$ transitions		
1′	$\frac{1}{3}\lvert f_0\rvert^2$	$\uparrow$	$I^{\sigma-}$
2′	$\frac{1}{3}\lvert g_0\rvert^2$	$\downarrow$	$I^{\sigma+}$
3′	$\frac{1}{6}\lvert g_1\rvert^2$	$\downarrow$	$I^{\sigma-}$
4′	$\frac{1}{6}\lvert f_1\rvert^2$	$\uparrow$	$I^{\sigma+}$
5′	$\frac{1}{2}\lvert f_1\rvert^2$	$\uparrow$	$I^{\sigma-}$
6′	$\frac{1}{2}\lvert g_1\rvert^2$	$\downarrow$	$I^{\sigma+}$
I′	$\frac{1}{3}\lvert f_0 - g_0\rvert^2$	$\uparrow$	$I^{\sigma+}$
II′	0		
III′	0		
IV′	$\frac{1}{6}\lvert f_1 - g\rvert^2$	$\uparrow$	$I^{\sigma-}$
V′	$\frac{1}{2}\lvert f_1 - g_1\rvert^2$	$\uparrow$	$I^{\sigma+}$
VI′	0		

[a] One-electron atoms.
[b] Labels of the transitions as in Figs. 1 and 2.

$$= \tfrac{1}{2}\sigma({}^2S_{1/2} \to {}^2P_{1/2,3/2})(1 - P_e'/P_e) \,. \quad (49)$$

In exciting the fine-structure states separately, an analysis similar to the above leads to the following relations:

$$\sigma_{ex}({}^2S_{1/2} \to {}^2P_{1/2,3/2}) = \tfrac{3}{2}\sigma({}^2S_{1/2} \to {}^2P_{1/2})(1 - P_e'/P_e) \,, \quad (50)$$

$$\sigma_{ex}({}^2S_{1/2} \to {}^2P_{1/2,3/2}) = \tfrac{3}{4}\sigma({}^2S_{1/2} \to {}^2P_{3/2})(1 - P_e'/P_e) \,. \quad (51)$$

Note that Eqs. (49)–(51) are similar to Eqs. (35)–(37).

As in Sec. IV A, one can also use the observation of the circularly polarized components of the resonance lines, excited by polarized electrons, in order to obtain some details about integral excitation amplitudes. Again the excitation of the ${}^2P_{1/2}$ state gives the simplest results in terms of excitation amplitudes. For the case of excitation of the ${}^2P_{1/2}$ state with completely polarized electron, the number of transitions associated with the emission of circularly polarized light is then (see Table III)

$$I^{\sigma+} = \tfrac{1}{9}|G_0|^2 + \tfrac{2}{9}|F_1|^2 + \tfrac{1}{9}|F_0 - G_0|^2 \,,$$

$$I^{\sigma-} = \tfrac{1}{9}|F_0|^2 + \tfrac{2}{9}|G_1|^2 + \tfrac{2}{9}|F_1 - G_1|^2 \,.$$

Taking a partially polarized beam of electrons (degree of polarization P_e) to excite the ${}^2P_{1/2}$ state, one gets the following expression for the integral amplitudes:

$$\tfrac{1}{6}|F_0|^2 + \tfrac{1}{3}|G_1|^2 + \tfrac{1}{3}|F_1 - G_1|^2 = \tfrac{1}{2}Q({}^2S_{1/2} \to {}^2P_{1/2})(1 - P^{\sigma t}/P_e) \,, \quad (52)$$

where $P^{\sigma t}$ is the circular polarization of the ${}^2P_{1/2} \to {}^2S_{1/2}$ transition.

With the knowledge of those amplitudes one also obtains

$$\tfrac{1}{3}|F_1|^2 + \tfrac{1}{6}|G_0|^2 + \tfrac{1}{6}|F_0 - G_0|^2 = \tfrac{1}{2}Q({}^2S_{1/2} \to {}^2P_{1/2})(1 + P^{\sigma t}/P_e) \,. \quad (53)$$

The analysis of the circular polarization of the ${}^2P_{3/2} \to {}^2S_{1/2}$ transition gives the following result:

$$\tfrac{1}{3}|F_0|^2 + \tfrac{1}{2}|F_1|^2 + \tfrac{1}{6}|G_1|^2 + \tfrac{1}{6}|F_1 - G_1|^2 = \tfrac{1}{2}Q({}^2S_{1/2} \to {}^2P_{3/2})(1 - P^{\sigma t}/P_e) \,, \quad (54)$$

$$\tfrac{1}{6}|F_1|^2 + \tfrac{1}{3}|G_0|^2 + \tfrac{1}{2}|G_1|^2 + \tfrac{1}{3}|F_0 - G_0|^2 + \tfrac{1}{2}|F_1 - G_1|^2 = \tfrac{1}{2}Q({}^2S_{1/2} \to {}^2P_{3/2})(1 + P^{\sigma t}/P_e) \,. \quad (55)$$

The resultant circular polarization of both fine-structure components ${}^2P_{3/2,1/2} \to {}^2S_{1/2}$ provides

$$|F_0|^2 + \tfrac{3}{4}\, F_1|^2 + \tfrac{5}{4}|G_1|^2 + \tfrac{5}{4}|F_1 - G_1|^2 = (1 - P^{\sigma t}/P_e)Q({}^2S_{1/2} \to {}^2P_{1/2,3/2}) \,, \quad (56)$$

$$\tfrac{5}{4}|F_1|^2 + |G_0|^2 + \tfrac{3}{4}|G_1|^2 + |F_0 - G_0|^2 + \tfrac{3}{4}|F_1 - G_1|^2 = (1 + P^{\sigma t}/P_e)Q({}^2S_{1/2} \to {}^2P_{1/2,3/2}) \,, \quad (57)$$

where $P^{\sigma t}$ is the circular polarization of the ${}^2P_{3/2,1/2} \to {}^2S_{1/2}$ transition.

The limiting cases of threshold excitation and excitation with infinite energy give

$$\frac{\frac{1}{2}|F_0|^2}{Q({}^2S_{1/2} \to {}^2P_{3/2}, E_{thr})} = \frac{3}{4}\left(1 - \frac{P^{\sigma t}}{P_e}\right) \,, \quad (58)$$

$$\frac{\frac{3}{4}|F_1|^2 + \frac{5}{4}|G_1|^2 + \frac{5}{4}|F_1 - G_1|^2}{Q({}^2S_{1/2} \to {}^2P_{1/2,3/2}, E_\infty)} = 1 - \frac{P^{\sigma t}}{P_e} \,. \quad (59)$$

It might be of some interest to follow up a sideline at this stage. Equations (52)–(59) show that, when the excitation amplitudes involved and the total cross section are known, the measurement of the polarization of the circularly polarized light emission can be used to determine the degree of polarization of the exciting electrons. This suggestion for the measurement of electrons-spin polarization was first made by Farago,[10] who proposed the use of Hg line excitation. It certainly might be interesting to compare the application of Farago's proposal by using either the alkali or the Hg lines. One important factor for giving preference to the alkali lines is the fact that at least the threshold behavior of the polarization of the Li and Na resonance radiation[11] is now in very good agreement with theory.[12] In addition, the polarization curves of these resonance lines do not show any dramatic behavior, caused by some kind of "structure," in a wide energy range above the threshold. The understanding, however, of the polarization of Hg lines, the threshold behavior, and possible structure is still not in a satisfactory state.[13]

C. Polarized Electrons Exciting Polarized Ground-State Atoms to ${}^2P_{1/2,\,3/2}$ States

We need not discuss the details of deriving the expressions for this case since everything can easily be obtained from Table IV and the previous sections. The observation of the intensity (S_e) of the inelastically scattered electrons already gives the important interference terms:

With

$$\sigma_e'({}^2S_{1/2}\to{}^2P_{1/2,3/2})+\sigma_i'({}^2S_{1/2}\to{}^2P_{1/2,3/2})=S_e$$

we obtain

$$\begin{aligned}\sigma_{\mathrm{int}}({}^2S_{1/2}\to{}^2P_{1/2,3/2}) &= \tfrac{1}{2}|f_0-g_0|^2+|f_1-g_1|^2\\ &=\tfrac{1}{2}\sigma({}^2S_{1/2}\to{}^2P_{1/2,3/2})\\ &\quad+\tfrac{1}{2}(1/P_AP_e)[S_e-\sigma({}^2S_{1/2}\to{}^2P_{1/2,3/2})]\ .\end{aligned} \tag{60}$$

By separating the scattering process into the two contributions with different orbital angular momentum components we derive

$$\begin{aligned}|f_0-g_0|^2 &= \sigma_{m_l=0}({}^2S_{1/2}\to{}^2P_{1/2,3/2})\\ &\quad+(1/P_eP_A)[S_0-\sigma_{m_l=0}({}^2S_{1/2}\to{}^2P_{1/2,3/2})]\ ,\\ |f_1-g_1|^2 &= \sigma_{m_l=\pm1}({}^2S_{1/2}\to{}^2P_{1/2,3/2})\\ &\quad+(1/P_eP_A)[S_1-\sigma_{m_l=\pm1}({}^2S_{1/2}\to{}^2P_{1/2,3/2})]\ ,\end{aligned}$$

with $S_0+2S_1=S_e$, where S_0 or $2S_1$ are the intensity contributions with the two possible orbital angular momentum components ($m_l=0$ or ±1).

The measurement of S_e, combined with that of the polarization P_e' of the scattered electrons, provides the direct and exchange terms

$$\begin{aligned}\tfrac{1}{2}|f_0|^2+|f_1|^2 &= \sigma_d({}^2S_{1/2}\to{}^2P_{1/2,3/2})\\ &=\frac{1}{2}\frac{S}{P_A-P_e}\left[\frac{1}{P_A}-P_e'-\frac{\sigma}{S}\left(\frac{1}{P_A}-P_A\right)\right]\ ,\end{aligned} \tag{61}$$

$$\begin{aligned}\tfrac{1}{2}|g_0|^2+|g_1|^2 &= \sigma_{\mathrm{ex}}({}^2S_{1/2}\to{}^2P_{1/2,3/2})\\ &=\frac{1}{2}\frac{S}{P_e-P_A}\left[\frac{1}{P_e}-P_e'-\frac{\sigma}{S}\left(\frac{1}{P_e}-P_e\right)\right]\ .\end{aligned} \tag{62}$$

The angular orbital momentum components for f and g separated from each other can be obtained from the following:

$$\begin{aligned}|f_0|^2 &= \frac{S_0}{P_A-P_e}\left[\frac{1}{P_A}-P_e'-\frac{\sigma}{S_0}\left(\frac{1}{P_A}-P_A\right)\right]\ ,\\ |f_1|^2 &= \frac{S_1}{P_A-P_e}\left[\frac{1}{P_A}-P_e'-\frac{\sigma}{S_1}\left(\frac{1}{P_A}-P_A\right)\right]\ ,\end{aligned} \tag{63}$$

TABLE IV. Various characteristics of the excitation process ${}^2S_{1/2}\to{}^2P_{1/2,3/2}$ by excitation with completely polarized electrons and atoms: $e(\uparrow)+A({}^2S_{1/2},\uparrow)$ or $e(\downarrow)+A({}^2S_{1/2},\uparrow)$.[a]

Transition[b]	Excitation amplitude	Spin direction of scattered electrons	Circular polarization of $\Delta m_j=\pm1$ deexcitation transition
	$e(\uparrow)+A({}^2S_{1/2},\uparrow)$		
I	$\frac{1}{3}\lvert f_0-g_0\rvert^2$	$\uparrow$	$I^{\sigma+}$
II	0		
III	0		
IV	$\frac{2}{3}\lvert f_1-g_1\rvert^2$	$\uparrow$	$I^{\sigma-}$
I′	$\frac{2}{3}\lvert f_0-g_0\rvert^2$	$\uparrow$	$I^{\sigma+}$
II′	0		
III′	0		
IV′	$\frac{1}{3}\lvert f_1-g_1\rvert^2$	$\uparrow$	$I^{\sigma-}$
V′	$\lvert f_1-g\rvert^2$	$\uparrow$	$I^{\sigma+}$
VI′	0		
	$e(\downarrow)+A({}^2S_{1/2},\uparrow)$		
I	$\frac{1}{3}\lvert f_0\rvert^2$	$\downarrow$	$I^{\sigma+}$
II	$\frac{1}{3}\lvert g_0\rvert^2$	$\uparrow$	$I^{\sigma-}$
III	$\frac{2}{3}\lvert g_1\rvert^2$	$\uparrow$	$I^{\sigma+}$
IV	$\frac{2}{3}\lvert f_1\rvert^2$	$\downarrow$	$I^{\sigma-}$
I′	$\frac{2}{3}\lvert f_0\rvert^2$	$\downarrow$	$I^{\sigma+}$
II′	$\frac{2}{3}\lvert g_0\rvert^2$	$\uparrow$	$I^{\sigma-}$
III′	$\frac{1}{3}\lvert g_1\rvert^2$	$\uparrow$	$I^{\sigma+}$
IV′	$\frac{1}{3}\lvert f_1\rvert^2$	$\downarrow$	$I^{\sigma-}$
V′	$\lvert f_1\rvert^2$	$\downarrow$	$I^{\sigma+}$
VI′	$\lvert g\rvert^2$	$\uparrow$	$I^{\sigma-}$

[a] One-electron atoms.

[b] Labels of the transitions as in Figs. 1 and 2.

$$|g_0|^2 = \frac{S_0}{P_e - P_A}\left[\frac{1}{P_e} - P_e' - \frac{\sigma}{S_0}\left(\frac{1}{P_e} - P_e\right)\right] ,$$
$$|g_1|^2 = \frac{S_1}{P_e - P_A}\left[\frac{1}{P_e} - P_e' - \frac{\sigma}{S_1}\left(\frac{1}{P_e} - P_e\right)\right] . \quad (64)$$

Finally, we discuss what information about the excitation process can be obtained by analyzing the components of the $^2P_{1/2,3/2} \to {}^2S_{1/2}$ transition. As will be seen the observation of the total intensity of the $P \to S$ emission directly provides the interference terms of the cross section. We will discuss in detail the case for the $^2S_{1/2} \to {}^2P_{1/2}$ excitation; the results for the $^2S_{1/2} \to {}^2P_{3/2}$ and $^2S_{1/2} \to {}^2P_{1/2,3/2}$ excitation can then easily be found.

The rate equations for the number of circularly and linearly polarized optical transitions from the $^2S_{1/2} \to {}^2P_{1/2}$ excitation are

$$I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) = \tfrac{2}{3}\{[j_P'n_P' + \tfrac{1}{2}(j_P'n_u' + j_u'n_P')]\tfrac{1}{3}|F_0 - G_0|^2 + \tfrac{1}{2}j_P'n_u'(\tfrac{2}{3}|F_1|^2 + \tfrac{1}{3}|G_0|^2)^2 + \tfrac{1}{2}j_u'n_P'(\tfrac{1}{3}|F_0|^2 + \tfrac{2}{3}|G_1|^2) + \tfrac{1}{2}n_u'j_u'Q(^2S_{1/2} \to {}^2P_{1/2})\} ,$$
$$I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2}) = \tfrac{2}{3}\{[j_P'n_P' + \tfrac{1}{2}(j_P'n_u' + j_u'n_P')]\tfrac{2}{3}|F_1 - G_1|^2 + \tfrac{1}{2}j_P'n_u'(\tfrac{1}{3}|F_0|^2 + \tfrac{2}{3}|G_1|^2) + \tfrac{1}{2}j_u'n_P'(\tfrac{2}{3}|F_1|^2 + \tfrac{1}{3}|G_0|^2) + \tfrac{1}{2}n_u'j_u'Q(^2S_{1/2} \to {}^2P_{1/2})\} .$$

We have $I^{\pi}(^2P_{1/2} \to {}^2S_{1/2}) = \tfrac{1}{2}(I^{\sigma+} + I^{\sigma-})$, and with the total number of transitions $I(^2P_{1/2} \to {}^2S_{1/2}) = I^{\sigma+} + I^{\sigma-} + I^{\pi}$ we obtain

$$I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) + I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2}) = \tfrac{2}{3}Q(^2S_{1/2} \to {}^2P_{1/2}) + \tfrac{2}{3}P_eP_A[Q(^2S_{1/2} \to {}^2P_{1/2}) - 2(\tfrac{1}{6}|F_0|^2 + \tfrac{1}{3}|F_1|^2 + \tfrac{1}{6}|G_0|^2 + \tfrac{1}{3}|G_1|^2)] , \quad (65)$$
$$I(^2P_{1/2} \to {}^2S_{1/2}) = Q(^2S_{1/2} \to {}^2P_{1/2}) + P_eP_A[\tfrac{1}{3}|F_0 - G_0|^2 + \tfrac{2}{3}|F_1 - G_1|^2 - Q(^2S_{1/2} \to {}^2P_{1/2})] ,$$
$$Q_{\text{int}}(S \to P) = \tfrac{1}{2}|F_0 - G_0|^2 + |F_1 - G_1|^2 = \tfrac{3}{2}(1/P_eP_A)[I(^2P_{1/2} \to {}^2S_{1/2}) + Q(^2S_{1/2} \to {}^2P_{1/2})(P_eP_A - 1)] . \quad (66)$$

The total intensity $I(^2P_{1/2} \to {}^2S_{1/2})$ of the emitted radiation determines the interference term of the cross section. It also follows from the above that the total intensity of the circularly or linearly polarized components provides the interference term

$$Q_{\text{int}}(S \to P) = (1/P_eP_A)\{\tfrac{9}{4}[I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) + I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2})] + \tfrac{3}{2}Q(^2S_{1/2} \to {}^2P_{1/2})(P_eP_A - 1)\}$$
$$= (1/P_eP_A)[\tfrac{9}{2}I^{\pi}(^2P_{1/2} \to {}^2S_{1/2}) + \tfrac{3}{2}Q(^2S_{1/2} \to {}^2P_{1/2})(P_eP_A - 1)] . \quad (67)$$

Obviously, the same kind of information can be obtained either from the fine-structure component $^2P_{3/2} \to {}^2S_{1/2}$ or from the total intensity of both doublet components

$$Q_{\text{int}}(S \to P) = \tfrac{3}{4} \cdot (1/P_eP_A)[I(^2P_{3/2} \to {}^2S_{1/2}) + Q(^2S_{1/2} \to {}^2P_{3/2})(P_eP_A - 1)]$$
$$= \tfrac{1}{2} \cdot (1/P_eP_A)[I(^2P_{3/2,1/2} \to {}^2S_{1/2}) + Q(^2S_{1/2} \to {}^2P_{1/2,3/2})(P_eP_A - 1)] . \quad (68)$$

Combining an analysis of the circular polarization with that for the total intensity of the circularly polarized components provides detailed information about the excitation cross sections only for special cases. The intensity difference of the two circularly polarized components for the $^2P_{1/2} \to {}^2S_{1/2}$ transition is given by

$$\tfrac{2}{3}[I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) - I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2})] = Q(^2S_{1/2} \to {}^2P_{1/2})(P_e + P_A) - \tfrac{2}{3}(P_e + P_A)(|F_1 - G_1|)^2 - \tfrac{2}{3}P_e(\tfrac{1}{2}|F_0|^2 + |G_1|^2) - \tfrac{2}{3}P_A(\tfrac{1}{2}|G_0|^2 + |F_1|^2) . \quad (69)$$

This equation combined with Eq. (65) does not provide excitation amplitudes. Setting, however, $P_e = P_A$ and combining Eqs. (65) and (69) lead to the following results:

$$Q_{\text{int}}(S \to P)_{m_l = \pm 1} = |F_1 - G_1|^2$$
$$= \frac{1}{P_e}\left[\frac{9}{8}\,\frac{I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) + I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2})}{P_e} - [I^{\sigma+}(^2P_{1/2} \to {}^2S_{1/2}) - I^{\sigma-}(^2P_{1/2} \to {}^2S_{1/2})]\right.$$

$$+\tfrac{3}{4}Q({}^2S_{1/2}\to{}^2P_{1/2})\left(P_e-\frac{1}{P_e}\right)\Big], \tag{70}$$

$$\tfrac{1}{2}|F_0|^2+|F_1|^2+\tfrac{1}{2}|G_0|^2+|G_1|^2=Q_d(S\to P)+Q_{ex}(S\to P)$$
$$=(1/P_e^2)\{\tfrac{3}{2}Q({}^2S_{1/2}\to{}^2P_{1/2})-\tfrac{9}{4}[I^{\sigma+}({}^2P_{1/2}\to{}^2S_{1/2})+I^{\sigma-}({}^2P_{1/2}\to{}^2S_{1/2})]\}\ . \tag{71}$$

Knowing $|F_1-G_1|^2$ from Eq. (70) enables one to determine $\frac{1}{2}|F_0-G_0|^2$ by means of Eq. (66) and also to determine the following quantities from Eqs. (65) and (69):

$$|F_1|^2+\tfrac{1}{2}|G_0|^2=\tfrac{3}{2}[1/P_A(P_A-P_e)]\{\tfrac{3}{2}[I^{\sigma+}({}^2P_{1/2}\to{}^2S_{1/2})+I^{\sigma-}({}^2P_{1/2}\to{}^2S_{1/2})]-\tfrac{3}{2}P_A[I^{\sigma+}({}^2P_{1/2}\to{}^2S_{1/2})-I^{\sigma-}({}^2P_{1/2}\to{}^2S_{1/2})]$$
$$+Q({}^2S_{1/2}\to{}^2P_{1/2})(P_A^2-1)-\tfrac{2}{3}(P_eP_A+P_A)|F_1-G_1|^2\}\ , \tag{72}$$

$$|G_1|^2+\tfrac{1}{2}|F_0|^2=\tfrac{3}{2}[1/P_e(P_e-P_A)]\{\tfrac{3}{2}[I^{\sigma+}({}^2P_{1/2}\to{}^2S_{1/2})+I^{\sigma-}({}^2P_{1/2}\to{}^2S_{1/2})]-\tfrac{3}{2}P_e[I^{\sigma+}({}^2P_{1/2}\to{}^2S_{1/2})-I^{\sigma-}({}^2P_{1/2}\to{}^2S_{1/2})]$$
$$+Q({}^2S_{1/2}\to{}^2P_{1/2})(P_e^2-1)-\tfrac{3}{2}(P_eP_A+P_e)|F_1-G_1|^2\}\ . \tag{73}$$

A similar analysis for the circular polarization of the ${}^2P_{3/2}\to{}^2S_{1/2}$ transition provides the following result:

$$\tfrac{2}{3}|F_0|^2+|F_1|^2+\tfrac{1}{3}|G_1|^2=\frac{1}{P_A-P_e}\Big[3[I^{\sigma+}({}^2P_{3/2}\to{}^2S_{1/2})-I^{\sigma-}({}^2P_{3/2}\to{}^2S_{1/2})]-\frac{3[I^{\sigma+}({}^2P_{3/2}\to{}^2S_{1/2})+I^{\sigma-}({}^2P_{3/2}\to{}^2S_{1/2})]}{P_e}$$
$$+\tfrac{1}{3}(P_e+P_A)|F_1-G_1|^2+Q({}^2S_{1/2}\to{}^2P_{3/2})\left(\frac{1}{P_e}-P_e\right)\Big], \tag{74}$$

$$\tfrac{1}{3}|F_1|^2+\tfrac{2}{3}|G_0|^2+|G_1|^2=\frac{1}{P_e-P_A}\Big[3[I^{\sigma+}({}^2P_{3/2}\to{}^2S_{1/2})-I^{\sigma-}({}^2P_{3/2}\to{}^2S_{1/2})]-\frac{3[I^{\sigma+}({}^2P_{3/2}\to{}^2S_{1/2})+I^{\sigma-}({}^2P_{3/2}\to{}^2S_{1/2})]}{P_A}$$
$$+Q({}^2S_{1/2}\to{}^2P_{3/2})\left(\frac{1}{P_A}-P_A\right)+\tfrac{1}{3}(P_e+P_A)|F_1-G_1|^2\Big]\ . \tag{75}$$

From observing the circularly polarized intensities of both fine-structure components ${}^2P_{1/2,3/2}\to{}^2S_{1/2}$ one obtains

$$\tfrac{4}{9}|G_0|^2+\tfrac{1}{3}|G_1|^2+\tfrac{5}{9}|F_1|^2=\frac{1}{P_e-P_A}\Big[I^{\sigma+}({}^2P_{1/2,3/2}\to{}^2S_{1/2})-I^{\sigma-}({}^2P_{1/2,3/2}\to{}^2S_{1/2})$$
$$-\frac{I^{\sigma+}({}^2P_{1/2,3/2}\to{}^2S_{1/2})+I^{\sigma-}({}^2P_{1/2,3/2}\to{}^2S_{1/2})}{P_A}+\frac{4}{9}\left(\frac{1}{P_A}\right)Q(S\to P)+\tfrac{5}{9}(P_e+P_A)|F_1-G_1|^2\Big]\ , \tag{76}$$

$$\tfrac{4}{9}|F_0|^2+\tfrac{1}{3}|F_1|^2+\tfrac{5}{9}|G_1|^2=\frac{1}{P_A-P_e}\Big[I^{\sigma+}({}^2P_{1/2,3/2}\to{}^2S_{1/2})-I^{\sigma-}({}^2P_{1/2,3/2}\to{}^2S_{1/2})$$
$$-\frac{I^{\sigma+}({}^2P_{3/2,1/2}\to{}^2S_{1/2})+I^{\sigma-}({}^2P_{3/2,1/2}\to{}^2S_{1/2})}{P_e}+\frac{4}{9}\left(\frac{1}{P_e}-P_e\right)Q(S\to P)+\tfrac{5}{9}(P_e+P_A)|F_1-G_1|^2\Big]\ . \tag{77}$$

The results of all Eqs. (72)–(77) depend upon knowing the interference term $|F_1-G_1|$. The special case $P_e=-P_A$ would cancel the interference term in the above equations. On the other hand, the interference term $|F_1-G_1|$ could also be determined from setting $P_e=P_A$ in Eqs. (72)–(77).

V. CONCLUSIONS

It follows from Sec. III that the analysis of the spin polarization for elastic scattering of polarized electrons on atoms provides the magnitude of the direct (f), exchange (g), and interference ($f-g$) amplitudes, and also the phase difference between f and g.

The analysis of the spin of polarized electrons which have excited polarized atoms from an S to P state gives the direct, exchange, and interference cross sections. However, these cross sections cannot be resolved in terms related to the magnetic quantum numbers $m_l=0$ or ± 1. In order to obtain the interference terms of the integral excitation amplitudes distinguished by the magnetic quantum number m_l, one can use the analysis of the total intensity and the circular polarization of the light emitted from the P state of atoms having the same degree of polarization as the exciting electrons. It also follows from Sec. IV that different sets of sums for integral excitation amplitudes squared can be obtained from all the different types of collision processes analyzed by observing the circularly polarized line components. This means, for example, that combining the results of Eqs. (40) and (42) enables one to extract the exchange amplitude $|G_1|$ for comparison with theory. Combining Eq. (71) with Eq. (75), Eq. (52) with Eq. (56), and Eq. (71) with Eq. (74) produces the direct excitation amplitudes $|F_0|$ and $|F_1|$, and also the exchange excitation amplitude $|G_0|$, which can be compared with theory. In other words, combining different types of experiments with polarized electrons and atoms should enable one to have all the excitation amplitudes separated. Table IV reveals how coincidence experiments for observing photons and electrons simultaneously can be used in order to distinguish the differential excitation amplitudes labeled by the magnetic quantum number $m_l = 0$ or ± 1.

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